The Parallel Implementation of the Adaptive Mesh Technology in Poroelasticity Problems

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Abstract. The actual process of oil and gas field development is associated with a large amount of numerical modeling. This is due to unreliable data used in modeling. For example, these are a small amount of reliable measurement information on a geological structure, reservoir and geomechanical properties of rocks forming a given field. There is a need to solve a large number of inverse problems to determine the structure and properties that satisfy the whole set of reliable measurement results, taking into account the interinfluence of physical processes occurring in the course of development.

The poroelasticity problem in question is of essential practical interest, when a value of the pore pressure is affected by the stress-strain state of a reservoir, depending on the same pressure. The process of solving the inverse problem is associated with solving a large number of direct problems, while a major challenge is in reducing the calculation time of each direct problem. A large amount of computation requires the usage of supercomputer technologies.

This paper discusses the adaptive mesh usage for building hydro-geomechanical proxy models and solving poroelasticity problems with an effective strategy for adapting the computational grid for parallelization. Parallelization is performed with the computing cluster of the Siberian Supercomputer Center.

Keywords: Proxy model · Poroelasticity · Adaptive mesh · Fluid filtration · Flow simulation · Geomechanical simulation

1 Introduction

Nowadays the numerical modeling of oil & gas reservoirs takes a big part of the whole reservoir development process. In particular, models are used by geologists to build a layered structure, reflecting a real reservoir to a certain amount. Every layer consists of relatively small cells extending tens of kilometers across the reservoir area. The total amount of cells may reach several millions. Every cell has certain properties such as permeability, porosity, saturation, etc. The accuracy of such models strictly depends on assumptions attracted. In particular, there is no unique technique to distribute properties among the cells located between the wells. The problem is the lack of measurement techniques capable of investigating media far beyond the radius of a borehole. In other

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words, one needs to attract the tools of geostatistics such as kriging or variograms. On the other hand, seismic models are only the tools to take an image of the ground structure between wells, but the vertical accuracy of a layer location may take up to 50 m, while the thickness of this layer may be about tens of centimeters or even smaller. Nevertheless, a geological model is the basis for building other models such as hydrodynamic models or geomechanical models. It is clear that the accuracy of such models cannot be higher than geological model. Moreover, due to a huge amount of cells, the time required to calculate a change in the reservoir pressure, across the whole reservoir or part of it, caused by the production of wells, become huge as well. Taking into account relatively a small significance of every particular calculation, the engineers need a tool capable of estimating a change in the reservoir characteristics (pressure, temperature, stress, strain, etc.) in a matter of minutes or even seconds for each particular calculation. By combining the results of several runs of slightly different models (different assumptions) one can estimate required characteristics with certain probability. This workflow has become a standard in the industries.

The frontier of modeling techniques is now shifting to the so-called proxy models [1]. These models are based on the reduced amount of cells using the so-called upscaling of the cells [2]. Nevertheless the accuracy of upscaled models may become too poor in some regions.

Even the most popular commercial software uses different tricks to overcome the problem of a low accuracy. In particular, the inflow into a certain well is calculated by sewing a numerical solution in far field zone (where the error between the numerical and the exact solutions is small) and the analytical solution near the borehole [3]. Such technique helps one to use relatively simple finite difference models (FDM) to calculate the pressure across a reservoir while blocks with the wells are treated separately.

Another problem comes to the foreground when one tries to estimate strains that are close to geological faults or fractures. In this case, a stress field may change dramatically leading to the risks of a wellbore instability as well as an unpredictable behavior of hydraulic fractures.

It is important to mention that a change in a stress filed may cause a change in the pore pressure and vice versa. This effect is known as poroelasticity. In other words, building separate models for hydrodynamics and geomechanics increases the risks to get negative impact in the development process or even in people.

A finally, it is worth to mention that there is no universal tool to solve poroelastic problems of sufficiently a large size. To efficiently solve the problems under consideration, it is necessary to choose such an approach to constructing numerical solutions, in which a computational grid would have the smallest size (by the number of grid nodes), and the accuracy of calculations would be maximum possible.

The commercial software tries to solve a problem of an accuracy by increasing the number of cells in the zones of a rapid change of a certain field (pressure, temperature etc.). At the same time, the total amount of cells is changed for certain problems, as well as the number of equations needed for a solution. It is difficult to find an optimum solution in terms of a number of cores in a supercomputer, as well as a memory volume required.

Of great practical interest are such algorithms of building a grid for which the total number of elements would remain constant. A constant number of elements allows one

uniform loading of computational nodes in the course of parallel computing. The main difficulty here is to preserve the numbering of grids (nodes and elements): if all the nodes and grid elements, after a cycle of mesh adaptation retain their numbers, it is relatively easy to organize a uniform breakdown of a single computational domain into subdomains, thereby ensuring a uniform loading of cluster computing nodes. It is obvious that one can save the numbering of the grid only if the reduction in the size of some elements occurs due to an increase in the size of others. In a finite element method, such an approach is called r-adaptation technique. Moreover, in the case of the usage of high order polynomials for the shape functions, the hybrid technique can be called as rp-adaptaion.

Currently, there are relatively a few publications on the application of the rp-adaptation technique for solving problems of the oil and gas hydrodynamics, and in particular, problems of estimating the production of hydrocarbons. One of the main reasons for a low popularity of the method lies in the well-established methodology for the distribution of environmental properties in terms of computational volume. Standard grid algorithms with an increase in the total number of computational elements (nodes) and an analytical solution in the near-wellbore area mentioned above are already embedded to commercial simulators. With a simple increase in the number of elements, the error of the final result will only decrease if we assume that the initial distribution of properties on a coarse grid is close to a real one.

The idea of this research is to present a technique that is appropriate for solving large poroelastic problems with the help of a supercomputer with an effective strategy for adapting computational grid for parallelization.

2 Some Aspects of Using Adaptive Grids

Let us note that at any time step, an adaptive grid can be considered as an adaptive fixed grid with a non-uniform distribution of nodes. In the theory of the finite element method, it is proved that reducing the size of elements leads to an increase in the accuracy of the numerical solution. Based on this fact, we can conclude that the preservation of nodes of the original grid and the n-fold addition of intermediate nodes will not worsen the accuracy of a numerical solution.

Figure 1 shows the solution of the problem of a fluid inflow into the well with different step sizes of the computational grid and at different time instants. In this case, the one-dimensional problem of the non-stationary filtration of reservoir fluids through a non-deformable reservoir with specified properties has been solved. The direct problem has been solved by the finite element method, for which the so-called weak formulation of the boundary value problem has been obtained under given initial conditions.

$$\int_V \frac{\varphi \rho_f}{\mathrm{K}} \frac{\partial p}{\partial t} \delta p \, dV + \int_V \rho_f \frac{k}{\mu} \nabla p \cdot \nabla \delta p \, \mathrm{d}V = \int_{A^m} m \delta p \, \mathrm{d}A, \tag{1}$$

where φ is the porosity, ρ_f is the fluid density, K is the compressibility of a reservoir fluid, k is the reservoir permeability, μ is the viscosity of the reservoir fluid, p is the

desired pressure, m is the mass flow through the part A^m of the outer boundary A. From the analysis of the solution shown in Fig. 1, we can conclude that even with constant porosity and permeability properties for arbitrary finite element sizes, a computational error can be sufficiently significant (it is enough to estimate the error using the Euclidean L2 norm for solutions on a grid with a constant step and on an adaptive grid).

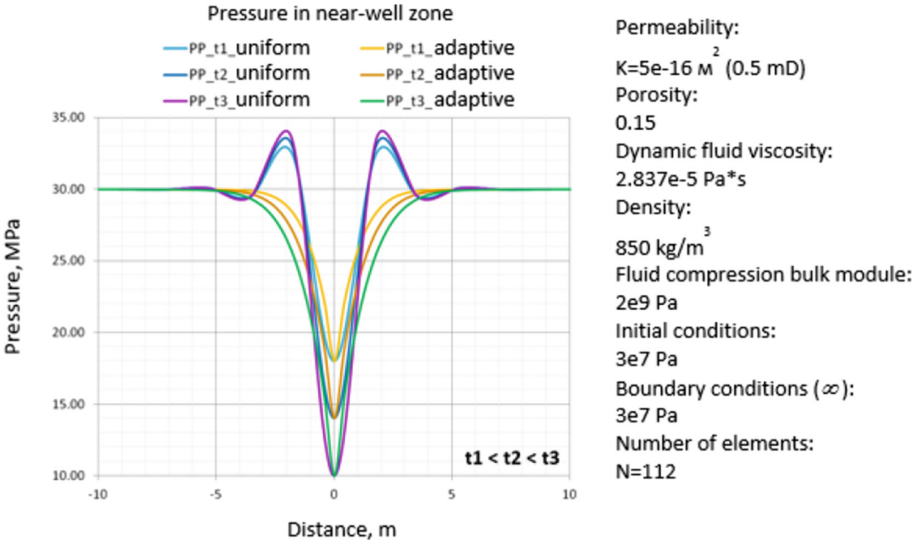

Fig. 1. The pressure distribution in a near-well zone for uniform and adaptive grids.

3 The Algorithm of Adaptive Mesh Construction

There are two main methods for constructing an adaptive mesh [4]. The first method is based on the principle of an equidistributing grid: the step of such a grid is chosen in such a way that the error in estimating a desired function (for example, pressure or temperature) is the same for each element, for which the so-called error density function is chosen (the grid density function). The second method is based on writing and finding a solution of the grid differential equation. Both methods lead to a system of related equations both for determining the position of nodes of a moving grid and for determining a sought for function reflecting the distribution of a certain physical quantity (for example, pressure). The second method can be used both with keeping equal-to-error principle, and without keeping this principle. In practice, strict keeping the principle of an equally distributed error leads to considerable difficulties in constructing a stable computational algorithm for solving multidimensional problems. For this reason, one of the most useful methods of formulating and solving the grid equation in the multidimensional case is the use of the variational method. In this case, the Euler-Lagrange equations are written down with a “grid” functional of a special form.

As an example, which is well suited for solving practical problems of hydrodynamics, we will consider the Euler-Lagrange differential equation used to form an adaptive grid:

$$-\nabla \cdot \left(\frac{1}{w}\nabla \xi_i\right) = 0, \quad i = 1, 2 \ldots d, \tag{2}$$

where $w = w(\boldsymbol{x}) > 0$ is the defined weight function.

Such a principle of forming the «grid» equation is called the «variable diffusion» method [4, 5]. It should be noted that the first results with the use of adaptive moving meshes were obtained by Godunov et al. [6], when considering a problem of impact a certain volume of water against a rigid wall. In essence, formula (2) is a stationary diffusion equation, in which the spatial-variable diffusion coefficient affects the concentration distribution (in this case, the «density» of the mesh lines).

The coefficient in Eq. (2) depends on the so-called «physical» solution and the variable ξ depends on unknown coordinates $\boldsymbol{x}$ of nodes of the mesh on which a solution to the «physical» differential equation is sought. There is no need a direct solution of Eq. (2). Since in practice, it is required to find the distribution of $\boldsymbol{x} = \boldsymbol{x}(\boldsymbol{\xi})$, since it changes the roles of the independent and dependent variable. If we set $w = 1$, then in a two-dimensional case such a role changing in (2) leads to the «grid» equations:

$$\left(x_\eta^2 + y_\eta^2\right)x_{\xi\xi} - 2\left(x_\xi x_\eta + y_\xi y_\eta\right)x_{\xi\eta} + \left(x_\xi^2 + y_\xi^2\right)x_{\eta\eta} = 0,$$
$$\left(x_\eta^2 + y_\eta^2\right)y_{\xi\xi} - 2\left(x_\xi x_\eta + y_\xi y_\eta\right)y_{\xi\eta} + \left(x_\xi^2 + y_\xi^2\right)y_{\eta\eta} = 0.$$

If in formula (2), we accept that $w = w(\boldsymbol{x}) > 0$, then we can obtain more cumbersome expressions given in [5].

In the case of a variation approach, the general form of the Euler-Lagrange equation can be obtained in the following form:

$$-\nabla \cdot \left[\frac{\partial F}{\partial \boldsymbol{a}^{\mathrm{i}}} - J\frac{\partial F}{\partial J}\boldsymbol{a}_i\right] = 0, \quad \mathrm{i} = 1, 2, 3, \tag{3}$$

where the corresponding functional is as follows:

$$I[\xi] = \int_\Omega F\left(\boldsymbol{a}^1, \boldsymbol{a}^2, \boldsymbol{a}^3, J, \boldsymbol{x}\right)\mathrm{d}x. \tag{4}$$

In Eqs. (3) and (4), it is assumed that J is the Jacobian of the transformation, and the corresponding vectors $\boldsymbol{a}^i$ are the columns of the inverse Jacobi matrix:

$$\boldsymbol{J} = \frac{\partial \boldsymbol{x}}{\partial \boldsymbol{\xi}} = \frac{\partial(x_1, x_2, x_3)}{\partial(\xi_1, \xi_2, \xi_3)} = [\boldsymbol{a}_1, \boldsymbol{a}_2, \boldsymbol{a}_3]. \tag{5}$$

For practical purposes, the function F in the integral can be represented as follows:

$$F\left(\boldsymbol{a}^1, \boldsymbol{a}^2, \boldsymbol{a}^3, J, \boldsymbol{x}\right) = F_1(\rho, \beta) + F_2(\rho, J) \tag{6}$$

$$F_1(\rho, \beta) = \frac{1}{2}\sum\nolimits_i (\nabla\xi_i)^T M^{-1}\nabla\xi_i = \frac{1}{2}\beta \tag{7}$$

$$F_2(\rho, J) = 0 \tag{8}$$

$$M = w(\boldsymbol{x})I \tag{9}$$

$$w = \sqrt{1 + |\nabla p|^2}. \tag{10}$$

It should be noted that in Eq. (9), $w(\boldsymbol{x})$ is the function that determines the «density» of grid lines used to solve the physical Eq. (1). In Eq. (10), the explicit form of the grid density function is shown, depending on the gradient of the unknown function. After performing rather a cumbersome chain of transformations aimed at changing the roles of the independent and dependent variables in Eq. (3), we can obtain a compact form of the «grid» differential equation for determining the function $\boldsymbol{x} = \boldsymbol{x}(\boldsymbol{\xi})$.

$$\sum\nolimits_{i,j} A_{i,j}\frac{\partial^2 \boldsymbol{x}}{\partial\xi_i \partial\xi_j} + \sum\nolimits_i B_i \frac{\partial \boldsymbol{x}}{\partial\xi_i} = 0. \tag{11}$$

In this equation:

$$A_{ij} = \left(\left(\boldsymbol{a}^{\mathrm{i}}\right)^T M^{-1}\boldsymbol{a}^j\right)I \tag{12}$$

$$B_i = I\sum\nolimits_k \left(\left(\boldsymbol{a}^k\right)^T \frac{\partial M^{-1}}{\partial\xi_k}\boldsymbol{a}^{\mathrm{i}}\right). \tag{13}$$

When solving Eq. (7) with a finite elements method, it is necessary to obtain a weak formulation of the boundary value problem

$$\sum\nolimits_{i,j}\int_{\Omega_C} \frac{\partial \boldsymbol{x}}{\partial\xi_i}\cdot\frac{\partial}{\partial\xi_j}\left(A_{ij}\boldsymbol{v}\right)\mathrm{d}\boldsymbol{\xi} + \sum\nolimits_i \int_{\Omega_C}\frac{\partial \boldsymbol{x}}{\partial\xi_i}\left(B_i\boldsymbol{v}\right)\mathrm{d}\boldsymbol{\xi} = 0. \tag{14}$$

Here it should be noted that as the boundary conditions for Eq. (11), it is often sufficient to set the immobility of the nodes on the boundary of the region.

4 Using an Adaptive Grid for Geomechanical Problems

Let us consider using the adaptive mesh method (Fig. 2) for a problem of subsurface fluids filtering. Figure 3 shows the pressure distribution field around a separate production well for an adaptive grid with the number of elements $N^2 = 2500$ and for a grid

with constant mesh spacing ($(3N)^2 = 22500$). It is possible to note a change in the position of the nodes of the computational grid, as well as a characteristic change in the size and shape of the elements while preserving the total number of nodes. The calculations are performed with the use of the open-source FreeFem++ software package [7]. It should be noted that formula (10) depends on the derivative of an unknown function, which itself is numerically calculated, and, therefore, with a certain error. This error can be quite substantial. For this reason, the practical application of adaptive grids requires a suitable choice of function (10), and here the researcher is provided with a wide field for creativity. In this study we used an ordinary Gaussian function whose approach to the delta-function while the variance (in the Gaussian function definition) approach is close to zero. The numerical solution in the near-well zone using the such function is close to the analytical solution given in [8].

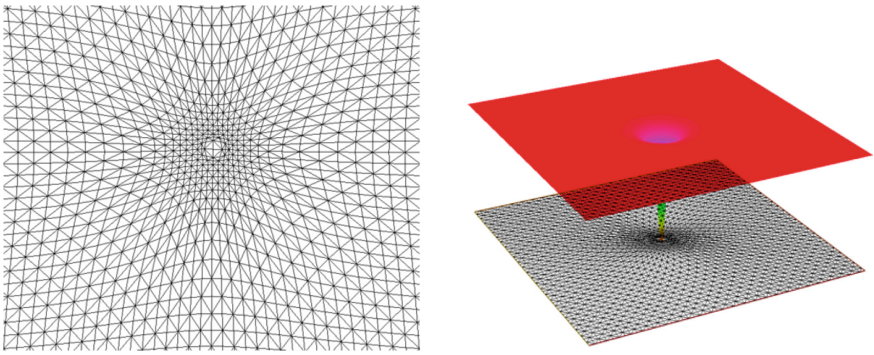

Fig. 2. An adapted grid (left) and a pressure distribution field (right).

5 The Statement of Poroelasticity Problem

The numerical solution of hydrodynamic problems has been carried out under the assumption of incompressibility of a rock. In most cases, this assumption was used to simplify the problem being solved and to reduce the calculation time, since the contribution of the compressibility of rocks to the estimation of the volume of a produced fluid was often insignificant. However in some cases it is necessary to take into account the deformability of rocks, since a change in a pore volume affects the pressure field and may lead to an underestimation of the level of production. In general, there is a reverse effect of the level of the reservoir pressure on the magnitude of the deformations, and an incorrect account of these deformations can lead to the collapse of the walls of a wellbore when performing various technological operations. Using the method of adaptive grids allows us not to separate the problems of hydrodynamics and geomechanics, since it suffices to use a single computational grid both for estimating the pressure fields and estimating the stress and strain fields.

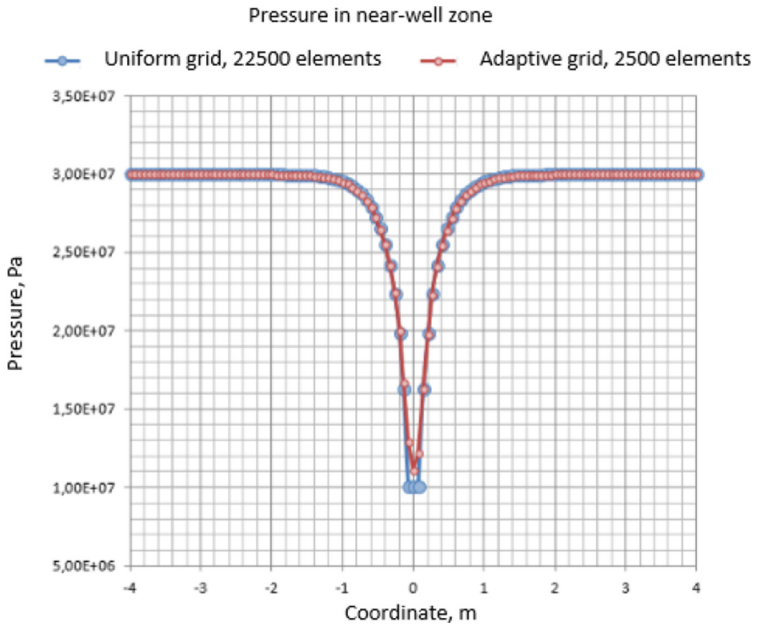

Fig. 3. The 2D pressure distribution for uniform and adaptive grids

The poroelasticity problem can be formulated by the following system of differential equations:

$$-G\nabla\cdot\left(\nabla u+(\nabla u)^T\right)-G\frac{2v}{(1-2v)}\nabla(\nabla\cdot u)+\alpha\nabla p=F \qquad \text{in } \Omega\times(0,T), \quad (15)$$

$$\frac{\partial}{\partial t}(Se\ p+\alpha\nabla\cdot u)-\nabla\cdot\left(\frac{k}{\mu}\nabla p\right)=Q \qquad \text{in } \Omega\times(0,T), \quad (16)$$

$$u=0 \qquad \text{on } \Gamma_c, \quad (17)$$

$$\left[G\left(\nabla u+(\nabla u)^T\right)+G\frac{2v}{1-2v}\nabla\cdot uI\right]\widehat{n}-\beta\alpha p\widehat{n}\chi_{tf}=0 \qquad \text{on } \Gamma_t, \quad (18)$$

$$p=0 \qquad \text{on } \Gamma_d, \quad (19)$$

$$-\frac{\partial}{\partial t}((1-\beta)\alpha u\cdot\widehat{n})\chi+\frac{k}{\mu}\nabla p\cdot\widehat{n}=h_1\chi_{tf} \qquad \text{on } \Gamma_f, \quad (20)$$

$$\text{e } p+\alpha\nabla\cdot u=v_0 \qquad \text{in } \Omega\times\{0\}, \quad (21)$$

$$(1-\beta)\alpha u\cdot\widehat{n}=v_1 \qquad \text{on } \Gamma_{tf}\times\{0\}. \quad (22)$$

Equation (15) is responsible for determining the displacements u of points of a poroelastic medium given by the elastic constants G, v as well as the constants of the poroelasticity α (the Biot constant) and Se (the Skempton constant). The meaning of the remaining notation is presented in [9]. Equation (16) is responsible for determining the pressure in the process of the filtration of a fluid through a deformable poroelastic medium, taking into account the contribution of the compressibility of the rock matrix.

One needs to add (2) or (3) to the system of Eqs. (15, 16) with the corresponding boundary conditions for obtaining a complete system of equations for solving the poroelasticity problem using adaptive grids. As usual, it is assumed that a poroelastic medium is continuous, isotropic and homogeneous. To solve the poroelasticity problem numerically using the finite element method, it is necessary to write down a weak formulation of the boundary value problem:

$$\int_{\Omega}\left[G\left(\nabla u^{n+1}+\left(\nabla u^{n+1}\right)^{T}\right):\nabla v+G\frac{2v}{1-2v}\left(\nabla\cdot u^{n+1}\right)(\nabla\cdot v)\right]+\int_{\Omega}\alpha\nabla p^{n+1}v$$
$$=\int_{\Omega}F^{n+1}v+\int_{\Gamma_t}G\left(\nabla u^{n+1}+\left(\nabla u^{n+1}\right)^{T}\right)\cdot\widehat{n}v+\int_{\Gamma_t}G\frac{2v}{1-2v}\left(\nabla\cdot u^{n+1}\right)\widehat{n}\cdot v, \tag{23}$$

$$-\int_{\Omega}\alpha u^{n+1}\cdot\nabla q+\int_{\Omega}\left(Se\,p^{n+1}q+\frac{k\tau}{\mu}\theta\nabla p^{n+1}\cdot\nabla q\right)$$
$$=\int_{\Omega}\left(\tau\left(\theta Q^{n+1}+(1-\theta)Q^{n}\right)+\alpha\nabla\cdot u^{n}+Sep^{n}\right)q-\int_{\Omega}\frac{k\tau}{\mu}(1-\theta)\nabla p^{n}\nabla q \tag{24}$$
$$-\int_{\Gamma_f}\alpha u^{n+1}\cdot\widehat{n}q+\int_{\Gamma_f}\frac{k}{\mu}\left(\theta\nabla p^{n+1}+(1-\theta)\nabla p^{n+1}\right)\cdot\widehat{n}q$$

$$\sum\nolimits_{i,j}\int_{\Omega_C}\frac{\partial\boldsymbol{x}^{n+1}}{\partial\xi_i}\cdot\frac{\partial}{\partial\xi_j}\left(A_{ij}\boldsymbol{\omega}\right)\mathrm{d}\boldsymbol{\xi}+\sum\nolimits_{i}\int_{\Omega_C}\frac{\partial\boldsymbol{x}^{n+1}}{\partial\xi_i}\left(B_i\boldsymbol{\omega}\right)\mathrm{d}\boldsymbol{\xi}=0. \tag{25}$$

Thus, system (23, 24) is a weak formulation of the boundary value problem of poroelasticity using adaptive grids. It should be noted that rebuilding the adaptive grid according to Eq. (25) is determined only by the fluid pressure gradient. In many cases, such a formulation will be sufficient, since the applied tasks of the oil and gas geomechanics relate to the effects occurring in the near-well zone, where large gradients of both pressures and displacements are observed in the first place.

6 The Parallel Implementation of a Finite Element Method

To speed up the computations of the 2D poroelasticity problem and to make possible large grid calculations, the parallelization of the numerical solution of the equation system with a finite element method was carried out using the FreeFem++ solver.

Let us note that an extended interface with MPI has been added to FreeFem++. The Schwarz algorithm [10] with overlapping and a coarse grid preconditioner is used to decompose the computational domain. The grid of triangles is adapted once before the calculations in the main time cycle. The Metis graph partitioner [11] is used for partitioning into an equal (according to the number of elements) subdomain among computational nodes (cores). At each time step, the problem is first solved on a coarse grid, and then this solution is used as an initial approximation in each subdomain of the partitioning. For the numerical solution of the coupled problem of poroelasticity, each of the equations, rewritten in a matrix form, is solved one after another iteratively in all subdomains at the same time.

The considered approach of grid adaptation is convenient because it keeps the number of grid elements unchanged and does not require additional solving the problem of load balancing between MPI processes during computing. With the initial selection of a regular grid of triangles, the problem of its optimal decomposition between computational nodes is solved trivially. With a more complex initial organization of the grid of triangles, the functions of the Metis package are used to divide it.

A strong scalability study has been conducted on a node of the Siberian Supercomputer Center (SSCC) cluster equipped with two 16-core Intel processors on Broadwell architecture. The calculations were carried out on a grid consisting of 240000 triangles.

The results (Fig. 4) show that acceleration has been reached about 14 times on 32 cores as compared to a single core.

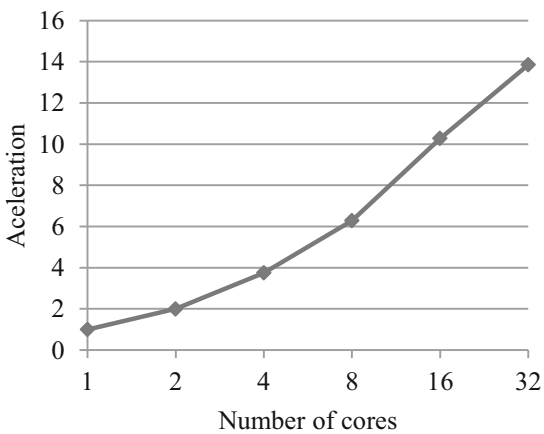

Fig. 4. The results of strong scalability research at Intel Broadwell

The possibility of using a node of SSCC equipped with Intel Memory Drive Technology (IMDT) called Optane was also investigated. Intel Optane is a new SSD product based on the novel 3D XPointTM technology, which can be used instead of DRAM, albeit as a slow memory [12]. It can be still an attractive solution given that Intel Optane is notably cheaper than the random access memory (RAM) per gigabyte. The novel Intel Memory Drive Technology (IMDT) allows one to use Intel Optane drives as a system memory. Various benchmarking results [13] for large dense tasks show different efficiencies. We have used two memory configurations (Table 1): hybrid IMDT DDR4/Optane and DDR4 only for systems of different sizes of a grid for the numerical solution of filtering problem (16) with our parallel implementation.

Table 1. The results of the two memory configurations usage: hybrid IMDT DDR4/Optane and DDR4 only systems for filtering problem.

Memory configurations	Number of triangles	Used memory, Gb	Execution time, s
DDR4	$12{,}5 \cdot 10^6$	51	941
DDR4	$32 \cdot 10^6$	125	5508
DDR4/Optane	$50 \cdot 10^6$	293	8373
DDR4/Optane	$72 \cdot 10^6$	466	19614

7 Conclusion

In this paper, the use of adaptive grids for solving geomechanical problems is discussed. An algorithm for constructing an adaptive grid with the Jacobian coordinate transformation is presented. The adaptation algorithm changes the grid density, which depends on the gradient of a desired function. The number of nodes of the adaptive mesh remains unchanged and is equal to the number of nodes of the initial grid. The results of the simulation by the finite elements method of the pressure distribution for the fluid filtration problem in the near-well zone using adaptive mesh are presented. The calculations were carried out using the freeware and open-source Freefem++ software package. The formulation of the poroelasticity problem in the integral form is presented. The studies of the parallel implementation for poroelasticity problems show the acceleration of about 14 times in 32 cores.

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References

1. Filippov, D.D., Vasekin, B.V., Mitrushkin, D.A.: Multiphase filtration modeling of complex structure on dynamic adaptive PEBI-grid. PROneft. Professionals Oil **4**(6), 48–53 (2017)
2. Qi, D., Hesketh, T.: An analysis of upscaling techniques for reservoir simulation. Petrol. Sci. Technol. **23**(7–8), 827–842 (2005)

3. Peaceman, D.W.: Interpretation of well-block pressures in numerical reservoir simulation. SPEJ **23**(03), 183–194 (1978)
4. Huang, W., Russell, R.D.: Adaptive Moving Mesh Methods. Springer, New York (2011)
5. Winslow, A.M.: Adaptive mesh zoning by the equipotential method. Technical report UCID-19062. Lawrence Livemore Laboratory (1981)
6. Godunov, S.K., Prokopov, G.P.: The use of moving meshes in gas-dynamical computations. USSR Comput. Math. Math. Phys. **12**, 182 (1972)
7. Hecht, F.: New development in FreeFem++. J. Numer. Math. **20**(3–4), 251–265 (2012)
8. Basniev, K.S., Kochina, I.N., Maksimov, V.M.: Podzemnaya gidromekhanika: Uchebnik dlya vuzov. Nedra, Moscow (1993). (in Russian)
9. Aji, C.A.: Poroelasticity. Auburn, Alabama (2007)
10. Toselli, A., Widlund, O.: Domain Decomposition Methods - Algorithms and Theory. Computational Mathematics, vol. 34. Springer, Heidelberg (2004). https://doi.org/10.1007/b137868
11. Karypis, G., Kumar, V.: A fast and high quality multilevel scheme for partitioning irregular graphs. SIAM J. Sci. Comput. **20**(1), 359–392 (1998)
12. Shenoy, N.: What happens when your PC meets intel optane memory? (2018). https://newsroom.intel.com/editorials/what-happens-pc-meets-intel-optane-memory/
13. Mironov, V., Kudryavtsev, A., Alexeev, Y., Moskovsky, A., Kulikov, I., Chernykh, I.: Evaluation of Intel Memory Drive Technology performance for scientific applications. In: MCHPC 2018: Workshop on Memory Centric High Performance Computing (MCHPC 2018), pp. 14–21. ACM, New York (2018)