

PRACTICAL ASPECTS THE RESTORING OF VELOCITY MODELS OF ELASTIC MEDIA IN THE PROBLEMS OF THE MONITORING OF THE ZONES THE UNDERGROUND NUCLEAR EXPLOSIONS

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ABSTRACT

The paper considers the problem of velocity model reconstruction of an elastic medium in the problem of monitoring the zones of underground nuclear explosions. The inverse problem can be solved with help of finding solutions for a set of forward geophysical problems. Main problem is to find the location coordinates and treat to find the characteristic size and shape of cavernous object in entire isotropic elastic media. To realize it the problem statement and the description of basic methods to perform full seismic field numerical modeling are presented. Inclusion is a cavern presented by oval shape object formed in the result of an underground nuclear explosion. The software for 2D and 3D models building (defining values of elastic properties into mesh and model geometry construction) of inhomogeneous elastic media and for simulation of elastic waves propagation was developed. The synthetic results of experiments for 3D full seismic field simulation with cavity inclusion described by elastic parameters with zero values are presented. In terms of the use of neural networks, the advantages of using such an approach are shown. The results of subsurface structure restoration for 2D elastic media with cavernous inclusions are presented.

Keywords: seismic field, elastic waves, simulation, neural network, finite-difference method, cavern

INTRODUCTION

The present study and obtained results are actual and useful in developing approaches to solve problem of model reconstruction. The main tasks of modelling are connected with developing software for monitoring the cavernous and adjacent areas of underground nuclear explosions in order to track the changes in subsurface structure of areas under study. The environmental aspect of tasks to be solved is connected with the need to control the underground pathways of possible spread of radioactive products produced during underground nuclear explosions. To build elastic medium model, one can solve the inverse problem or find solutions for a set of forward geophysical problems. In second case, it is possible to vary calculation parameters (subsurface geometry of model and values of elastic parameters) to verify model in order to achieve the best fit between the simulation results and the experimental data. We developed algorithms and software to build mesh models of elastic media with inclusions and to perform simulation of

seismic field in order to study its structure and distinctive features. The research work to perform simulation for 3D inhomogeneous media with hollow cavity inclusion was conducted. We modify numerical algorithm to obtain results for 3D cavity, described by zero values of elastic parameters. In this case, boundary conditions are implemented on the surface of the cavity located inside 3D elastic media. We describe results in area of using neural networks to reconstruct model of 2D elastic media. It was carried out more than 100 computational experiments for 2D seismic field simulation that are used for training the developed neural network. Such models are presented in form of homogeneous media that contains only one cavernous inclusion. Models differ from each other in location and size of the cavity inside 2D elastic media. In the paper the description of neural network is presented. The results of elastic model reconstruction for 2D case using the trained developed neural network are also presented. The paper covers description of problem statement of seismic field simulation and finite-difference method for problem solving.

PROBLEM STATEMENT

Elastic media is described by a part of subsurface area having linear sizes, Fig.1. We perform simulation in Cartesian coordinate system. In such case modeling area has rectangular form in 2D or cubic in 3D. In the paper we study only isotropic media. Such inhomogeneous media is described by three elastic parameters: density and two speeds of elastic waves. One of the main restrictions is a flat geometry of free surface, where zero boundary conditions are realized.

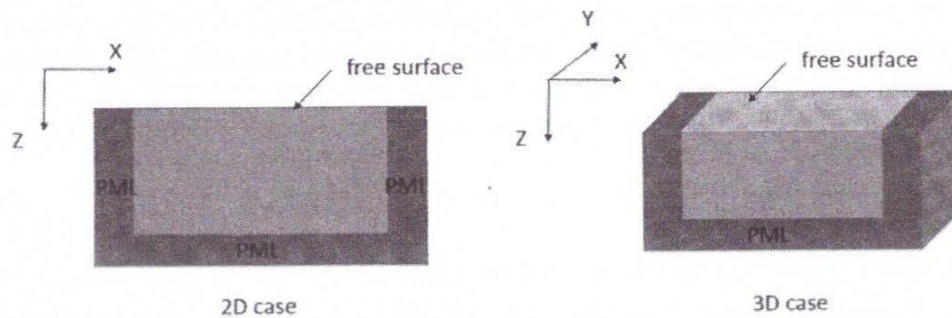


Fig. 1 View of model area in case of 2D and 3D simulation

Propagation of seismic waves in 3D elastic medium is described by a system of equations of the elasticity theory written in vector form:

$$\rho \frac{\partial \vec{u}}{\partial t} = [A] \vec{\sigma} + \vec{F}(t, x, y, z), \quad \frac{\partial \vec{\tau}}{\partial t} = [B] \vec{u},$$

$$A = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} & 0 \\ 0 & \frac{\partial}{\partial y} & 0 & \frac{\partial}{\partial x} & 0 & \frac{\partial}{\partial z} \\ 0 & 0 & \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \end{bmatrix}, \quad B = \begin{bmatrix} (\lambda+2\mu)\frac{\partial}{\partial x} & \lambda\frac{\partial}{\partial y} & \lambda\frac{\partial}{\partial z} \\ \lambda\frac{\partial}{\partial x} & (\lambda+2\mu)\frac{\partial}{\partial y} & \lambda\frac{\partial}{\partial z} \\ \lambda\frac{\partial}{\partial x} & \lambda\frac{\partial}{\partial y} & (\lambda+2\mu)\frac{\partial}{\partial z} \\ \mu\frac{\partial}{\partial y} & \mu\frac{\partial}{\partial x} & 0 \\ \mu\frac{\partial}{\partial z} & 0 & \mu\frac{\partial}{\partial x} \\ 0 & \mu\frac{\partial}{\partial z} & \mu\frac{\partial}{\partial y} \end{bmatrix}$$

where $\vec{u} = (U, V, W)^T$ is velocity vector and $\vec{\tau} = (\tau_{xx}, \tau_{yy}, \tau_{zz}, \tau_{xy}, \tau_{xz}, \tau_{yz})^T$ is stress vector, F is external force (depicting seismic source function). $\rho(x, y, z)$ is a density, $\lambda(x, y, z)$, $\mu(x, y, z)$ are Lamé parameters. We find solution of elasticity problem under zero initial and boundary conditions:

$$\vec{\tau}|_{t=0} = 0, \quad \vec{u}|_{t=0} = 0, \quad \tau_{xz}|_{z=0} = 0, \quad \tau_{yz}|_{z=0} = 0, \quad \tau_{zz}|_{z=0} = 0.$$

In case of 2D media, we deal with next system:

$$\begin{aligned} \rho \frac{\partial u}{\partial t} &= \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} + f_x \\ \rho \frac{\partial w}{\partial t} &= \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{zz}}{\partial z} + f_z \\ \frac{\partial \tau_{xx}}{\partial t} &= (\lambda + 2\mu) \frac{\partial u}{\partial x} + \lambda \frac{\partial w}{\partial z} \\ \frac{\partial \tau_{zz}}{\partial t} &= \lambda \frac{\partial u}{\partial x} + (\lambda + 2\mu) \frac{\partial w}{\partial z} \\ \frac{\partial \tau_{xz}}{\partial t} &= \mu \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right) \end{aligned}$$

One of the main features of research work is to simulate elastic waves propagation in 3D medium with cavern inclusion that is described with zero values of elastic parameters. So cavern can be treated as vacuum subdomain. In such case cavern boundary is treated as addition free surface. To perform calculations using supercomputers we use finite-difference numerical methods. It is well known and widely used methods that use staggered grids (Virieux, Levander). To make amplitudes of waves reflected from side and bottom boundaries of modeling area negligible we apply Perfectly Matched Layers (PML) in form of «Convolutional PML» [1]. In such case classical schema works in interior area and in PML zones works finite-difference approximations with damping coefficients.

ALGORYTHMS AND METHODS

To solve elasticity system numerically in 3D we use Virieux scheme [2]. The scheme has the second order of approximation respect to space and time. In present study we deal only with uniform grids of equal step size, therefore $\Delta x = \Delta y = \Delta z$. W component is placed on free surface ($z=0$) in the center of face, $\tau_{xx}, \tau_{yy}, \tau_{zz}$ are placed in the center of 3D cell. Indexes (i, j, k) are assigned to the center of 3D cell. The finite-difference approximations looks like:

$$\begin{aligned} \frac{\rho_{i,j,k} + \rho_{i-1,j,k}}{2} \frac{u^{n+1}_{i-\frac{1}{2},j,k} - u^n_{i-\frac{1}{2},j,k}}{\Delta t} &= \frac{(\tau^{n+\frac{1}{2}}_{xxi,j,k} - \tau^{n+\frac{1}{2}}_{xxi-1,j,k})}{\Delta x} + \frac{(\tau^{n+\frac{1}{2}}_{xyi-\frac{1}{2},j+\frac{1}{2},k} - \tau^{n+\frac{1}{2}}_{xyi-\frac{1}{2},j-\frac{1}{2},k})}{\Delta y} + \\ &+ \frac{(\tau^{n+\frac{1}{2}}_{xzi-\frac{1}{2},j,k+\frac{1}{2}} - \tau^{n+\frac{1}{2}}_{xzi-\frac{1}{2},j,k-\frac{1}{2}})}{\Delta z} + f^n_{xi,j,k}, \mu_{i-\frac{1}{2},j-\frac{1}{2},k} = \left(\frac{1}{4} \left(\frac{1}{\mu_{i,j,k}} + \frac{1}{\mu_{i-1,j,k}} + \frac{1}{\mu_{i,j-1,k}} + \frac{1}{\mu_{i-1,j-1,k}} \right) \right) \\ \frac{\tau^{n+\frac{1}{2}}_{xyi-\frac{1}{2},j-\frac{1}{2},k} - \tau^{n+\frac{1}{2}}_{xyi-\frac{1}{2},j+\frac{1}{2},k}}{\Delta t} &= \mu_{i-\frac{1}{2},j-\frac{1}{2},k} \left(\frac{u^n_{i-\frac{1}{2},j,k} - u^n_{i-\frac{1}{2},j-1,k}}{\Delta y} + \frac{v^n_{i,j-\frac{1}{2},k} - v^n_{i-1,j-\frac{1}{2},k}}{\Delta x} \right) \end{aligned}$$

As was mentioned it was interesting to perform simulation for a cavern with zero elastic parameters. It means the boundary of 3D cavern can be treated as free surface where we need to realize zero boundary conditions. As we apply finite-difference method than boundary has stair form. Elastic parameters are set in centres of each grid cell. So to recognize that we have free boundary surface inside entire elastic media we special need algorithm. To make this we compare density values for each two neighbouring grid cells in each direction of coordinate axes. For example consider vertical boundary and free surface boundary condition $\tau_{xx} = 0$ in Oxz plane, Fig.2. Finite-difference approximation

has form: $(\tau^{n+\frac{1}{2}}_{xxi,j,k} + \tau^{n+\frac{1}{2}}_{xxi-1,j,k}) / \Delta x = 0$. From this we conclude that $\tau^{n+\frac{1}{2}}_{xxi,j,k} = -\tau^{n+\frac{1}{2}}_{xxi-1,j,k}$ on the free surface of the cavity along the Ox axis. When substituted into system of equations, this will give twice the voltage value for the corresponding stress component to be calculated. Depending on the direction of the coordinate axis, zero boundary conditions are satisfied for the corresponding stress components $\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xy}, \sigma_{xz}, \sigma_{yz}$. In 2D one can use similar to 3D schema or Levander schema that is fourth order of accuracy [3].

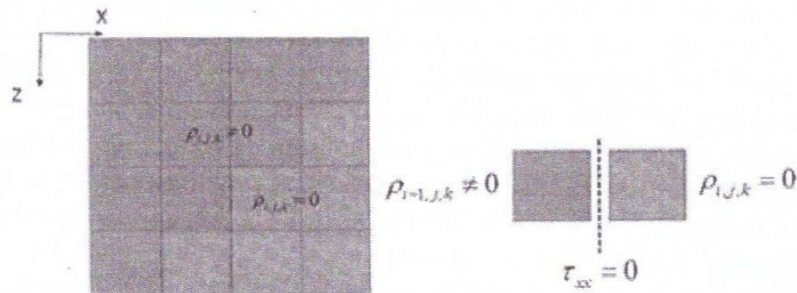


Fig.2 Realization of boundary condition on free surface of cavern in Oxz plane.

Due to the large amount of data in the case of 3D modeling it is necessary to use supercomputers to perform calculations. To enable calculations of detailed models, the computational domain is decomposed into subdomains, see Fig.3. One of the simplest methods is a one-dimensional decomposition, along the Oz axis, which was used in present work. For three-dimensional modeling of seismic wave propagation, a software implementation has been developed for calculations on clusters with MPP and SMP architectures. To implement parallel calculations on multi-core computing systems, MPI and OpenMP capabilities are used. MPI is used to organize decomposition of modeling area, to realize interconnections between neighboring sub-areas in the topology and for data exchange between computing devices. OpenMP can be used to parallelize calculations within a single multi-core computing device (cluster node). Intel Xeon Phi coprocessors can be easily used to perform simulation. Main principles of parallel realization are shown in Fig.3. Presented schema is similar for 2D and 3D simulation. In case of 3D layers are 3D subdomains. Using second order of accuracy finite-difference schema one need to organize data exchange for one layer of points. Such layers (1D in 2D case and 2D in 3D case) are organized on the boundaries of each subdomain.

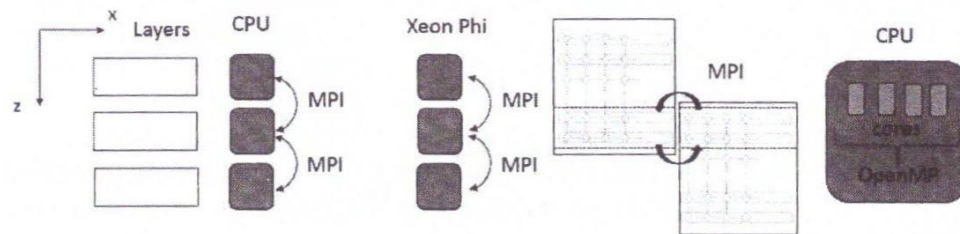


Fig.3 Parallel organization of computations

APPLICATION OF NEURAL NETWORK

Dealing with neural network we need set of pictures of seismic waves propagation for a given set of 2D elastic models. They are presented in the form of a sequence of two-dimensional colour images. Such images is used as input data for the neural network, see Fig. 4. The output data of network work should be proposed geophysical model of the media with cavern. More than 100 models with 15 calculated snapshots were used to train neural network. In such models cavity was described with non-zero values of elastic parameters. It was a part of elastic medium with values of parameters differs from values of parameters for entire area.

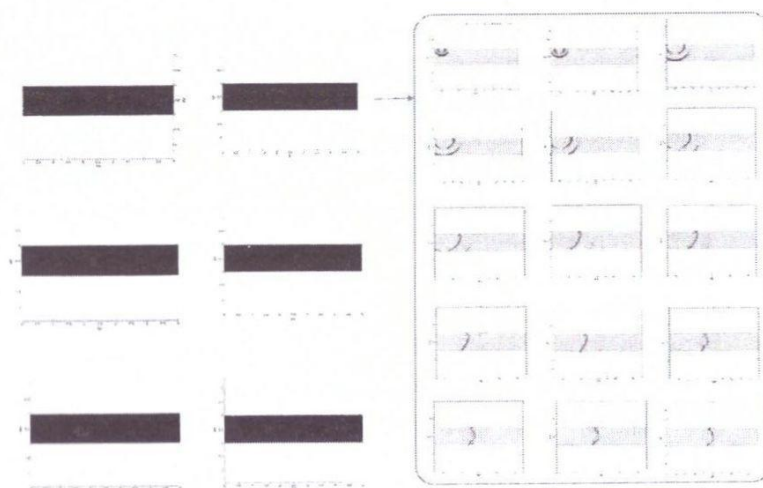


Fig.4 Input models and seismic field snapshots for one of them used to train neural network

When designing the neural network several network architectures were used that have proven themselves to solve various problems. Due to the need of image processing, it was decided to use a convolutional neural network (CNN) [4]. This type of network allows one to obtain high accuracy models when solving problems of classification and detection of objects in image [5]. Since the input data are a sequence of images, an assumption was made about the possibility of separating the dependencies that characterize the presence of a cavern. LSTM networks were used as a determinant of dependencies [6]. As a possible architecture, it is proposed to use CNN for converting input images into a numerical vector that characterizes the input data. This NN is planned to be used as «features extractor» that uniquely characterizes each picture with the help of a data vector. Further, the sequence of the resulting vectors must go to the LSTM layer to identify dependencies in them. After receiving some response from the LSTM layer, it is necessary to interpret it into an image of the environment, similar to the original. For this purpose, it is proposed to use the NN with a scan — an operation similar to convolution in CNN, however, the input vector gradually unfolds into a colour image of a given medium. To implement the «features extractor», it is proposed to train a full-convolutional neural network to repeat its own input. This network will be a convolution from the size of the input image to the size of the desired vector and scan from the shape of the vector to the size of the input image. It is assumed that NN that has trained in reproducing its input with regard to its architecture is able to most accurately describe the input image with a numerical vector in centre [7]. In process of NN training there are some problems: great number of local min and saddle points; difficult surface of loss function, speed of learning is too low. We apply «Adam» optimizing algorithm [4] to train NN. Such algorithm applies two ideas: idea of accumulation of motion to perform smoothing between steps of gradient descent algorithm; idea of weakly updating weights for typical signs. Second one means that rare, but playing a great role signs in training models have more weight than signs typical of the entire training set. The input data for the NN is a sequence of vectors formed by the «feature extractor» over several seismic field snapshots. The implementation of the NA is realized using the Python language and the Tensorflow library. To work with images we use the OpenCV library. The Tensorboard tool is used to draw graphs of loss functions.

THEORETICAL RESULTS

In case of 3D simulation by algorithm with zero valued cavity inclusion we build test model of homogeneous isotropic medium $4.0 \times 1.0 \times 1.0$ km with one cavern inclusion. The cavern is represented by a sphere with a centre in coordinates (2.0 km, 0.5 km, 0.52 km) and with radius 0.22 km, fig. 3. The cavity is represented with zero values of elastic parameters. The entire media is described with next parameter values: $V_p = 2.2$ km/s, $V_s = 1.1$ km/s, density = 2.65 km/s. 3D simulation was carried out with a point source of seismic waves with dominant frequency of 10 Hz. The source was located near the free surface with coordinates (0.3 km, 0.5 km) along the axes O_x and O_y , respectively. The simulation results in the form of wave field images for different times of seismogram calculation are presented in Fig. 5.

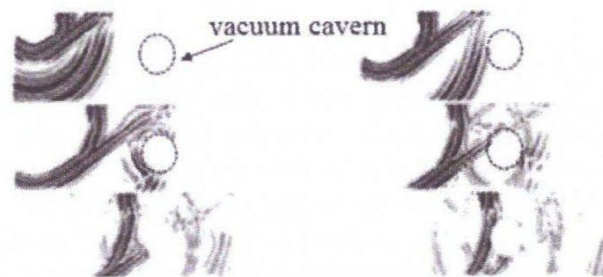


Fig.5. Results of mesh model building (top-initial data, mid-point in polygon algorithm, bottom-spline interpolation algorithm)

One can see significant changes in the elastic wave field picture due to the presence of a cavern. Different elastic wave types can be observed: reflected, refracted etc. Such results can help to study seismic field in presence of cavity and to determine its location and size. Simulation was performed on the NKS-30T cluster of Multipurpose Siberian Supercomputer Centre of SB RAS.

NN was trained on the available amount of 2D data of seismic field evolution process. According to the obtained results, it can be said that the implemented model rather accurately determines the geometric shape and location of the cavity, Fig. 6. Note that comparable results were obtained for cavities of various shapes and sizes [8].

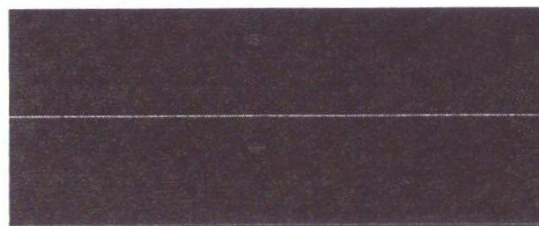


Fig.6. Results of mesh model reconstruction (top - input model, bottom- model reconstructed by the NN)

CONCLUSIONS

The main problem of proposed study is to define shape, size and location of cavern placed in isotropic elastic media using results of finite-difference simulation and neural network training. First task was to perform simulation of 3D elastic waves propagation in elastic media with 3D cavern described by vacuum object. Such approach was

realized by zero valued boundary conditions on the surface boundary of 3D cavern. Second task was having series of 2D results (snapshots of full seismic field and elastic model) of finite-difference simulation to train neural network to recognize cavern. This approach for solving a set of forward geophysical problems and restoration of the structure of a geophysical model is described in the paper. We developed programs for building mesh models: specifying the geometrical structure of elastic media and determining the distribution of values of elastic parameters into mesh. The applicability of developed software and algorithms for calculating various models of elastic media is shown. The results of parallel algorithms and programs work in the form of seismic field snapshots are presented. The developed neural network was successfully trained. The possibility of using neural networks to solve the problem of determining the position and size of cavernous inclusions for the 2D case is demonstrated.

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