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Numerical Simulation of Acoustic Waves Propagation in an "Atmosphere–Forestland–Ground" System

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Abstract—Under study is the problem of numerical simulation of acoustic waves propagation in a two-dimensional inhomogeneous medium represented by the "atmosphere–forestland–ground" model. A specific feature of the simulation is the introduction into the basic equations of acoustics of a linear damping function that characterizes the energy loss of the acoustic wave with respect to afforestation. The problem is considered of interaction between the acoustic waves incident at a given angle from the atmosphere to the "forestland–ground" boundary and the seismic waves arising in the ground. The issue of the forestland influence on the levels of acoustic and seismic waves is investigated. In particular, the impact of the friction coefficient on the attenuation rate of acoustic oscillations in the forestland is estimated. The algorithm and software are developed and implemented for calculating the acoustic pressure levels in various media, by using the wave equation for the atmosphere, Euler's gas dynamics equations for the forestland, and the elasticity equation for the ground. The results of numerical experiments are presented as instantaneous images of the wave field.

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Keywords: *technogenic noise, infrasonic wave, geoecological danger, noise absorption, forestland, numerical experiment, equations of gas dynamics, numerical result*

INTRODUCTION

An important place among the today's geo-environmental problems is occupied by the problem of preventing geo-environmental risks for the surrounding social and natural environment from the effects of noise generated by natural and technogenic sources of increased danger. These include acoustic noise generated by the surface quarry and test site explosions [1, 2], major highways [3], as well as the noise from airports [4] and rocket launch sites [5]. In recent time, the disposal of the accumulated expired ammunition stocks by exploding has been carried out. An effective way to reduce the impact of the ammunition disposal processes for residential buildings may be the placement of explosion fields in far forestlands [6].

The negative impact of noise can multiply under the influence of meteorological factors, in particular, causing the formation of phenomenon of spatial focusing of acoustic oscillations in a certain azimuth direction relative to the noise source [7–9]. Designing measures to reduce the "noise risk" is one of the urgent tasks, especially with respect to the noise at infra-low frequencies that is most threatening the human brain.

One of the measures to combat noise is creation of protective afforestation along major highways and railways, as well as near other sources of noise [10–12]. In connection with this, the problem arises of studying the process of interaction of acoustic wave fronts with permeable obstacles, which includes a forestland. The problem of propagation of acoustic waves in the forestland, from the standpoint of their

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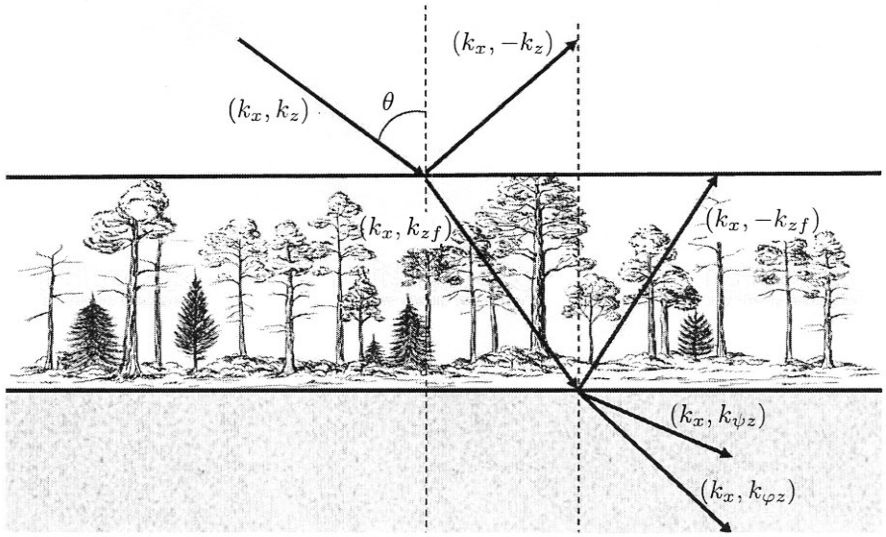

Fig. 1. Three-layer "atmosphere—forestland—ground" model with the scheme of incident and reflected waves in the layers.

properties of environmental protection from harmful technogenic acoustic disturbances, is topical but yet poorly studied today.

In this paper, the special feature of the numerical simulation approach for solving the problem under consideration is the use of acoustic vibrations from seismic vibration sources with the precision characteristics which allows us further comparison of the results of numerical simulation and field experiments. This technology of vibroseismic-acoustic sounding of complex organized media is well developed today [13, 14] and provides high repeatability and accuracy of the obtained estimates of the acoustic field parameters.

1. FORMULATION OF THE PROBLEM AND ANALYTICAL SOLUTIONS

The problem is considered of the acoustic wave incident at a given angle on an elastic half space with a forestland on the surface for the case of long-range propagation of acoustic waves from an infra-low frequency source. To solve the problem, a three-layer model is considered, whose domain consists of air, forestland, and ground (Fig. 1).

An acoustic wave propagates in the atmosphere and falls on the forest edge at some angle θ to the vertical, $0 \le \theta \le 90°$. In this model, *air* occupies the upper half-space with the following parameters: the sound velocity c and the density ρ. The *forestland* is characterized by the friction coefficient α whose physical meaning is connected with the effect of absorption of the acoustic oscillation energy. In the forestland, α depends on the aerodynamic resistance coefficient c_d and the specific density S of the forest surface (leaves and branches) [10]. The *ground* occupying the lower half-space is characterized by the density ρ_g and the velocities of longitudinal and transverse waves V_p and V_s. The issue is studied of the extent to which the forestland affects the absorption of acoustic oscillations and reduces the acoustic wave level in dependence on the characteristics of the massif. Wherein, the assumption is made about the source location at a distance much greater than the length of the acoustic wave. Therefore, the wavefront is assumed locally flat, and the analysis is carried out within the framework of $2D$ simulation. First of all, note that the measure of geoecological risk from the impact of acoustic waves is the specific density of acoustic energy

$$E = \frac{1}{\rho_c} \int\limits_T^0 p^2(t)\, dt$$

Characteristics of the forestlands

Type of forest stand	Height h, m	S, m^2	c_d
Small-leaved forest	7–30	1.2	0.02
Pine	10–15	1.2	0.03
Brushwood	1–5	7	0.05

accepted by the sanitary norms. Here, ρ_c is the specific acoustic resistance of air equal to 42 g/(cm^2·s), $p(t)$ is acoustic pressure recorded at a point in space, and T is the duration of the acoustic wave. Further calculation results are related to the acoustic pressure levels.

For air, the wave equations are considered with constant density and sound speed. The equations for the air pressure as well as the relation between velocities and pressures have the form

$$\frac{1}{c^2}\frac{\partial^2 p}{\partial^2 t} - \Delta p = 0, \qquad \rho\frac{\partial \vec{u}}{\partial t} + \nabla p = 0, \tag{1}$$

where p, $\vec{u} = (u_x, u_z)$, ρ, and c are the pressure, the velocity vector of the air particle displacements, the air density, and the sound speed respectively. A solution of (1) can be represented as a superposition of two-dimensional harmonic oscillations. Thus, the resulting pressure in the air is represented as the sum of the pressures of the incident and reflected waves:

$$p = P_0 e^{-i\omega t + ik_x x + ik_z z} + P_1 e^{-i\omega t + ik_x x - ik_z z}. \tag{2}$$

Here, k_x and k_z are the projections of the wave vectors on the x and z axes.

The model of the forestland is based on the Euler system of equations and can be written as

$$\begin{gathered}
\frac{\partial \rho}{\partial t} + \rho\left(\frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z}\right) = 0,\\
\frac{\partial u_x}{\partial t} = \frac{1}{\rho}\frac{\partial p}{\partial x} - \frac{\alpha}{\rho}\,u_x,\\
\frac{\partial u_z}{\partial t} = \frac{1}{\rho}\frac{\partial p}{\partial z} - \frac{\alpha}{\rho}\,u_z,
\end{gathered} \tag{3}$$

where u_x and u_z are the components of the velocity vector of the air particle displacements, while α is the damping coefficient of friction for the forestland. The presence of a nonuniform component in (3) is expressed by an additional term with the coefficient α. In result, we arrive at the equation for the pressure $p(x, z, t, \alpha)$:

$$\frac{\partial^2 p}{\partial t^2} - \left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial z^2}\right) + \frac{\alpha}{\rho}\,\frac{\partial p}{\partial t} = 0. \tag{4}$$

The solution of (4) can also be represented as the sum of pressures of the refracted and reflected waves:

$$p = P_2 e^{-i\omega t + ik_x x + ik_{zf} z} + P_3 e^{-i\omega t + ik_x x - ik_{zf} z}, \tag{5}$$

where k_x and k_{zf} are the projections of the wave vectors on the x and z axes in the forestland.

The friction coefficient α depends on the aerodynamic resistance coefficient c_d and the specific surface density S of vegetation (leaves and branches). The table shows the characteristics of the forestlands. Real forestlands are characterized by the distribution of the vegetation surface density S nonuniform with respect to the height h.

The boundary conditions at the "air–forestland" interface are the equality of pressures and velocities with respect to the component z in both media:

$$u_{za}|_{z=0} = u_{zf}|_{z=0}, \qquad p_a|_{z=0} = p_f|_{z=0}.$$

In the boundary conditions, the acoustic refracted wave falling on the"forestland—ground" boundary with the constant speed c and frequency ω is taken into account as follows: The pressure of the acoustic wave is equal to the normal stress of the ground, and the z components of the velocities are equal in both media. And so, we arrive at the equalities

$$\sigma_{xz}^{\mathrm{gr}}\big|_{z=h} = 0, \qquad \sigma_{zz}^{\mathrm{gr}}\big|_{z=h} = pe^{-i(\omega t - kx)}, \qquad u_{\mathrm{air}}\big|_{z=h} = u_z^{\mathrm{gr}}\big|_{z=h}. \tag{6}$$

For ground, the dynamic equations of elasticity are solved with constant values of λ^{gr}, μ^{gr}, and ρ^{gr}:

$$(\lambda^{\mathrm{gr}} + \mu^{\mathrm{gr}})\mathrm{grad}\ \cdot \mathrm{div}\, \mathrm{u}^{\mathrm{gr}} + \mu_{\mathrm{gr}}\Delta \mathrm{u}^{\mathrm{gr}} - \rho^{\mathrm{gr}}\frac{\partial^2 \mathrm{u}^{\mathrm{gr}}}{\partial \mathrm{t}^2} = 0. \tag{7}$$

The solution of (7) will be sought in the form of potentials:

$$\begin{aligned} \varphi &= \varphi(x, z, t) = P_4 e^{-i\omega t + ik_x + ik_{\varphi z} z}, \\ \psi &= \psi(x, z, t) = P_5 e^{-i\omega t + ik_x + ik_{\psi z} z}, \end{aligned} \tag{8}$$

where $k_{\varphi z}$ and $k_{\psi z}$ are the wave numbers of the form

$$k_{\varphi z} = k_z\sqrt{c^2/V_p^2 - 1}, \qquad k_{\psi z} = k_z\sqrt{c^2/V_s^2 - 1},$$

V_p and V_s are, respectively, the speed of the longitudinal and transverse waves.

The potentials φ and ψ are connected with the displacement field by the following formulas in general form:

$$u_x = \frac{\partial \varphi}{\partial x} - \frac{\partial \psi}{\partial z}, \qquad u_z = \frac{\partial \varphi}{\partial z} + \frac{\partial \psi}{\partial x}.$$

Inserting into (6) the potentials φ and ψ (8), we obtain an inhomogeneous system of equations for the coefficients P_4 and P_5 in general form:

$$\begin{aligned} \sigma_{xz}|_{z=0} &= \mu\left[2\frac{\partial^2 \varphi}{\partial x \partial z} + \frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi}{\partial z^2}\right] = 0, \\ \sigma_{zz}|_{z=0} &= \left[\lambda\frac{\partial^2 \varphi}{\partial x^2} + (\lambda + 2\mu)\frac{\partial^2 \varphi}{\partial z^2} + 2\mu\frac{\partial^2 \psi}{\partial x \partial z}\right] = \rho e^{-i(\omega t - kx)}. \end{aligned} \tag{9}$$

Having the value of pressure amplitude P_0 of the incident wave, we can estimate the contribution of the forestland to the energy of acoustic wave transferred to the ground. To this end, we should

(1) to determine the coefficients P_1, P_2, and P_3 which reflect the amplitudes of pressure of the reflected and refracted waves in the forestland;

(2) to determine the coefficients P_4 and P_5 which reflect the pressure amplitudes of longitudinal and transverse waves arising in the ground;

(3) to construct a $2D$ model of inhomogeneous "air-forestland" medium and obtain a numerical solution of the Euler equations of gas dynamics.

2. CALCULATION OF ACOUSTIC PRESSURE LEVELS

To calculate acoustic pressure levels, a special code is developed basing on equations (2)–(9). The values of acoustic pressure are calculated for refracted and reflected waves in the case of passage of a harmonic acoustic wave through the forestland and the ground, by taking into account the friction coefficient. As an example, the graphs of acoustic pressure are depicted in Fig. 2 for both types of waves respectively, depending on the incidence angle θ in the case of the acoustic wave passing through the forestland at various heights 0, 5, 10, and 50 m; the frequency $f = 10$ Hz.

The graphs illustrate the pronounced absorption effect with increasing incidence angle θ; i.e., when the direction of the wave front propagation approaches the horizontal. This is quite consistent with a priori ideas. For $h = 50$ m and $\theta = 90°$, the amplitude of acoustic pressure can decrease by more than an order of magnitude.

In Fig. 3, the graphs of acoustic pressure are depicted for refracted and reflected waves, respectively, in its dependence on the incidence angle θ at the frequencies $f = 8$, 15, and 80 Hz. As can be seen from the figure, with the frequency increase by an order of magnitude, the attenuation rate of acoustic pressure is almost 10 times higher.

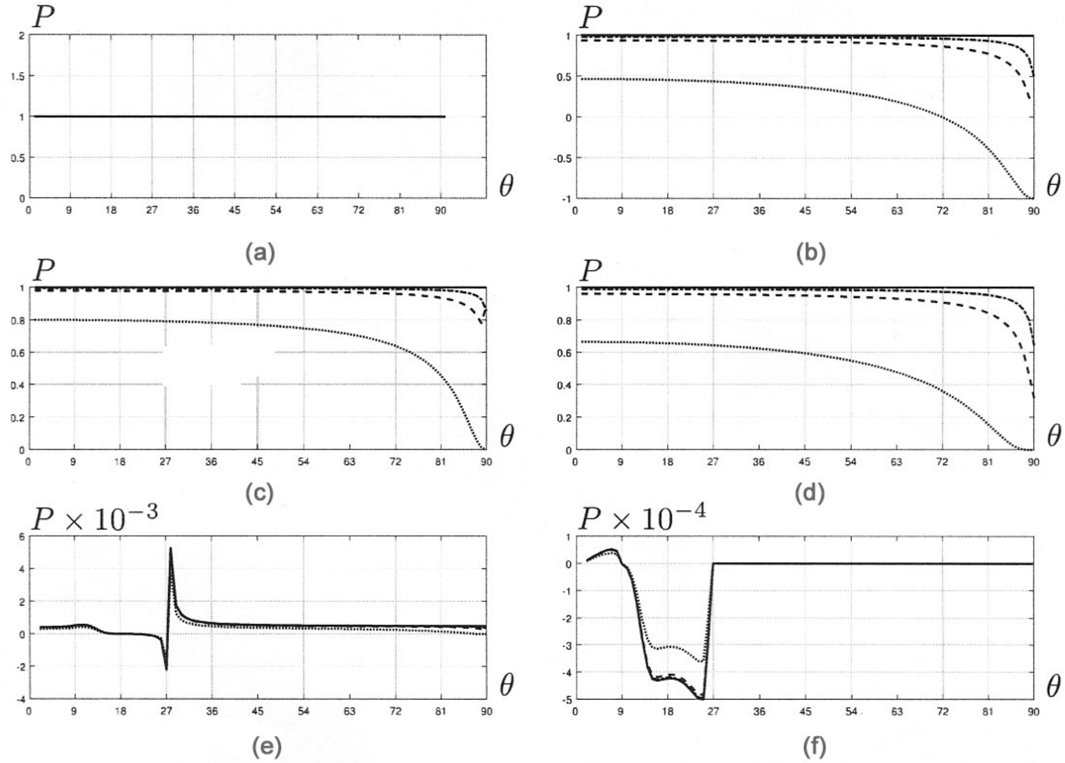

Fig. 2. Graphs of the acoustic pressure dependence on the incidence angle θ (degrees) for refracted and reflected waves in the case of passage of an acoustic wave through the forestland with the frequency $f = 10$ Hz and various heights h (m): $h = 0$ (solid line), 5 (dashed curve), 10 (dash-dotted curve), and $h = 50$ (dotted curve). We have the incident (a) and reflected (b) waves in the air, the refracted (c) and reflected (d) waves in the forestland, and the longitudinal (e) and transverse (f) waves in the ground.

3. NUMERICAL SIMULATION OF THE DYNAMICS OF THE WAVE PATTERN OF ACOUSTIC OSCILLATION PROPAGATION

To solve the problem under discussion in the article, it is expedient to perform numerical simulation of the acoustic wave propagation from a point source located outside the forestland near the border with the ground. The forestland has finite size. Investigation of the acoustic wave propagation is conducted by numerical simulation in this case. The latter is connected with solving the system of equations of gas dynamics (3) in the two-dimensional case under presence of an inhomogeneous component characterizing the presence of tree cover. Let the computational domain be a rectangle in the two-dimensional coordinate system Oxz (Fig. 4).

The system of equations (3) is solved with the corresponding zero boundary and initial conditions. To eliminate the unwanted waves reflected from the side and upper boundaries of the computational domain, the methodology of perfectly matched layers (PML) is applied [15, 16]. In this way, some three subdomains of small size are identified on three borders, where the calculation formulas for PML approach are realized. To generate a signal and simulate the propagation of acoustic waves, some point source is applied. The source can be located at an arbitrary point inside the domain, excluding the PML zone. In the system of equations (1) the source is presented as a component of force. For numerical simulation of the acoustic wave propagation in the nonhomogeneous "atmosphere—forestland" model, the finite difference method on shifted grids is used [17]. The method has the second order of accuracy in space. We implemented a finite difference scheme with the same spatial grid sizes Δx and Δz, respectively. The finite-difference approximation of equations can be represented in the form

$$p_{i,j}^{n+1} = p_{i,j}^{n} - \rho_{i,j} c_{i,j}^{2} \frac{1}{2}\frac{\Delta t}{\Delta x}\big(\big(u_{x,i+1/2,j}^{n} - u_{x,i-1/2,j}^{n}\big) + \big(u_{z,i,j+1/2}^{n} - u_{z,i,j-1/2}^{n}\big)\big),$$

$$u_{x,i+1/2,j}^{n+1} = u_{x,i+1/2,j}^{n} - \frac{1}{\rho_{x,i+1/2,j}}\,\frac{1}{2}\frac{\Delta t}{\Delta x}\,\big(p_{i+1,j}^{n+1} - p_{i,j}^{n+1}\big) - \frac{1}{\rho_{x,i+1/2,j}}\,\Delta t\,\alpha_{x,i+1/2,j} u_{x,i+1/2,j}^{n},$$

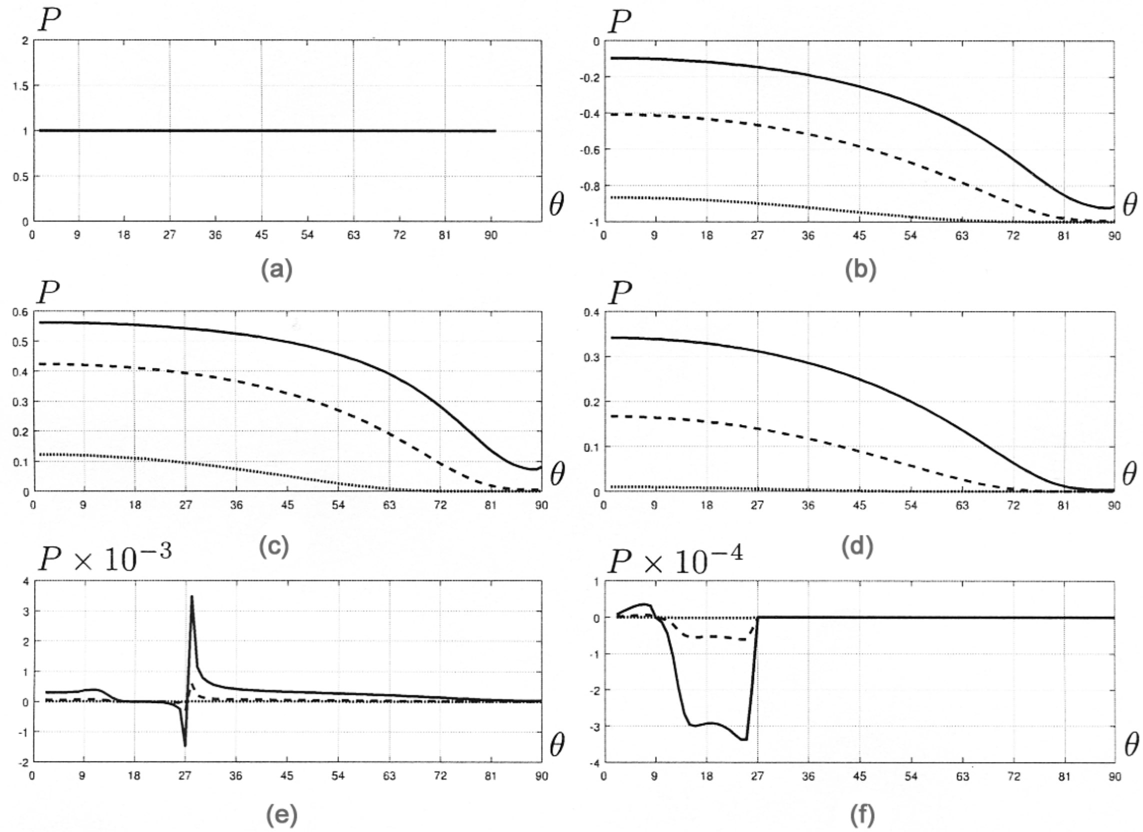

Fig. 3. Dependences of acoustic pressure for refracted and reflected waves on the incidence angle θ (degrees) at various frequencies f (Hz): $f = 8$ (solid line), 15 (dashed curve), and $f = 80$ (dash-dotted curve). We have the incident (a) and reflected (b) waves in the air, the refracted (c) and reflected (d) waves in the forestland, and the longitudinal (e) and transverse (f) waves in the ground.

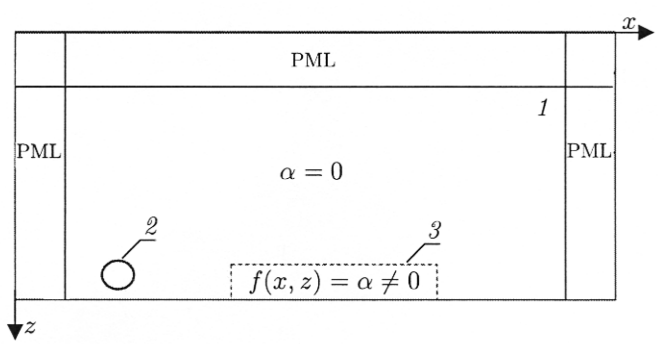

Fig. 4. Schematic representation of the two-dimensional simulation domain: atmosphere (*1*), the source (*2*), and the forestland (*3*).

$$\rho_{x,i+1/2,j} = \frac{1}{2}(\rho_{i+1,j} + \rho_{i,j}),\ \alpha_{x,i+1/2,j} = \frac{1}{2}(\alpha_{i+1,j} + \alpha_{i,j}).$$

To exclude any wave reflections from the boundaries of the computational domain, the PML implementation methodology is applied in the form of convolutional PML (CPML). In connection with this, the equations in PML zones are solved for the approximation that takes into account the additional factor and damping coefficients [16]. This approach is applied successfully to solving the dynamic problem of elasticity theory of the numerical simulation of seismic fields [18, 19].

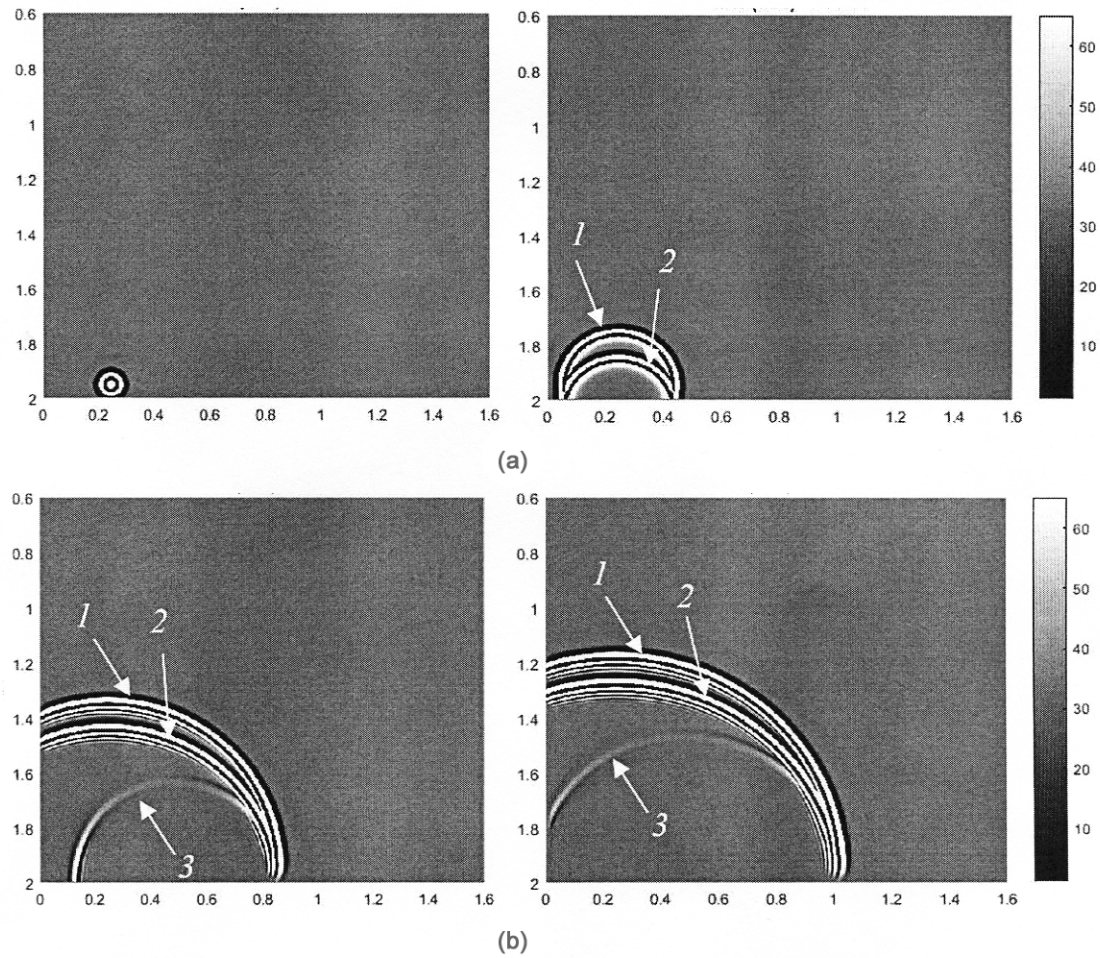

Fig. 5. Wave field images for acoustic pressure with various types of acoustic waves: the incident wave (*1*), the waves reflected from the lower rigid boundary (*2*), and the refracted waves due to the presence of the forestland (*3*).

4. RESULTS OF NUMERICAL SIMULATION

To conduct theoretical experiments on simulating the propagation of acoustic waves from a point source in an inhomogeneous media, a computational code is developed which implements the proposed finite difference scheme. In this case, the parallelizing technology using OpenMP (open multiprocessing) is used in multicore processors.

To study the interaction of the acoustic wave with the forestland, numerical simulation using the developed program is carried out for the inhomogeneous "air—forestland" model. A two-dimensional simulation domain is selected with the linear dimensions of 2.04×2.04 km^2 (see Fig. 4). The inhomogeneous medium is represented by two objects: *1* corresponds to the enclosing air medium (atmosphere) and *3*, to the forestland. The values of parameters for the atmosphere are selected as $c = 340$ m/s and $\rho = 1.2$ g/m^3. The grid model for calculations has 1200×1200 nodes. The point source of acoustic waves with the frequency of 5 Hz is located in the air and has the coordinates 0.25 km along the axis Ox and 1.95 km along the Oz axis. As a forestland, a subdomain is created in the enclosing medium with length from 0.5 to 1.5 km along the Ox axis and height of 0.05 km along the Oz axis.

As a result of application of the implemented program for calculating the acoustic wave propagation in the two-dimensional model of inhomogeneous medium in the presence of forestland, a set of wave field images is taken which were shot at the calculation times $t = 0.5$, 1.5, 4.0, and 5.0 s (Fig. 5). The images show various types of acoustic waves: the incident wave (*1*), the waves reflected from the lower rigid boundary of the ground (*2*) (see Figs. 5, a and b), and the refracted waves due to the presence of the forestland (*3*) (see Fig. 5, b). The results are obtained by the Matlab package.

In the presented images we can trace the evolution the pressure field. In particular, we can clearly observe the formation of reflected wave after the acoustic wave coming on the front boundary of the forestland (Fig. 5, b).

CONCLUSION

In a three-layer "atmosphere—forestland—ground" model, the problem was considered of the interaction of the acoustic wave incident at some given angle on the elastic half-space with a forestland on the surface. We discussed the case of long-range propagation of acoustic waves from an infra-low frequency source. For this problem, the conditions are analyzed and the results are presented of solving the problem of acoustic propagation through a permeable obstacle in the form of a forestland.

Mathematical simulation was performed of the processes of infrasonic acoustic wave distribution in the atmosphere in the presence of forestland, taking into account the friction force introduced in this case. It was supposed that, at long distances from the source, the spherical wave field was locally flat, which allowed $2D$ simulation. The influence of the friction coefficient on the attenuation rate of acoustic oscillations in the forestland was evaluated.

There were developed and implemented an algorithm and a code for calculating the acoustic pressure levels in various media using the wave equation for the atmosphere processes and the Euler gas dynamics equations for the forestland. Within the framework of the mathematical model, the choice of initial parameters and boundary conditions was substantiated and the test calculations that use the developed codes were performed.

A system of equations of acoustics in the two-dimensional case was presented to study the interaction of the acoustic waves with some forestland. A method was proposed for solving the system of equations describing the propagation of acoustic waves from a point source in an inhomogeneous "atmosphere—forestland" model. In this case, the linear damping function characterizing the energy loss of the acoustic wave in the forestland was introduced into the basic acoustics equations. For numerical realization, a finite difference scheme was developed and presented.

Basing on the algorithm, the software implementation was performed by using OpenMP parallel programming technology. This approach allows us to carry out calculations on the multiple processors and use the multicore computing systems with shared memory, even on a PC. With the help of this program, an inhomogeneous "atmosphere—forestland" model was designed in the article and numerical experiments on simulating the acoustic wave propagation from a point source were conducted. The theoretical results of model calculations were obtained and presented in the form of two-dimensional images of the acoustic wave field. The developed code can be used for calculation on SMP systems, as well as on computing devices like Xeon Phi.

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REFERENCES

1. M. S. Khairetdinov, S. A. Avrorov, G. M. Voskoboinikova, and G. F. Sedukhina, "Estimation of Seismic-Acoustic Effects of Technogeneous Explosions by Using Seismic Oscillators," Tekhnologii Seismorazvedki No. 2, 98–105 (2012).
2. M. S. Khairetdinov, V. V. Kovalevsky, G. M. Voskoboinikova, and G. F. Sedukhina, "Estimation of Meteodependent Geo-Ecological Risks from Explosions by Using Seismic Oscillators," Tekhnologii Seismorazvedki No. 3, 132–138 (2016).
3. T. Van Renterghem et al., "Using Natural Means to Reduce Surface Transport Noise During Propagation Outdoors," Appl. Acoustics No. 92, 86–101 (2015).
4. S. V. Klein and D. N. Koshurnikov, "An Estimate of Noise Exposition and Related Public Health Hazard for Population in the Influence Zone of an Airport," Izv. Samarsk. Nauchn. Tsentr. Ross. Akad. Nauk **15** (3(6)), 1806–1812 (2013).
5. V. M. Krasnov, Ya. V. Drobzheva, and A. N. Maslov, "Acoustic Field on the Earth under Explosion of a Carrier Rocket," Vestnik N.Ya.Ts. No. 2, 79–85 (2006).

6. S. A. Orlov, *Mathematical Modeling of Aerodynamic Processes in Forestlands and Green Plantations*, Candidate'S Dissertation in Physics and Mathematics (Tomsk, 2012).
7. L. M. Brekhovskikh, *Waves in Stratified Media* (Nauka, Moscow, 1973) [in Russian].
8. V. V. Gubarev, V. V. Kovalevsky, M. S. Khairetdinov, S. A. Avrorov, and G. M. Voskoboinikova, "Prediction of Environmental Risks from Explosions Based on a Set of Coupled Geophysical Fields," Optoelectronics, Instrumentation and Data Processing **50** (4), 3–13 (2014).
9. M. Khairetdinov, V. Kovalevsky, G. Voskoboinikova, and G. Sedukhina, "Vibroseismoacoustic Method in Studying of Geophysical Fields Interaction in Ground Atmosphere," in *Proceeding. 14th Internatioinal Multidisciplinary Scientific Geoconference "Informatics, GeoInformatics, and Remote Sensing SGEM-2014,"* Vol. 1 (Albena, 2014), pp. 925–931.
10. V. V. Sitnik, "Modeling of Influence of a Natural Growth Tract on Propagation of Acoustic Perturbations," Mat. Model. **19** (8), 90–96 (2007).
11. P. Chobeau, *Modeling of Sound Propagation in Forests Using the Transmission Line Matrix Method. Study of Multiple Scattering and Ground Effects Related to Forests* (Univ. du Maine, Le Mans, 2014).
12. E. Johansson, *The Sound Amplifying Forest with Emphasis on Sounds from Wind Turbines* (Chalmers Univ. of Technology, 2010).
13. A. S. Alekseev et al., "Effect of Seismic-Acoustic Induction in the Course of Vibroseismic Probing," Dokl. Akad. Nauk **346** (5), 664–667 (1996).
14. B. G. Mikhailenko and A. A. Mikhailov, "Numerical Modeling of Seismic and Acoustic-Gravity Waves Propagation in an 'Earth-Atmosphere' Model under Presence of Wind in the Air," Numer. Analysis and Applications **7** (2), 124–135 (2014).
15. D. Komatitsch and J. Tromp, "A Perfectly Matched Layer (PML) Absorbing Condition for the Second-Order Elastic Wave Equation," Geophys. J. Internat. No. 154, 146–153 (2003).
16. D. Komatitsch and R. Martin, "An Unsplit Convolutional Perfectly Matched Layer Improved at Grazing Incidence for the Seismic Wave Equation," Geophysics **72**, 155–167 (2007).
17. J. Virieux, "P-SV Wave Propagation in Heterogeneous Media: Velocity-Stress Finite-Difference Method," Geophysics No. 51, 889–901 (1986).
18. D. A. Karavaev, B. M. Glinsky, and V. V. Kovalevsky, "A Technology of 3D Elastic Wave Propagation Simulation Using Hybrid Supercomputers," in *CEUR Workshop, Proceedings of 1st Russian Conference on Supercomputing Days*, Vol. 1482, pp. 26–33 (2015).
19. A. A. Yakimenko, A. E. Morozov, and D. A. Karavaev, "Practical Aspects of Using a Neural Network to Solve Inverse Geophysical Problems," IOP Conf. J. Physics. Conf. Ser., Vol 1015, 032148 (2018); DOI 10.1088/1742-6596/1015/3/032148.