

Towards a Statistical Adaptation of Order Filters for Processing Periodic and Frequency-Modulated Signals Using a Graphical Interface^{1,2}

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Abstract—Order filters possess a number of advantages. However such filters are related to nonlinear ones and therefore their analytical projection and the analysis of their behavior are rather complicated. This depends both on the kind of signal and on the character of noise in each concrete case. The given circumstance allows us to consider the project of an order filter as a casual event defining the quality of processing a signal. At the same time, the peculiarity of the standard order filter is that the response of the filter tends to zero as the filter length approaches signal period. In this paper a technique for the adaptation of order filters for the processing of periodic and frequency-modulated signals using a graphical interface is offered.

Keywords: order filters, statistical trials, graphical innerface.

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INTRODUCTION

Harmonic-analysis methods, as a rule, are used in processing periodic and frequency-modulated (FM) signals. In this paper, it is proposed to use weighted-order statistics (WOS) filters for improving the quality of the periodic and FM signals. In this case, a signal is considered as a one-dimensional time series $X = \{x_1, \dots, x_N\}$ recorded at discrete instants of time t_1, \dots, t_N , where $t_{i-1} - t_i = \Delta t = \text{const}$, $i = 2, \dots, N$.

The problem of processing noise signals using WOS filters is not new, as such filters have some merits over others: a—their remarkable ability to remove the impulse noise accompanied by a reduction of other disturbances, b—the stability of the noise character (robustness), and c—the preservation of steps for a signal in the form of a telegraphic sequence.

However, there is an important restriction to the wide appeal of WOS filters for processing periodic signals: as the length of a filter approaches an integer quantity of signal periods, the filter response approaches zero. A number of publications consider the problem of overcoming this restriction. Thus, the problem is solved in [1] by making use of hybrid (WOS and linear finite-impulse response—FIR) filters, while in [2] negative weights in the median filter

project are admitted. Finally, in [3] it is proposed to use zero weights, among other things. A similar technique (decimation) is widely used in image processing, in particular when median filters are employed as, for example, in [4]. The latter generalization allows us to define a co-phased (CoPh WOS) filter. As is shown in [3, 5], the CoPh WOS filters make possible to retain the waveform in the frequency range $\Delta f(\text{CoPh WOS}) \sim 1.0\text{--}1.5$ Hz. Furthermore, for processing of periodic and FM signals we will understand the processing by the co-phased and the standard WOS filters.

Another peculiarity of the order filters is their non-linearity. A special case of such filters, the median filter, was investigated in [6]. Owing to the nonlinearity of such filters, the analytical estimation of their behavior is a very complicated process. Generally, its behavior appreciably depends on the filter project (weight values, size of aperture or window of analysis, sequence of operations in the multistage process of filtration). We can say here that the quality of signal processing by the WOS filter is a casual case. At the same time, in [7] the author has come to the conclusion that the response of the median filter to the step signal is close to the response of the linear filter at a high noise level. In addition, the importance of the filter project for suppressing noise without unduly distorting the signal is noted. In these conditions, the numerical method of solving mathematical tasks through attracting the modeling of casual events, i.e., the method of statistical trials, for selecting the most efficient WOS filter project is of interest.

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As noted earlier, CoPh WOS filters make it possible to retain the waveform in a limited frequency range. In the general case, the frequency band of a real FM signal can exceed $\Delta f(\text{CoPh WOS})$. Thus the requirement of the processing of this signal throughout the whole frequency band and the corresponding restoration arises. Here, it is proposed to involve the cluster analysis considered in [8]. The results of such an analysis allow us to solve the task of restoring the status of a FM signal with a wide frequency band, as demonstrated in [5, 9].

In this paper, the specialized graphical interface that provides statistical processing by WOS filters of a noised FM signal, its analysis, and its restoration throughout the whole frequency band is proposed. Choosing the method of statistical trials for selecting the WOS filter project allows us to achieve the quality of processing periodic and FM signals that would satisfy a user. The model of a FM signal with a wide frequency band is chosen for illustrating the possibilities of the method proposed.

First of all, we will consider the basic definitions and situations of WOS filters and of cluster analysis.

THE BASIC DEFINITIONS OF THE WOS FILTERS AND OF THE CLUSTER ANALYSIS

Let the sequence $X = \{x_j; j = 1, \dots, N\}$ represent a certain source signal. Thus a formal definition of the order-filtration procedure can be presented as a sequence of the following operations:

1. $Y = \{y_i, i = 0, \pm 1, \dots, \pm v, \}$ is the sampling of $n = 2v + 1$ signal values, where $Y \subset X$ (n is odd),
2. $\tilde{Y} = \tilde{y}_{c-v} \leq \dots \leq \tilde{y}_c \leq \dots \leq \tilde{y}_{c+v}$ is a construction of a variational row, where the term \tilde{y}_i is the statistics of a corresponding order, $i = 1, \dots, n$.
3. $\text{RANK}(y_1, \dots, y_{(n-1)/2}, \dots, y_n) = \tilde{y}_r$ is the operation of replacement of the central term (CE) $y_v \in Y$ by the statistics $\tilde{y}_r \in \tilde{Y}$ ($v, c, i \in Z$).

In the special case where $r = (n - 1)/2$ (i.e., $@ = 0.5$), we have a median filter $\text{MED}_n(y_1, \dots, y_{(n-1)/2}, \dots, y_n) = \tilde{y}_c$.

The locus of the term of the variational row can be defined also, using relative values $\alpha = (r - 1)/(n - 1)$, $\beta = 1 - \alpha$ (α is the relation of the quantity of previous terms to their general quantity without the present term, which further is called procentage). Such a mould of the order filter is called a procentage.

In the general case, it is supposed that each element of a sequence Y is correlated with the rational number $w_i \in W, i = 1, \dots, n$ (interpreted as the number of copies $y_i \in Y$). Then $N = \sum_{i=1}^n w_i$ is the length of the sequence. At the same time, N is, also, odd.

Let the signal for a filter input be a harmonic function $y_i = \sin(\varphi + \omega i \Delta t)$ with the period T . We will equate the weights of all the terms of the sequence to

zero except for the weight of the central element CE and the weights of such terms (the cophased terms) that are apart from one another by the length of the period. At the same time, let filter length be $R = m + 1$, where m is even ($m = 2, 4$, and so on) and is the quantity of the function periods. In this case, the value $V_m = mT/\Delta t = m2\pi/\omega\Delta t$ can be used as the “cost” (or the estimation) of m periods of the signal in quanta of time. Figure 1 illustrates this reasoning.

The cophased values of a harmonic function are equal to one another in the case of the equality of the cophased estimations V_m to an integer and in the case of the absence of noise. The latter circumstance allows us to use averaging for the whole filter length as means of decreasing the noise level. However, in the general case, the relation $R/\Delta t$ will not be an integer. Here, we can turn to averaging drawing on the remainders of the relations $mT/\Delta t$ for estimating the weighted coefficients of the function values. However, in any case, the averaging will be within linear operations. This means that the problem of wrestling with impulse noise is not overcome.

In our case, within order filtration, it is proposed to widen the quantity of signal values for a filter input, as shown in Fig. 1.

We will use the algorithm proposed in [3] to estimate filter length and weight values:

1. Computation of the filter length as \hat{n} , where $n = 2RT/\Delta t + 1$;
2. Realization of the assignments: $w_0 = 1$ for CE and $w_{\pm i} = 0$ for the other components of the sequence Y ;
3. Computation in the loop $j = 1, \dots, R$ of the indices of components of the sequence Y according to the rules:

- i— $V_j = jT/\Delta t$;
- ii—if $V_j = [\dot{V}_j] = [\check{V}_j]$, then $w_{\pm V_j} = 1$; else:
- iii— $w_{\pm[\dot{V}_j]} = V_j - [\dot{V}_j]$;
- iv— $w_{\pm[\check{V}_j]} = [\dot{V}_j] - V_j$.

Here, the symbol $[\dot{x}]$ denotes the smallest integer which is the nearest from above to a value x , and $[\check{x}]$ is the largest integer which is nearest from below. Such a WOS filter will be called the cophased (CoPh WOS) filter, and the frequency which is associated with the period T will be called the work frequency.

Let us turn to cluster analysis. Here, a periodic signal, especially one corrupted by noise, is convenient to consider as a black-and-white image on a two-dimensional plane. At the same time, we will call on the estimation of dispersion on the sliding basis

$$\sigma_k(L) = \sqrt{\sum_{j \in L} (x_j - M_k(L))^2 / L}, \tag{1}$$

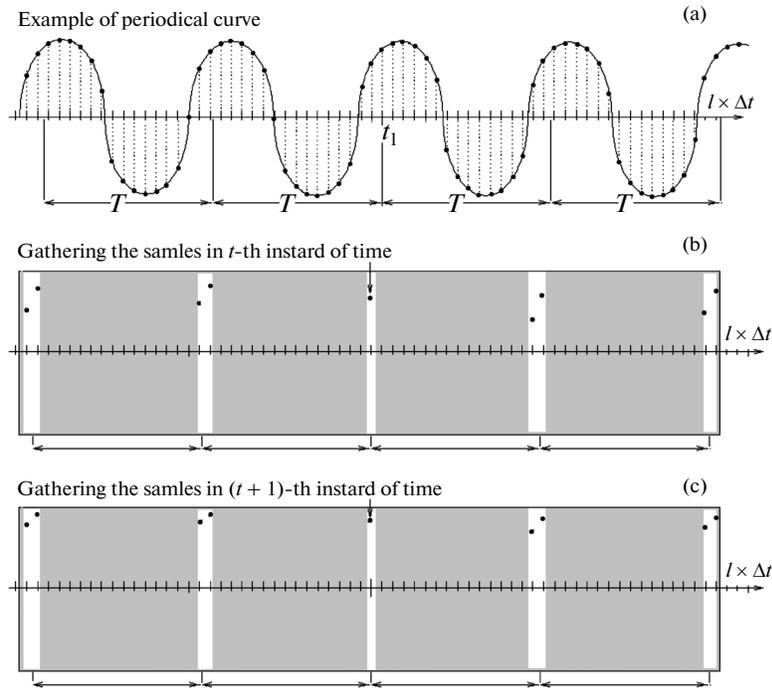


Fig. 1. Sampling values of a periodic signal with the same CE phase (or closest to it) at sequential instants of time.

where L is odd, x is the signal value, and $k = (L - 1)/2, \dots, N - (L - 1)/2$ and σ_k are accepted integer values. Let $\hat{\sigma}$ be the uppermost dispersion value, $0 \leq \sigma_k \leq \hat{\sigma}$. Thus, the temporal sequence $X = \{x_1, \dots, x_N\}$ is converted to the sequence $\{\sigma_k, k = (L - 1)/2, \dots, N - (L - 1)/2\}$, where ‘threshold’ $h, h = 1, \dots, \hat{\sigma}$ is assumed to be the axis of ordinates. This $\hat{\sigma}$ -coordinate grid can be regarded as the field of events $Q = h_l \times t_k$, where an event is defined as

$$q_k(h) = \begin{cases} 1, & \text{if } \sigma_k \geq h_l, \\ 0, & \text{if } \sigma_k < h_l. \end{cases} \quad (2)$$

A subset of events $Q_{r(h)} \subset Q$ which satisfies the condition $\forall q \in Q_{r(h)}: q = 1$ will be called a cluster with a cardinal number $\beta_{r(h)} = \sum_{q \in Q_{r(h)}} q, r = 1, \dots, m(h)$. Let us say that the cluster $Q_{r(h+1)}$ is generated by the cluster $Q_{r(h)}$ if they both have common instants of time on the thresholds $Q_{r(h)}, Q_{r(h+1)}$. The pool of clusters in question will be called a cluster family with the cardinal number $b_{r(h)} = \beta_{r(h)} + \sum_{s=1}^{n(h+1)} \beta_{s(h+1)}$, etc. The locus of such a cluster $Q_{r(h)}$ is presented as $\Delta t_r(h_l) = \{t_{1(r)}(h_l) - t_{n(r)}(h_l)\}$. Let $B^{(h)} = \sum_{r=1}^{n(h)} b_{r(h)}$ be a common cardinal number of the cluster families on the threshold h . Then the relation:

$$P_{r(h)} = b_{r(h)} / B^{(h)} \quad (3)$$

will be called a representative probability of the cluster family $\Delta t_r(h_l)$.

As was established in [8], the behavior of cluster families and functions $P_{r(h)}, r = 1, \dots, m(h), h = 1, \dots, \hat{\sigma}$ reflects the degree of the presence of a signal in noise. The tie of the cluster families with their locus in terms of time enables us to formulate the tasks of estimating temporal parameters and specificity of signals.

ALGORITHM OF ANALYZING THE STRUCTURE OF CLUSTER FAMILIES AND ESTIMATING THEIR CHARACTERISTICS

Let us consider an image of a cluster formation of some signal, presented in Fig. 2, accompanying it by the following notes:

1—The process of signal recording ($t = 0$) precedes by an instant its real arrival, i.e., $T_0 > 0$.

2—A signal subject to transformation (1) is $y = s + \xi$, where s is a source signal, and ξ is additive white noise with zero mean Gaussian distribution. Let t_N be the signal recording period, and ΔT the signal existence period. The following conclusion is a consequence of such a supposition: the probability of localizing of the uppermost dispersion value in the interval ΔT is proportional to the ratio $t_N / \Delta T$ and to the signal-to-noise ratio, i.e., $P(\hat{\sigma}(k) \in \Delta T) \sim f(t_N / \Delta T, s / \xi)$ (a more exact dependence requires a separate consideration).

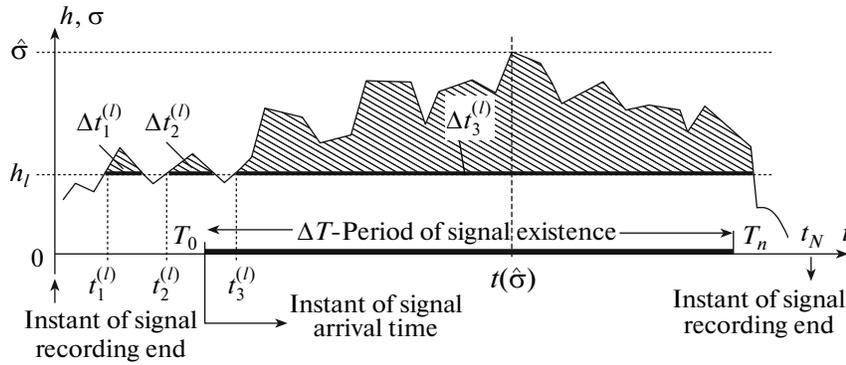


Fig. 2. Example of an image of signal dispersion estimation on some sliding basis.

Now it is possible to formalize the statement of the problem of estimating parameters and characteristics of a signal. It can be supposed, in the general case, that:

- i—one or more of such instants of time, where $t_{1(r)}(h_l) > 0$, exists;
- ii—a certain number of probabilities which satisfies the condition $P_r(h_l) = \max$ exists;
- iii—there exists a non-empty set of such periods of existence of cluster families which satisfies the condition $t(\hat{\sigma}) \in \Delta t_r(h_l), 1 \leq r \leq m(h_l)$.

In these conditions the problem can be reduced to an estimation of the locus of such a cluster family that the cluster $\Delta t_r(h_l) = \{t_{1(r)}(h_l) - t_{n(r)}(h_l)\}$, which generates this family, satisfies the following conditions [9]:

$$\left. \begin{aligned} t_{i(r)}(h_l) &> L, \\ t_{n(r)}(h_l) &< N - L, \\ P_{i(r)}(h_l) &= \max, \\ t_i(\hat{\sigma}) &\in \Delta t_{i(r)}(h_l), \\ 1 \geq i(r) &\geq m(h). \end{aligned} \right\} \quad (4)$$

The corresponding algorithm of the cluster family analysis for solving the problem of estimating parameters and features of the signal presented in [5] (see Fig. 3) includes the following two stages:

1. Search for informative thresholds (h_{\min}^l) in the course of the research into the cluster families beginning with the threshold $h = 1$ (movement upwards from below);
2. Calculation of representative probabilities of clusters families, beginning with the threshold $h = \hat{\sigma}$ (movement downwards from the top).

Here, such a threshold is designated as an informative one with the locus of the cluster $Q_{i(h)}$, which is limited by the instants of time $t_1(h_l) - t_{n(i)}(h_l), i = 1, \dots, m(h)$, provided that this threshold satisfies the requirements $t_{i(r)}(h_l) > L, t_{n(r)}(h_l) < N - L; 1 \geq i(r) \geq m(h)$; at

the same time, the cluster $Q_{r(h_1)}$, which generates the above cluster, does not satisfy the noted requirements. The cluster $Q_{i(h)}$ will be called basic and designated as $\bar{\Delta}_{i(r)}(h_l)$. The contents of other variables and parameters are the following:

- h is the current value of the threshold,
- Ξ is the set of basic clusters,
- S_Δ is the set of all clusters located at the current threshold,
- \aleph_i is the rating of the i th cluster family,
- $Texistence = t_h^{k(j)} - t_h^1$ is the period of signal existence, where $t_h^{k(j)}, t_h^1$ are time instants of the j th cluster of the current threshold.

THE ALGORITHM OF PROCESSING, ANALYSIS AND RESTORATION OF A WIDEBAND FM SIGNAL

As noted earlier, the CoPh WOS filter can save an FM signal only in the limited frequency range Δf (CoPh WOS). If the frequency band of a FM signal surpasses the named range ($\Delta f(\text{FM}) > \Delta f(\text{CoPh WOS})$), then the requirement to restore an initial signal throughout its whole frequency band arises. Here we will use a technique offered in [5]. The flowchart of the corresponding algorithm is presented in Fig. 4.

Apparently, the algorithm structure has a parallel organization whereby are N_{ff} -order filters are attracted. At the same time, the sequence of operations includes three stages in each assembly: signal filtration, cluster analysis, estimation of time and probability characteristics of a signal in each branch (assembly), and, lastly, the restoration of the status of an initial signal in all its completeness, i.e., pooling separate results.

Filter weights of each separate branch are counted under condition $\sum_{i=1}^{N(ff)} \Delta f_i(\text{CoPh}) = \Delta(\text{FM})$, where $\Delta f_i = f_i^b - f_i^k$ and f_i^b, f_i^k are initial, final values of a cor-

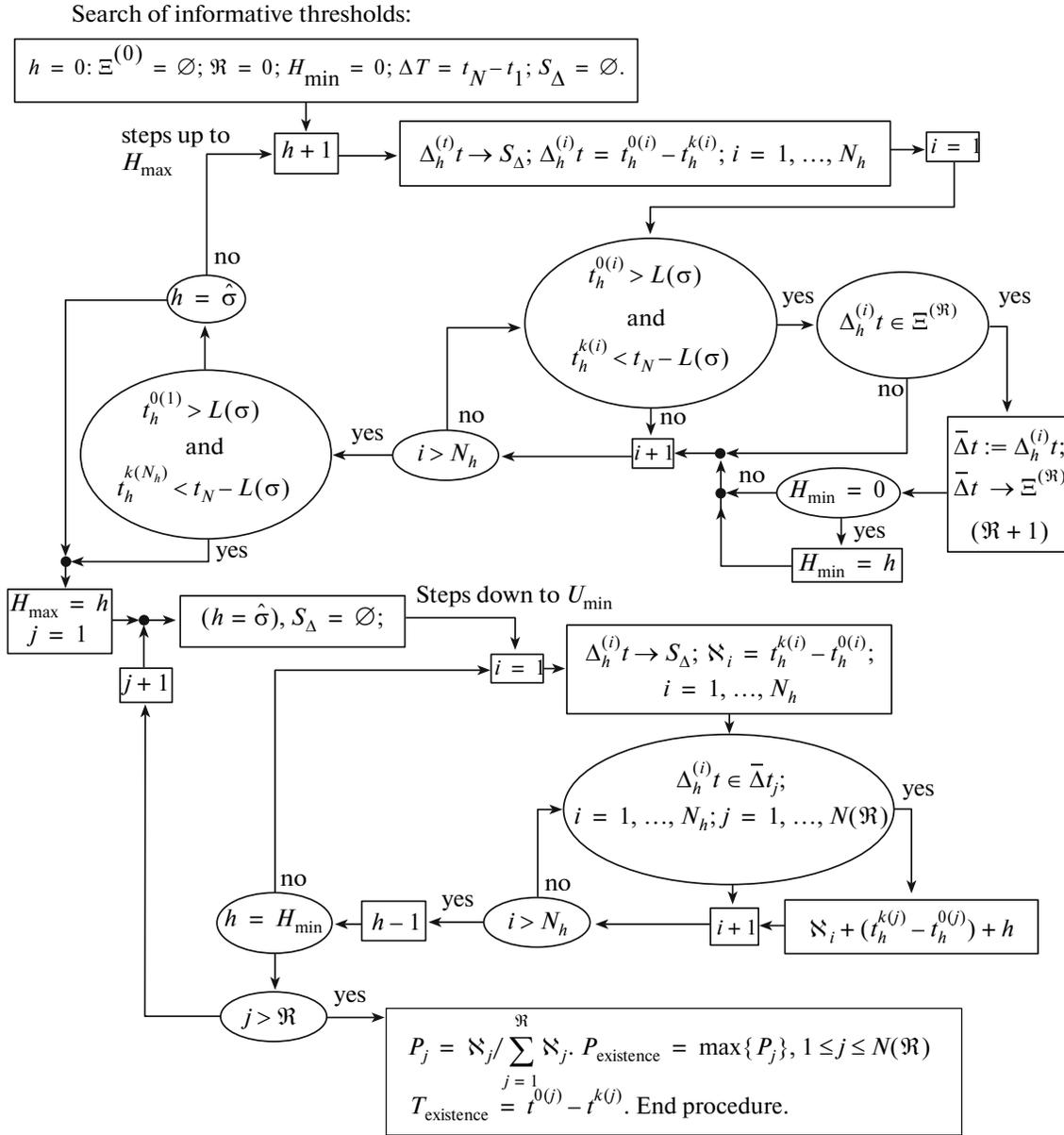


Fig. 3. Flowchart of the algorithm of analysis of clusters families.

responding frequency band. The values $f_i^{(w)} = (f_i^k - f_i^b)/2$ will be called tabular frequencies or a list of work frequencies ($i = 1, \dots, N_{\mathfrak{F}}$).

A locus of corresponding frequency zones on the temporal axis and their representative probabilities (ratings) in each branch of the algorithm allows us to restore the status of an initial signal in all its completeness in the case of processing a signal with a high signal-to-noise ratio. The problem becomes complicated in the case of a low signal-to-noise ratio. In this connection, it is more reasonable to use prominent features of cluster formations for estimating their ratings.

In our case, three forms of estimating representative probabilities are used:

$$P_j^1 = \mathfrak{N}_j^1 / \sum_{j=1}^{\mathfrak{R}} \mathfrak{N}_j^1, \quad P_j^{21} = \mathfrak{N}_j^2 / \sum_{j=1}^{\mathfrak{R}} \mathfrak{N}_j^2, \quad (5)$$

where $\mathfrak{N}_j^1 = \sum_{h_i=H_{\max}}^{H_{\min}} t_{m(h_i)} - t_{l(h_i)}$, $\mathfrak{N}_j^2 = \sum_{h_i=H_{\max}}^{H_{\min}} (t_{m(h_i)} - t_{l(h_i)}) h_i$,

$$P^3 = (H_{\max}/\hat{\sigma})(H_{\max} - H_{\min})/\hat{\sigma}.$$

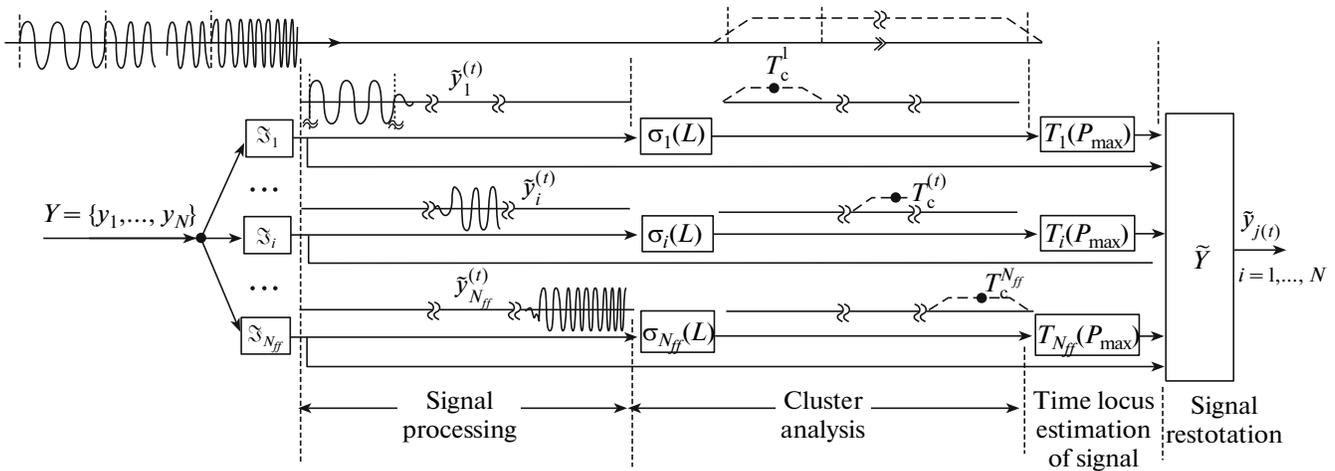


Fig. 4. Flowchart of the algorithm of processing, analysis and restoration of the FM signal.

Here H_{max} is a value of the threshold which limits a cluster family from above, and H_{min} is a value of the threshold which limits a cluster family from below.

Thus, at the restoration stage of a source signal, a set of basic clusters with corresponding families and, also, the estimations of their ratings are accessible. We will add here one more parameter: the “peak” of the clusters family $\hat{T}_i = \sum_{j=1}^n \hat{t}_j^{h(max)} / n$, which characterizes its locus on a temporal axis. Here summation is carried out at all instants of time which are localized on H_{max} .

Let us suppose that the values of the processed signal on the i th branch of the considered algorithm are close to those of the source signal in the vicinities of \hat{T}_i . Then the algorithm of restoring a source signal can be presented in the form:

$$\begin{cases} \text{if } \sigma_{k,j(i)} \geq \sigma_{k,j(i+1)}, & \tilde{y}_{k,j} = \tilde{y}_{k,j(i)}, \\ \text{else,} & \tilde{y}_{k,j} = \tilde{y}_{k,j(i+1)}, \\ j = \hat{T}_i, \dots, \hat{T}_{i+1}; & i = 1, \dots, N_{ff} - 1, \\ k = 1, \dots, N, \end{cases} \quad (6)$$

where $k = (L - 1)/2, \dots, N - (L - 1)/2$, i is the order number of algorithm branch, $j = (L - 1)/2, \dots, N - (L - 1)/2$ is the order number of a corresponding signal value.

THE ALGORITHM OF STATISTICAL TRIALS OF THE ORDER FILTER PROJECT

Basically the list of parameters and characteristics of the WOS filter that defines the quality of signal processing depends on the form of the signal. However, the filter structure, all the details of the corresponding

components (weight values, the size of aperture or analysis window), and, also, the sequence of operations in their specificity have crucial importance in a multistage filtration process.

We will note the most significant parameters and characteristics of the WOS filter without giving separate attention to all the possibilities of their variation.

1. The set of filter weights is supposed to be the most important for signals with a low signal-to-noise ratio. According to the algorithm of their calculation, filter weights are a function of the work frequency $f^{(w)}$, i.e., $W = W(f^{(w)})$. At the same time, the quality of the processing of a signal is rather sensitive to the change of $f^{(w)}$ even in the presence of a small increment/decrement. Thus, a list which includes a certain set of corresponding values $f^{(w)} \in \{f_1^{(w)}, \dots, f_n^{(w)}\}$ can be tied with a separate work frequency of the filter. The use of N_{ff} work frequencies is required in the case of processing a wideband FM signal. Then there are required N_{ff} lists of $f_i^{(w)} \in \{f_{1(i)}^{(w)}, \dots, f_{n(i)}^{(w)}\}, i = 1, \dots, N_{ff}$.

2. The size of the sliding basis of dispersion in the course of the analysis of the clusters families has a great enough value. The given parameter can also be used as a frequency function: $L = \text{Factor} * T / \Delta t$.

3. Fundamentally, the possibility of estimating representative probabilities of cluster families is not limited to an estimation of ratings of the form of (3). Some possible forms are pooled in the list (5). They take into consideration the nuances of the location and structure of the cluster families in a different manner and, in this connection, are of interest for the purposes of statistical tests of filter projects.

4. Finally, in the case of the use of procentile filters, the composition of results of the processing of a signal obtained by a pair under the condition of satisfying the parity $\alpha + \beta = 1$ is of interest. The change of values of the corresponding procentiles also leads to a change of

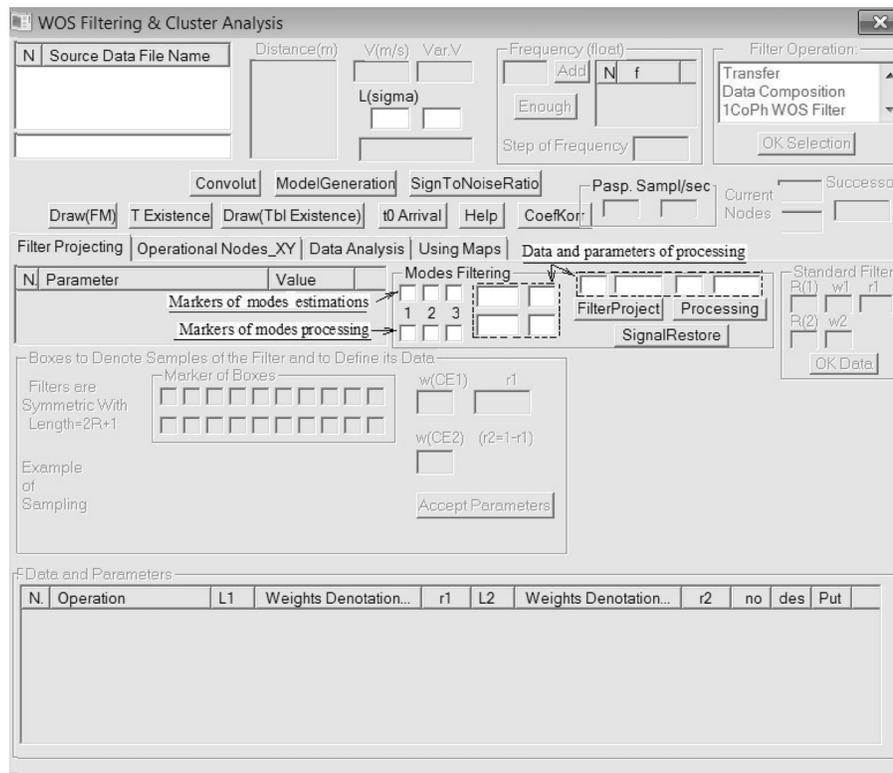


Fig. 6. The right side of the graphical interface.

for signal processing (signal processing using only one frequency).

GRAPHICAL INTERFACE OF THE SPECIALIZED COMPUTER SYSTEM

The right side of the graphical interface intended for statistical trials of the WOS filters is presented in Fig. 6.

The basic functions of projecting the order filters that are accessible by means of this system are considered in [9]. Therefore, here we will only consider the possibilities of the interface which are of interest in our case.

Two aggregates of the markers providing a choice of the method of signal processing and two aggregates of “windows” for the control of the parameter values of processing and their variations (both types of aggregates are outlined by dashed lines, and are supplied by “emerging” explanations) are used in the considered interface.

The basic functions of creating the filter project are accessible using the button “FilterProject,” while the button “Processing” allows us to start the process of processing a signal.

The reference to the button “Processing” opens the interactive console which allows us to input necessary parameters and characteristics of the processing. The

image of the console and a dialogue fragment are presented in Fig. 7.

Either the mode “throughout” (across the whole list of frequencies), or the mode “divided” (attracting a unique frequency from the corresponding list) is accessible to the user. The reference to the button “signal restoration” (“SignalRestore”), after obtaining intermediate results, allows us “to collect” an initial signal using obtained fragments of the processing.

The check of the quality of a signal at different stages of the processing and after its restoration with its full status is made by addressing appropriate functions of interface either the convolution (the button “Convolution”), and/or correlation coefficient (the button “Coefficient Correlation”).

SOME RESULTS OF THE ADAPTATION OF THE WOS FILTERS

The methodology in question was tested on the model of a linear frequency-modulated signal. The parameters and characteristics of the corresponding signal are the following: $T_0 = 4$ sec, $\Delta T = 90$ sec, $\Delta f(\text{FM}) = 15.0$ Hz – 25.0 Hz, $\Delta t = 0.08$ sec.

White noise with zero mathematical expectation when the signal-to-noise ratio $s/\xi = 0.066$ was provided.

ties of cluster families. The methodology proposed was tested on the model of a linear frequency-modulated signal. The comparison of the results obtained enables us to suppose that the method of statistical trials of WOS filter projects enables us to choose more effective values of parameters of a signal processing and, also, of the data involved for the analysis of the results of processing and restoration of a wideband FM signal. Research into the numerical model of such a signal has shown the possibility of an approach in the case of a signal-to-noise ratio of $s/\xi \ll 1$.

In this paper, the obtaining of parameters of WOS filters and their structure, recommended for the universal or a wide use was not declared. Each separate case of the processing of a signal corrupted by noise demands a separate investigation of the WOS filter adaptation to the character of a signal and specificity of noise. This paper offers a corresponding tool. However, the further improvement of the quality of signal processing in the considered case demands additional studies with attracting wider aggregation of the WOS filters structure and a more representative statistical material. In this sense, the idea of creating an original WOS filters bank is of interest, which can considerably increase the representative basis of the processing and analysis.

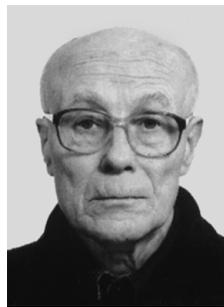
The method of statistical trials presupposes gathering considerable statistical material, i.e., it is rather time-consuming and requires high computer costs. The latter circumstance brings about the idea of attracting more efficient computation structures.

Finally, the problem of the “intelligent” analysis of results of the statistical trials arises. That is, there is the question of the formalized strategy of selecting filter projects with using those or other estimations of the results obtained, and finally, we should note that the considered computer system does not represent the final version. It is assumed that its development with new functions demands reflection, also, in a graphical interface.

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