Algorithm for Fast Evaluation of Tsunami Danger for Near Field Event

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Abstract **- In this paper we discuss two algorithms designed to reduce the time required for tsunami danger evaluation. First one makes it possible to provide a realistic approximation of tsunami source parameters provided that the sensors (bottom pressure recorders) are properly located. It is based on minimizing the difference between actual tsunami record and the linear combination of synthetic mareograms preliminary obtained by numerical modeling of tsunami generated by so-called unit sources. Only a part of the wave first period is used. This can help to restore the tsunami source few minutes earlier than using full wave length. Wave propagation through the water area under study is simulated at personal computer with the hardware acceleration. Specialized printed board with the Field Programmable Gates Array is exploited. The calculation is accelerated by using a computing pipeline implemented on several calculators available on the mentioned above printed board. In total, both algorithms require about two minutes to deliver wave heights maxima distribution along the coast.**

Keywords—tsunami wave simulation, orthogonal decomposition, computer code acceleration

I. Introduction

After the Great Tohoku Earthquake of March 11, 2011 offshore Japan, it become clear that tsunami warning systems leave much to be desired. For the local tsunami takes nearly 20 min for propagation from the source to the nearest coast. Modern software packages, like MOST (Method of Splitting Tsunamis, NOAA Pacific Marine Environmental Laboratory, Seattle, WA USA) [1,2] and TUNAMI-N1/TUNAMI-N2 (Tohoku University, Japan) [3,4], provide high accuracy simulation of tsunami wave generation, propagation and inundation. However, they both are not able to deliver analysis in a few minutes even using the supercomputer resources. It should be also noted that in case of disaster events, compared to the one of March 11, 2011, electric power shutdown is possible. At the same time, for evaluation of tsunami wave danger one does not need to know the exact wave amplitude at a particular populated coastal area or industrial site. Difference between 4.5-5.5 m and 19-21 m waves regarding expected casualties and economic loss is obvious. Approximation of the key wave parameters within a reasonably small error is acceptable.

So, the desired tsunami warning system must: (1) work very fast (within several minutes), (2) be independent from power supply (in case of emergency) and (3) provide realistic approximation of tsunami wave height (10% accuracy or even a bit more, is certainly acceptable).

In order to use the well-developed technique of computer (mathematical) simulation, we need to know the basic parameters of tsunami source. Seismic data provides coordinates of the earthquake hypocenter (and epicenter) along with its magnitude – integrated characteristic of energy release. This is not enough to know both the area (shape) of sea bed displacement at tsunami source and amplitude of displacement.

There are at least two approaches to evaluate the initial sea bed (or sea surface) displacement at tsunami source. First - direct mathematical modeling based on: (1) knowledge of geological structure of hypocenter area, or (2) comparison with historical event of the similar parameters (epicenter location and magnitude), or (3) GPS data analysis. The other methods are based on direct measurement of the wave at sea surface. It could be done using sea radar, satellite altimetry, wave profile measurement by seabed-based hydrostatic pressure recorders and so on. Each of these approaches has strong and weak points, but all provide an approximation of the desired parameters. Without going into details, we would like to use direct measurement of the wave profile, passing over pressure sensor, like cable network DONET (Dense Oceanfloor Network System for Earthquakes and Tsunamis, Japan) [5] or DART (Deep-ocean Assessment and Reporting of Tsunamis, USA) buoys [6] which is available online.

So, in order to evaluate tsunami wave heights along the coast one needs the following: (1) to measure the profile of moving wave; (2) to recalculate these measurements in terms of tsunami parameters at source; (3) to calculate wave propagation from source to coast.

Using several approaches, described below, it is possible to deliver the computed (based on approximation of the initial displacement at source) the distribution of tsunami height maxima along the coast in 1-2 min after the measured data of the wave profile at single point is available.

Rest of the paper is as follows. First of all, we provide the basic information about existing systems to measure wave profile. These are floating buoys and sensors along sea bed cables. Then the concept of measurement in advance is introduced. It is based on the so-called Unit Sources (UnSs), designed to simulate tsunami from typical sea bed disturbance. A set of such sources covers most of subduction zones worldwide. Such approach makes it possible to reduce the ill-posed problem of 2D function reconstruction (using just one 1D measurement of wave profile) to simpler one – identification of several amplification coefficients, associated with the corresponding UnSs. Description of the algorithm (based on Fourier series theory), which is able to pick up the desired coefficients within a few seconds, follows. Architecture of the specialized Calculator (hardware acceleration of code execution), designed for numerical solution of the shallow water system, is then introduced. Results of numerical tests are given, too. Finally, the obtained results are briefly discussed.

II. Description of Approaches

A. *Wave Profile Measurement*

There exists well developed system of the so-called DART buoys [6] around the Pacific Ocean. Image of the corresponding interactive map at NDBC is given in Fig. 1.

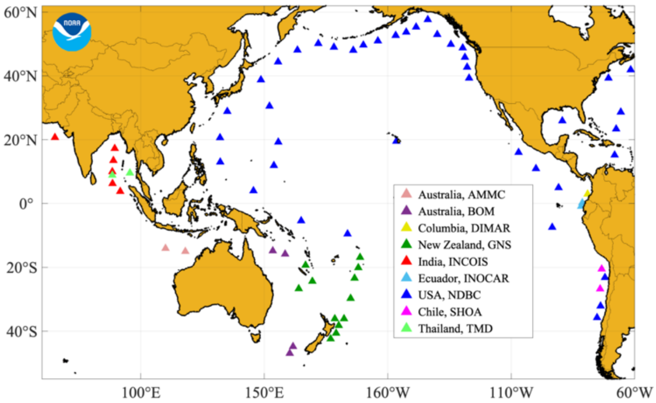

Fig. 1. Location of Dart buoys around the Pacific Ocean [6].

Measurements of the wave amplitude, passing over the bottom based pressure sensor, are available in a real time mode via satellite channels. In case of DONET and S-net sensor systems, developed in Japan, pressure sensors are connected with the data processing centers via bottom cables [5,7].

Suppose that the wave will approach the nearest sensor in ten minutes after the earthquake for any event in the considered subduction zone. It is possible to calculate the number and position(s) of additional sensors for the given subduction zone to achieve this time limitation.

B. *Source Parameters Identification*

It is not a problem to simulate tsunami wave propagation having the known initial field of sea surface disturbance at tsunami source. We believe that reasonably good approximation could be obtained using only one measurement of the wave profile. Moreover, numerical experiments show that just a part of the first wave period in enough.

Note that in general it is not possible to reconstruct correctly the 2D function (initial sea surface disturbance) using 1D function (measured wave profile at one point). Such an inverse problem is called “Ill-Posed” [8]. However, the problem could be correct (there exists a unique solution, continuously dependent on the input data) if we are solving it within a special class of functions [8].

Such class of function have been proposed [2, 9]. Let us describe briefly this approach. Based on historical records and geological information about the Earth crust structure of the given subduction zone, the entire area of possible earthquakes epicenters is covered by the so-called Unit Sources (UnSs). These are 50x100 km rectangles – typical size of sea bed disturbance area for M=7.5 magnitude earthquake. As a rule, for weaker events, no dangerous tsunami happens. Then we place initial displacement of the form, typical in the considered zone, to each UnS and simulate artificial tsunami wave propagation over the entire water area under interest.

In case of real seismic event, we approximate the measured wave profile, obtained at one of the sensors, as a linear combination of artificial wave signals (from all related UnSs), computed at the same point. In that way we have reduced the problem of 2D function reconstruction to much easier one (at least in terms of necessary computations) – to determine several coefficients of the above-mentioned linear combination of artificial waves at one given point.

There are a system of 1982 UnSs all over the world, 1338 of which are located around the Pacific Ocean [10], Fig. 2.

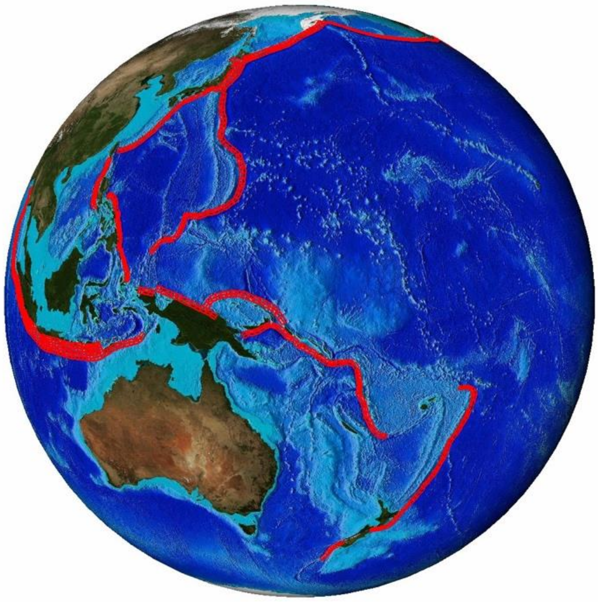

Fig. 2. Location of the Unit Sources (UnSs) for the Eastern Pacific area [10].

Computationally low-cost algorithm to obtain these coefficients by a part of the wave profile was proposed and numerically tested [11]. It is based on well-developed Fourier series theory.

Let $f(t)$ be the measured time series of the wave profile (marigram), obtained at any particular sensor (DART buoy, bottom cable sensor of DONET, or other). We will use notations $f_k(t)$, $k=1,\ldots,n$, for simulated wave profiles, calculated (according to the linear or nonlinear shallow water approximation [12]) at the same point of sensor location, by direct numerical modeling of the tsunami wave propagation. It is understood, that the subindex k refers to a tsunami wave initiated by the normalized sea surface initial profile of a given shape from the *k*-th UnS. We assume that all these functions, $f_k(t)$, are linearly independent. We are looking for coefficients, b_k, for the linear combination of these functions, which provide the best approximation of the function $f(t)$ in L_2 norm:

$$\int_0^T \left(\sum_{k=1}^{N} b_k f_k(t) - f(t) \right)^2 dt \to \min \quad (1)$$

So, we consider the problem of optimal approximation of a given function, *f(t)*, by the linear combination of the finite subset of functions from the system $\{f_k(t)\}$. Suppose that the system is orthogonal and normalized according to:

$$(f_i(t), f_j(t)) = \int_{t_0}^{T} f_i(t) f_j(t) dt = 0, \quad (2)$$

$$(f_i(t), f_i(t)) = 1. \quad (3)$$

As is well known in the Fourier series theory, the coefficients of such optimal approximation are nothing but the Fourier coefficients of expansion of *f(t)* in a series with respect to $\{f_k(t)\}$ (see, for example, [13]).

Based on the statement above, the following algorithm was proposed in [14] and tested in [15] for searching the coefficients of the linear combination allowing optimal approximation of the given function. It includes three steps.

First, the calculated system of "marigrams", $\{f_k(t)\}$, from the unit sources should be orthogonalized and normalized, according to equations (2)-(3).

Second, the measured marigram, *f(t)*, should be expanded to the Fourier series with respect to obtained orthonormal system of functions.

Finally, the so-determined Fourier coefficients should be recalculated in terms of initial functions $\{f_k(t)\}$.

Note that the described algorithm has a very low computational cost as mostly simple algebraic operations are involved. We consider calculation of integrals (scalar products and L_2 norms as simple operations, too.) During the performed numerical experiments it takes less than a second using regular personal computer.

C. Calculating Wave Propagation

Having the approximation of the initial sea surface displacement we now need to calculate wave propagation from source to coast line.

A version of nonlinear shallow water model was used for numerical tests. External forces (like Coriolis force and bottom friction) were neglected, except of the gravity [12]:

$$\frac{\partial H}{\partial t} + \frac{\partial (uH)}{\partial x} + \frac{\partial (vH)}{\partial y} = 0$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + g\frac{\partial H}{\partial x} = g\frac{\partial D}{\partial x} \quad (1)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + g\frac{\partial H}{\partial y} = g\frac{\partial D}{\partial y}$$

where $H(x,y,t)=\eta(x,y,t)+D(x,y)$ is the total height of water column, η being the sea surface disturbance (wave height), *D(x,y)* – depth (which is supposed to be known at all grid points), *u(x,y,t)* and *v(x,y,t)* are components of water flow velocity vector, *g* - acceleration of the gravity. Note that this approximation is widely used for tsunami simulation [2, 16].

Parallel version of the McCormack finite-difference scheme (which has second order approximation) was implemented on the FPGA platform [17]. Computation pipeline is based on the designed Processor Elements (PEs). One PE represents the implementation of one step of the algorithm. PEs are implemented using High-Level Synthesis (HLS) technology [18], which makes it possible to accelerate significantly the development process. A stream of values *H*, *u*, *v*, *D* at the *i*-th step comes to the PE input, which is a sequential traversal of the computational stack. The output is the same stream, with values at time step *i+1*. The PE operates in pipeline mode and allows to process one grid point per clock cycle. Connecting PE output with input of another PE it is possible to organize chains (pipelines) of different length that allow to calculate several algorithm time-steps simultaneously, as the results from the previous element arrive. The maximum length of the pipeline depends only on the capabilities (capacity) of the FPGA chip. In addition, each PE needs to store 3 lines of values in the internal FPGA memory, which imposes limitations on the maximum size (width) of the computation grid and on the parameters required from the FPGA.

The best results were achieved at ZCU106 platform. This is a standalone system based on the Zynq Ultrascale+ system-on-chip (SoC). The proposed architecture is given in Fig. 3. The SoC is a 4-core ARM processor (Cortex-A53) combined with an FPGA. The advantage of this solution is autonomy and high integration.

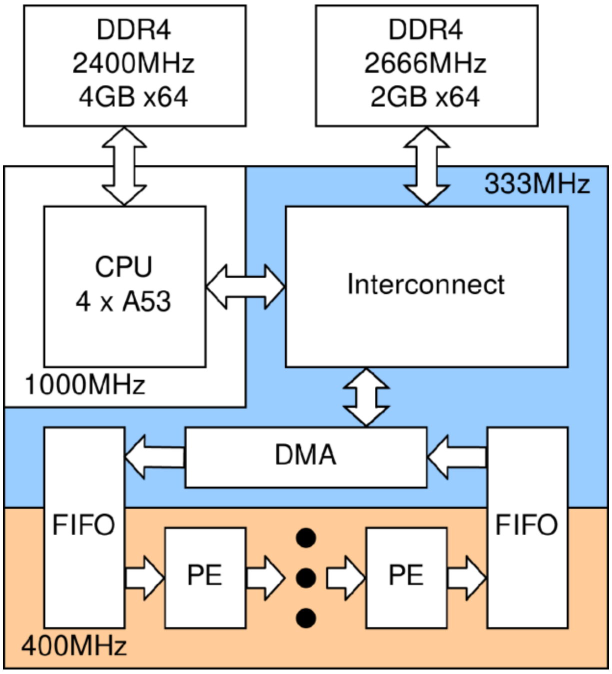

Fig. 3. Xilinx ZCU106-based implementation architecture [17].

To demonstrate the performance of the proposed approach to coastal tsunami hazard assessment, numerical calculations of tsunami wave propagation (initiated by the Great Tohoku Earthquake of March 11, 2011) were carried out. The reconstruction of the tsunami source (the field of initial vertical displacement of the water surface) proposed in [19] was used for numerical modeling. The area near the northeastern coast of Honshu Island (Japan), bounded by latitudes 36°N, 42°N and longitudes 140° and 146°E was selected for numerical modeling. A digital bathymetry array of 2401x2401 with spatial steps in both directions equal to 0.00248 arc degrees (214 m in the West-East direction and 276 m in the South-North direction) was constructed based on the JODC bathymetric data [20]. The bottom relief with reference to geographic coordinates is shown in Fig 4.

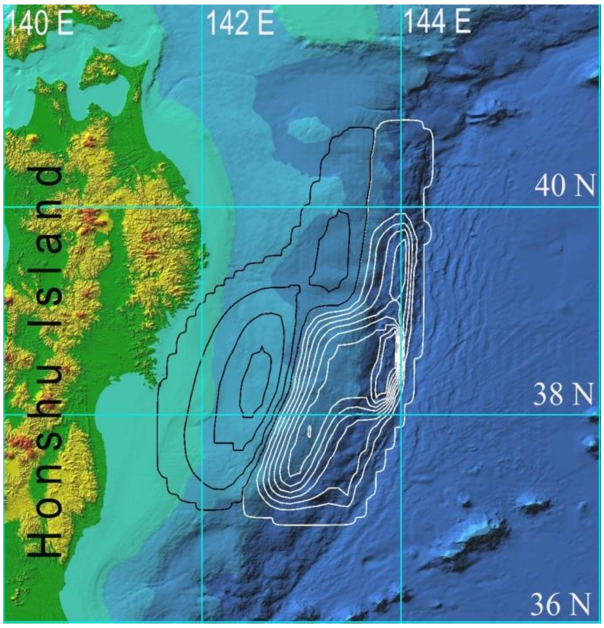

Fig. 4. Digital bathymetry and tsunami source of 11.03.2011 event. The isolines of the field of vertical elevation of the water surface with an interval of 1 m are drawn in white, and the isolines of the initial decrease of water level are drawn in black. The source is outlined by ±0.05 m level lines. The maximum elevation in source is limited by +9 m and the lowest value is equal to -4 m.

The first computational experiment simulated the propagation of tsunami wave of 11.03.2011, generated by the initial displacement of the water surface (Fig. 4) coinciding with the displacement of the ocean floor given in [19]. In the course of the numerical calculation, the maximum water surface level was fixed at each node of the computational grid for the entire simulation time. A 3D visualization of the tsunami height maximum distribution over the entire computational domain is shown in Fig. 5.

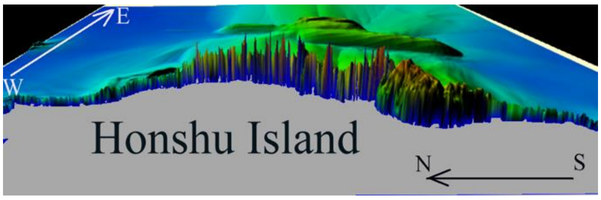

Fig. 5. 3D visualization of the distribution of tsunami height maxima along the coast of Honshu Island after 6000-second calculation of wave propagation.

It takes takes only 38.4 sec to simulate 1 hour (7,200 time steps) of the wave propagation for the considered grid. Fig. 5 demonstrates that the distribution of maximum wave heights (up to 22.4 m) along the coast has sharp spikes. These are caused by the presence of numerous harbors and capes at Sanriku coast (Fig. 4).

Note that results of numerical experiments are similar to those from [21], where the same initial sea-bed displacement was used.

III. Discussion

In order to support timely the decision making about evacuation measures in case of the near field event, several improvements have been proposed. First, it is possible to achieve rather fast (say, in nearly 10 min after quake) registration of tsunami wave at least on one sensor. Just a few additional sensors are needed to adjust the existing ones (DART buoys, DONET, S-net, and other). The proposed calculation in advance strategy provides approximation of the initial sea surface displacement at tsunami source within a few seconds. In some cases, nearly a quarter of the first wave profile, obtained at one sensor, is enough. Using the hardware acceleration (FPGA-based Calculator), it takes about 1 min to calculate wave propagation over several hundred kilometers wide water area. Resources of personal computer are needed, which could be regarded independent from electric power supply. In total, it is expected to obtain approximation of the expected tsunami wave maximal heights distribution along the coast within 11-12 min.

We do believe that in a few years tsunami warning systems will be able to save human life and reduce economic loss even in cases of strong offshore earthquakes.

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