Economical Sequential Algorithmic Convolutions in Problems of Active Vibroseismoacoustic Monitoring

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***Abstract*—Network technologies of data acquisition are introduced into systems of active geophysical monitoring of the environment. In this connection the development of methods and algorithms for online analysis of data is very important. As an example, some algorithms of sequential cross-correlation convolution of seismic and acoustic oscillations generated by seismic vibrators are considered. Such sources are widely used in the decision of active geophysical monitoring problems for Earth and atmosphere.**

***Keywords*—active monitoring of the environment, seismic vibrators, acoustic and seismic oscillations, sequential cross-correlation convolution, computational cost and memory characteristics.**

I. Introduction

The use of network technologies in systems of active geophysical monitoring of the environment [1, 2, 3] opens up possibilities for online analysis of geophysical data. This allows continuous data processing in active monitoring systems with the use of vibrational seismic and acoustic oscillations [4, 5]. It is well-known that in this case cross-correlation convolution of long (in time) recorded and reference signals is one of the major operations [5, 6]. The calculation of this operation in real time is possible only by sectioning of long time sequences. Also, the use of the fast Fourier transform (FFT) algorithm to process sectioned data makes it possible to perform high-performance calculations.

II. Problem Statement

Let Y be the sought-for correlotrace. With sectioning its estimate is calculated in iterative form: $Y^{i+1} = f\left(Y^i, X_{i+1}\right)$, where Y^i – approach of Y on *i*- th step and X_{i+1} are the current data. As sectioned data become available, they are accumulated in auxiliary buffers, and after the buffer in turn is full, they are processed using the FFT. Thus, to process signals in real time with the FFT, the aperiodic correlation convolution of two large data arrays must be presented as the sum of cyclic convolutions of parts (sections) of these arrays. This sectioning must be done so that the computational cost of processing of one section depends only on the size of this section but not on the set of all input data of size N.

III. Theory

In problems of processing of vibroseismic signals, the input array, *X*, and the reference array, *S*, are equally large, and the result of their convolution, *Y*, is represented by a comparatively small number of data *M (M<<N).* This is a basis for choosing an approach to sectioning the arrays *X* and *S*. With allowance for this, the algorithm of convolution of the both arrays can be represented in the following form:

$$\underset{m=0\ldots M-1}{Y_m} = \sum_{n=0}^{N-1} X_n \cdot S_{n-m} = \sum_{l=0}^{\frac{N}{M}-1} \sum_{n=0}^{M-1} X_{M\cdot l+n} \cdot S_{M\cdot l+n-m} = \sum_{l=0}^{\frac{N}{M}-1} \sum_{n=0}^{2L-1} A_n^l \cdot B^l_{<m-n>_{2M}} , \quad (1)$$

where *M* is the size of one section of the input array (it is assumed that *N* divides evenly into *M*). Otherwise the array S is increased by adding zeroes. The arrays *A* and *B* consist of 2L data and are defined as follows:

$$A_i^l = \begin{cases} X_{M\cdot l+i}, & \text{если } 0 \le i < M \\ 0, & \text{если } M \le i < 2M \end{cases}$$

$$B_i^l = S_{M\cdot(l-1)+\langle M-i\rangle_{2M}}$$

$$0 \le i < 2M$$

According to the convolution theorem for the discrete case, cyclic convolution of two arrays is equal to the inverse discrete Fourier transform (IDFT) of the product of direct DFTs of these arrays. Hence,

$$\underset{m=0\ldots M-1}{Y_m} = \frac{1}{N} \sum_{l=0}^{\frac{N}{M}-1} \sum_{k=0}^{2M-1} \left(\sum_{n=0}^{2M-1} A_n^l \cdot W^{-n\cdot k} \right) \cdot \left(\sum_{l=0}^{2M-1} B_n^l \cdot W^{-l\cdot k} \right) \cdot W^{k\cdot m},$$

где $W = e^{j\frac{2\pi}{2M}}$

Fig.1 presents a scheme showing the operations at the processing of the i-th section of the input sequence *X*. The symbol "*" here denotes the FFT of the arrays and multiplication of the spectrum of array A by the spectrum that is complex conjugate with the spectrum of array *B*. These operations are performed for every section of the input sequence. Once they are completed, the correlogram is calculated by performing the IDFT of the spectrum *Y*.

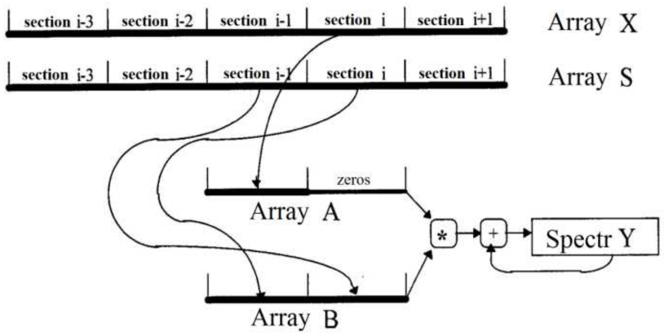

Fig. 1

IV. Calculation Aspects of the Programs

Computational cost of sectioned convolution on the basis of FFT. In this case the calculation of convolution includes calculating the direct FFT of each signal section, multiplying the spectra by the spectrum of the reference signal, and adding the product obtained to the current approximation of the correlogram spectrum. After the sections are processed, the inverse FFT of the obtained correlogram spectrum is performed. The total computational cost of processing for one channel is as follows:

$$T_\Sigma = N/M\cdot\left(T_{FFT}\left(2\cdot M\right)+6\cdot M\right)+T_{FFT}\left(2\cdot M\right)\approx$$
$$\approx N\cdot\left(5\cdot\log_2 M+2\right)+M\cdot\left(5\cdot\log_2 M-6\right)\approx 5\cdot N\cdot\log_2 M \text{ flops} \quad (2)$$

(it is assumed that *N* divides evenly into *M*).

The computational cost of convolution without sectioning is

$$T_1\approx 5\cdot(N+M)\cdot\log_2(N+M)\text{ flops.} \quad (3)$$

Here and below it is assumed that the numbers are presented in the single-precision floating-point (float) format.

The computational cost of the FFT coefficients and of the reference signal spectrum is also not taken into account here. A comparison of the obtained estimates of computational costs (2) and (3) shows that sectioning makes it possible to decrease the total computational cost of convolution (it should be remembered that it can be applied only if $M << N$).

The memory volume needed to calculate convolution with sectioning is calculated based on the fact that the data of all channels are processed in one algorithm cycle. The memory is used to process the array of the spectrum of one reference signal section, store the auxiliary array of the previous section of the reference signal, the array of FFT coefficients, and *P* arrays to accumulate the correlogram spectra. Here *P* is the number of channels. This is a total of ($22\cdot M+8\cdot M\cdot P$) bytes.

The memory volume needed to calculate convolution without sectioning includes the memory for the array being processed, the reference array, and the array of FFT coefficients, and is $10\cdot\left(N+M\right)$ bytes. Thus, sectioning also makes it possible to considerably decrease the memory volume.

V. Effect of Error in Specifying the Linear Frequency Variation Rate in Reference Signal on Quality of Cross-Correlation Convolution.

In real conditions, a sweep signal excited in the Earth by a vibro-module and a sweep signal reconstructed at the receiving point can differ due to instrumental errors. For instance, if the phase characteristics of the driving generator of radiated signals and the generator of reference signals at the receiving point are different.

Therefore, it seems reasonable to estimate the influence of the error in specifying the oscillation frequency variation rate of the driving generator (vibrator) on the response of the matching filter.

Let us consider a sounding LFM-signal (signal with linear frequency modulation) with the rectangular envelope

$$s(t)=a\ \cos\varphi(t). \quad (4)$$

Such signals are generated, for instance, by hydraulic vibro-modules [4]. Let the frequency $f(t)$ vary in the range from f_1 to f_2 according to the law

$$f(t)=f_0+\gamma\, t, \quad (5)$$

where $-\frac{T}{2}\le t\le\frac{T}{2}$, $f_0=\left(f_2+f_1\right)/2$, $\gamma=\left(f_2-f_1\right)/T$.

With (5), the expression for the phase takes the form

$$\varphi(t)=2\pi\left(\varphi_0+f_0 t+\frac{\gamma}{2}t\right). \quad (6)$$

Assume that $\varphi_0=0$, $a=1$. The response of a signal $s_\Delta(t)$ passing through a filter consistent with the signal $s(t)$ is expressed in terms of the cross-correlation function

$$y(t)=\int_{-T/2}^{T/2} s_\Delta(\lambda)\, s(\lambda-t+\tau)\, d\lambda, \quad (7)$$

where τ is the delay time (assume for simplicity that $\tau=0$).

Let the signal $s(t)$ be used as $s_\Delta(t)$ in one case, and the same signal with limiting frequencies f_2^*, f_1^* inconsiderably different from f_2 and f_1, that is

$f_2^*=f_2+\Delta f_2$, $f_1^*=f_1+\Delta f_1$, $\Delta f_2<<f_2$, $\Delta f_1<<f_1$, in the other case. Hence, the cross-correlation functions are, correspondingly,

$$y(t)=\int_{-T/2}^{T/2} s(\lambda)\, s(\lambda-t)\, d\lambda,\quad y^*(t)=\int_{-T/2}^{T/2} s_\Delta(\lambda)\, s(\lambda-t)\, d\lambda,$$

where $s_\Delta(t)=s(f_2^*,f_1^*)$.

It can be shown that when the conditions

$\left|\Delta\varphi(t)\right|=\left|\Delta\omega_0 t+\pi\,\Delta\gamma\, t^2\right|<<1$ rad,

where

$$\Delta\omega_0 = 2\pi(\Delta f_2 + \Delta f_1),\ \Delta\gamma = (\Delta f_2 - \Delta f_1)/T,\ t \in [-T/2,\ T/2],$$

are satisfied, we have the following approximate expansion for the error $\Delta y = y^*(t) - y(t)$ in powers of $\Delta\omega_0$ and Δy:

$$\Delta y(t) \approx \Delta\omega_0\ m_1(t) + \Delta\tilde{\gamma}\ m_2(t) + \Delta\omega_0^2\ m_3(t) + \\ +\Delta\omega_0\ \Delta\tilde{\gamma}\ m_4(t) + \Delta\tilde{\gamma}^2\ m_5(t),$$

where

$$m_1(t) = \frac{\cos q}{2\nu}\cdot\left[\cos\frac{\nu T}{2} - \frac{2}{\nu}\sin\frac{\nu T}{2}\right]$$

$$m_2(t) = -\frac{\sin q}{2\nu}\cdot\left\{\frac{T^2}{2}\sin\frac{T}{2} + \frac{2}{\nu}\left[T\cos\frac{\nu T}{2} - \frac{2}{\nu}\sin\frac{\nu T}{2}\right]\right\}$$

$$m_3(t) = -\frac{\cos q}{4\nu}\cdot\left[\left(\frac{T^2}{\nu} - \frac{4}{\nu^2}\right)\sin\frac{\nu T}{2} + \frac{2T}{\nu}\cos\frac{\nu T}{2}\right]$$

$$m_4(t) = -\frac{\sin q}{2\nu}\cdot$$

$$\cdot\left\{\frac{T^3}{4}\cos\frac{\nu T}{2} - \frac{3}{\nu}\left[\frac{T^2}{2}\sin\frac{\nu T}{2} + \frac{2}{\nu}\left[T\cos\frac{\nu T}{2} - \frac{2}{\nu}\sin\frac{\nu T}{2}\right]\right]\right\}$$

$$m_5(t) = -\frac{\cos q}{4\nu}\cdot$$

$$\left\{\frac{T^4}{8}\sin\frac{\nu T}{2} + \frac{4}{\nu}\left[\frac{T^3}{4}\cos\frac{\nu T}{2} - \frac{3}{\nu}\left[\frac{T^2}{2}\sin\frac{\nu T}{2} + \frac{2}{\nu}\left[T\cos\frac{\nu T}{2} - \frac{2}{\nu}\sin\frac{\nu T}{2}\right]\right]\right]\right\}$$

$$q = (\omega_0 t - \tilde{\gamma} t^2),\quad \nu = 2\tilde{\gamma} t^2,\quad \tilde{\gamma} = \pi\gamma,\ \Delta\tilde{\gamma} = \pi\Delta\gamma.$$

Fig. 2 shows some results of numerical implementation of the cross-correlation function $y^*(t)$ at different values of $\Delta f_1,\ \Delta f_2$. The reference signal duration T = 120 s (the figure presents the central part, 30 s).

The algorithm for calculating $y^*(t)$ was implemented using the FFT. The number of discretization points L = 4096, and the discretization step Δt = 0,0293 s, which provides the needed accuracy. The initial part of the frequency range is given without error (Δf_{min} =0,000), and for the upper frequency the error $\Delta f_{\max}$ constitutes 0, 0.5, 1, 2, 5, and 10% of the given value of f_2. It follows from the above plots that as the error Δf_2 increases (in the figure, Δf_2 is denoted by $\Delta f_{\max}$), the duration of responses increases and, hence, their maximum amplitude decreases. For instance, at Δf_2=0.5% f_2 the response duration T_y increases almost by a factor of 3 and the amplitude A decreases by a factor of 2, whereas at Δf_2=1% f_2 T_y increases by a factor of 5 and A decreases almost by a factor of 3, etc.

Calculations for various other error values have also been made. It follows from the results obtained that at inconsiderable (on the order of several tenths of a percent) differences in the frequency between the sounding signal and the reference signal reconstructed at the recording point the duration of response increases considerably and its amplitude decreases.

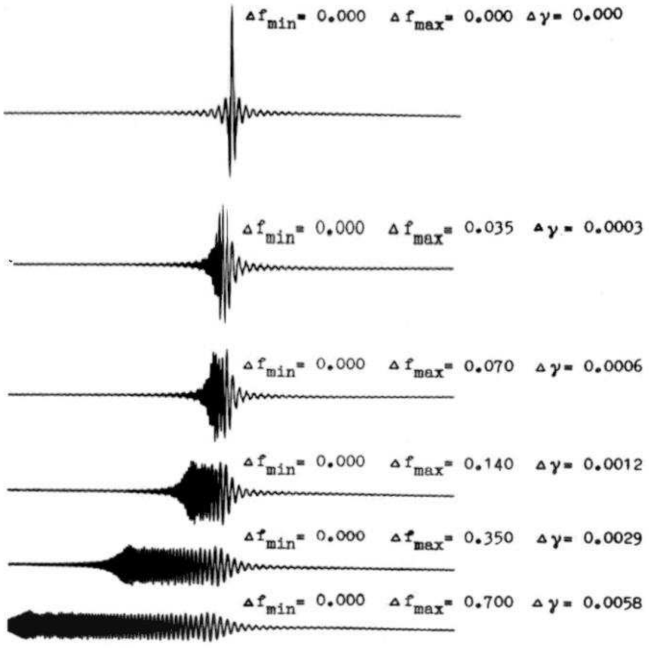

Fig. 2. Effect of the error in specifying the frequency in the reference signal on the quality of cross-correlation convolution

CONCLUSION

1. Some algorithms for the processing of signals as they arrive, with sectioning of input and reference signals, have been proposed within the framework of the network technology of data acquisition and analysis in vibroseismoacoustic monitoring systems. The computational cost of the algorithms and the memory volumes for their implementation have been estimated.
2. Estimates of the errors in calculating the parameters of the cross-correlation convolution at frequency differences between the sounding signal and the reference signal reconstructed at the recording point have been obtained. It has been shown that at inconsiderable errors in specifying the oscillation frequency variation rate of the driving generator the response duration increases considerably and its amplitude decreases. This determines the requirements to the accuracy of specifying the parameters of LFM oscillations in frequency: not worse than 0.1 %.

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