

# Optimization Algorithm for Seismic Location of Underground Objects

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**Abstract**—The paper considers the possibility of using the seismic event localization algorithm for localizing objects of the geophysical model of the environment. When adapting this algorithm the localization algorithm for point objects and the localization algorithm of objects of a certain form are proposed.

**Keywords**—localization algorithm, objects of the geophysical model of the environment, seismic

## I. INTRODUCTION

One of the important problems of the present-day geophysics is the solution of the inverse problem of geophysics. For its solution there is a lot of difficulties. First of all, it is a model parameterization and comparison of observations with theoretically calculated field characteristics for this model. It is also necessary to choose a criterion to which the solution will be submitted for an infinite set of solutions to the problem and a narrowing of the solution region in the case of instability of the solution of the inverse problem. Localization of underground objects and seismic events refers to a particular solution of the inverse problem of geophysics. There are many approaches to the search for underground objects depending on the methods of studying the Earth's crust, the chosen model of the environment, etc. Each of these approaches has a number of physical and computational limitations that narrow the range of its application. Thus, effective methods for the localization of geophysical objects are an important task of the present-day geophysics.

## II. THEORY

### A. Localization algorithm for point objects

In a simplified form the object of study can be taken as a point. In this case, it is necessary to take into account that the real size of the object must be at least twice the wavelength of the seismic vibrator. When the geophysical environment is probed by a seismic vibrator, the reflection of the generated wave from a point object at the initial moment of time will be considered the source of a seismic event. In this case, the algorithm for localizing seismic events can be applied to this task by the difference in the S-wave and P-wave arrival on one receiver and from the ratio of the difference between the P-wave arrival times on two neighboring receivers [1].

The first variant of the localization of a point object assumes the use of information about the difference in the arrival of S- and P-waves on each receiver:

$$Ts_i - Tp_i = L_i \cdot (1/V_s - 1/V_p); \quad L_i = \sqrt{(x_i - x)^2 + z^2}; \quad (1)$$

where  $Ts_i$  and  $Tp_i$  are the arrival times of S- and P-waves on the  $i$ -th receiver,  $i = 1, 2, \dots, n$  is the index of the corresponding receiver,  $L_i$  is the distance to the object,  $V_s$  and  $V_p$  are the propagation velocities of S- and P-waves, respectively;  $x_i$  are the coordinates of the  $i$ -th receiver,  $x, z$  are the coordinates of the object in the Cartesian coordinate system, the  $x$  axis is directed along the surface of the earth, and the  $z$  axis is down toward the center of the earth.

The second variant of determining the position of a point object involves the use of information only about the difference in the arrival of P-waves on two neighboring receivers:

$$Tp_{i+1} - Tp_i = \frac{1}{V_p} \cdot L_{p_{i+1}} - \frac{1}{V_p} \cdot L_{p_i}, \quad (2)$$

where  $Tp_{i+1}$  and  $Tp_i$  are the P-wave arrival times at two neighboring receivers,  $i = 1, 2, \dots, n$  is the index of the corresponding receiver,  $L_{p_{i+1}}$  and  $L_{p_i}$  is the distance to the object from two neighboring receivers.

Here, the P-wave distances are determined similarly to equation 1:

$$L_{p_{i+1}} = \sqrt{(x_{i+1} - x)^2 + z^2}; \quad L_{p_i} = \sqrt{(x_i - x)^2 + z^2}. \quad (3)$$

In both approaches, the required quantities are the coordinates of the object  $x$  and  $z$ . The propagation velocities of elastic waves are assumed to be known, and the time of entry of these waves to the receivers is determined from the seismograms. This task relates to the type of inverse problems on the problem of comparison of theoretical and observed data, because there are many parameters, but observations contain errors. Therefore, in both cases, it is advisable to reduce the task of finding the coordinates of an object to the least squares method.

Let  $m$  be the residual functions  $\mathbf{r} = (r_1, \dots, r_m)$  of  $n$  unknown variables (parameters)  $\boldsymbol{\beta} = (\beta_1, \dots, \beta_n)$ , and  $\mathbf{X} = (x_1, \dots, x_N)$  be the matrix of coordinates of receivers, where  $N$  is the number of receivers.  $f(x_i, \beta_j)$  is a set of functions from this set of variables;  $y_i$  is some values to which the corresponding values of functions are as close as possible.

Then the least squares method can be expressed as follows [2]:

$$F = \sum_i r_i^2(\beta_j) = \sum_i (y_i - f(x_i, \beta_j))^2 \rightarrow \min_{\beta} \quad (4)$$

In accordance with expressions 1–3, the residual functions  $r_i(\beta_j)$  have a non-linear parametric dependence. To find the minimum of the functional  $F$  in this case, the Gauss-Newton method and its modifications [3], the singular decomposition [3, 4], the Kacmage method [5], etc. are used.

### B. Localization algorithm of objects of a certain form

In order to take into account the shape of the object, we represent the object as a set of point objects. In this case it is necessary to impose a grid on the entire investigated area of the environment and check each grid node (Fig. 1). One grid node corresponds to a time window on the first receiver seismic trace. The window size is selected based on the requirement for the accuracy of the object definition and additional analysis of the seismogram. If there is a wave in one window with an amplitude exceeding the total noise level by a certain amount or more (the value of this amount is set based on the type of seismic trace obtained), then the possibility of a point object is recorded in this window. Further, the presence of this point object is checked on the remaining seismic traces. In the case of complete coincidence of the presence of a signal on all receivers, we assume that there is a point object in this grid node (Fig. 2). Thus checking each node of the grid and marking the nodes with the expected objects, we get a picture of the points of the object, i.e. we receive object of a certain form.

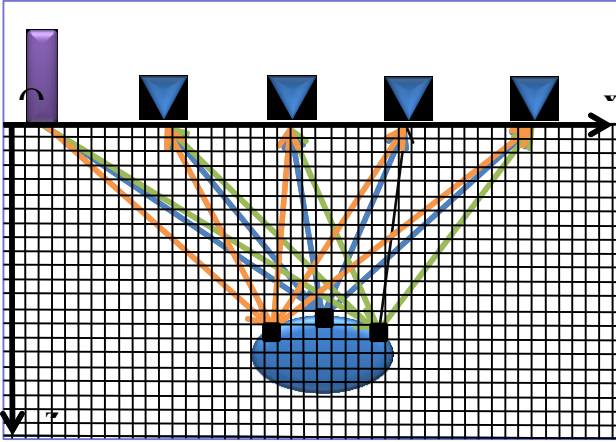


Fig. 1. Localization algorithm of objects of a certain form.

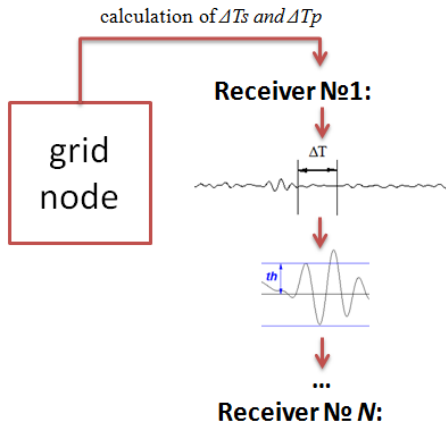


Fig. 2. Algorithm operation circuit for one grid node.

The calculations at each node of the grid are independent of each other, so a parallel approach can be applied to this task. To implement a parallel approach, two technologies will be considered: OpenMP + MPI and CUDA + MPI with schemes similar to those in [6]. In both cases, the computational domain is divided into layers along the direction of one of the coordinate axes. Inside each layer, parallel computations are performed using OpenMP or CUDA. The entire exchange of information between adjacent layers is performed using MPI technology.

### III. RESULTS AND DISCUSSION

The MATLAB application package was selected as the development environment. In the numerical experiment, a two-dimensional, homogeneous, isotropic model of the environment was used with elastic wave velocities:  $V_s = 1.4$  km/s and  $V_p = 2.0$  km/s and one point object at different distances from the receivers. The error of the time of entry of S- and P-waves on the receivers does not exceed 1 ms. The frequency of the seismic vibrator is 20 Hz.

For both approaches (for the difference in the arrival of S- and P-waves on one receiver and on the difference in the arrival of P-waves for two neighboring receivers) at known velocities of elastic waves, the same accuracy was shown by both the Gauss-Newton method and the Levenberg-Marquardt method (tab. I and II). In this case, it is preferable to choose the Gauss-Newton method, since it does not require the selection of a regularization parameter at each iteration step. And for a problem with three and four unknown parameters, it is preferable to use the Levenberg-Marquardt method (tab. III). However, in this case it does not give high accuracy and is quite sensitive to the choice of each step of the iterative process.

TABLE I. RESULTS FOR THE APPROACH BASED ON THE DIFFERENCE IN THE ARRIVAL OF S- AND P-WAVES ON ONE RECEIVER

The real position of the object: $x^* [km]; z^* [km]$	The calculated position of the object: $x \pm \Delta x [km]; z \pm \Delta z [km]$	The minimum value of the objective function, $F [s^2]$
0.5; 1.0	0.5±0.002; 1.0±0.000	1.531·10 <sup>-6</sup>
0.5; 10	0.5±0.01; 10.0±0.001	2.581·10 <sup>-6</sup>
2.0; 1.0	2.0±0.0; 1.0±0.001	2.394·10 <sup>-6</sup>
2.0; 10.0	2.0±0.0; 10.0±0.002	2.615·10 <sup>-6</sup>
3.0; 1.0	3.0±0.001; 1.0±0.001	1.725·10 <sup>-6</sup>
3.0; 10.0	3.0±0.003; 10.0±0.0	1.245·10 <sup>-6</sup>

TABLE II. RESULTS FOR THE APPROACH BASED ON THE DIFFERENCE IN THE ARRIVAL OF P-WAVES TO TWO NEIGHBORING RECEIVERS

The real position of the object: $x^* [km]; z^* [km]$	The calculated position of the object: $x \pm \Delta x [km]; z \pm \Delta z [km]$	The minimum value of the objective function, $F [s^2]$
0.5; 1.0	0.5±0.0; 1.0±0.0	3.751·10 <sup>-6</sup>
0.5; 10	0.5±0.013; 10.0±0.015	6.223·10 <sup>-6</sup>
2.0; 1.0	2.0±0.001; 1.0±0.001	4.947·10 <sup>-6</sup>
2.0; 10.0	2.0±0.008; 10.0±0.145	1.306·10 <sup>-6</sup>
3.0; 1.0	3.0±0.273; 1.0±0.003	6.917·10 <sup>-3</sup>
3.0; 10.0	the method does not converge	—

TABLE III. THE GAUSS-NEWTON METHOD AND THE LEVENBERG-MARQUARDT METHOD FOR THE CASE OF UNKNOWN PARAMETERS X, Z, VS AND VP

The real position of the object: $x^*=2.0$ km; $z^*=1.0$ km Initial approximation: $x^{(0)}=2.2$ km, $z^{(0)}=1.2$ km, $V_s^{(0)}=1.47$ km/s, $V_p^{(0)}=2.1$ km/s			
The Gauss-Newton method		The Levenberg-Marquardt method	
The calculated position of the object: $x \pm \Delta x$ [km]; $z \pm \Delta z$ [km]	The minimum value of the objective function, F [s <sup>2</sup> ]	The calculated position of the object: $x \pm \Delta x$ [km]; $z \pm \Delta z$ [km]	The minimum value of the objective function, F [s <sup>2</sup> ]
<i>For the approach based on the difference in the S-wave and P-wave arrival on one receiver</i>			
the method does not converge	—	2.0±0.072; 1.0±0.072; 1.4±0.058; 2.0±0.028	0.016
<i>For the approach based on the ratio of the difference between the P-wave arrival times on two neighboring receivers</i>			
2.0±0.222; 1.0±0.115; — 2.0±0.326	7.213·10 <sup>-3</sup>	2.0±0.034; 1.0±0.034; — 2.0±0.134	1.243·10 <sup>-3</sup>

From Tables I-III it can be seen that for some point objects for the approach based on the difference in P-wave arrival on two neighboring receivers, neither the Gauss-Newton method, nor the Levenberg-Marquardt method can calculate the position of the object. This may be due to the fact that the difference in the P-wave entry times on two neighboring receivers may be commensurate with the error in determining the P-wave entry on the receiver. In such cases, it is possible to carry out an experiment when P-waves enter the receivers that are at a greater distance from each other.

#### IV. CONCLUSION

This paper shows the applying features of the seismic event localization algorithm to the localization of objects of

the geophysical model of the environment. The possibilities of using numerical methods for approaches based on the use of information about the difference in the arrival of S- and P-waves on one receiver and on the difference in the arrival of S- and P-waves on two neighboring receivers are given. An approach to defining objects of a particular form as a set of point objects is proposed, and the possibility of applying a parallel approach to this task is described.

To solve the set tasks, the algorithm of localization of point objects of the geophysical model of the environment was developed, implemented in the MATLAB environment and tested on model data. A comparative analysis of the applied numerical methods for this algorithm is given.

#### ACKNOWLEDGMENT

This work was supported by the RFBR grant No. 19-07-00170.

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