THE FOCUSING EFFECT OF P-WAVE IN THE MOON'S AND EARTH'S LOW-VELOCITY CORE. ANALYTICAL SOLUTION

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Introduction:

The important aspect in the study of the structure of the interiors s of planets is the question of the presence and state of cores inside them. While for the Earth this task was solved long ago, the question of whether the core of the Moon is in a liquid or solid state up to the present is debatable up to present. If the core of the Moon is liquid, then the velocity of longitudinal waves in it should be lower than in the surrounding mantle. If the core is solid, then most likely, the velocity of longitudinal waves in it is higher than in the mantle. Numerical calculations of the wave field allow us to identify the criteria for drawing conclusions about the state of the lunar core.

In this report we consid the problem of constructing a stable analytic solution for wave fields in a layered sphere of arbitrary size. After the Fourier- Legendre transformations, the statement of the problem reduces to the consideration of a two-parameter family of boundary-value problems for ordinary differential equations. The solution of the latter problem in each spherical layer is in the form of a linear combination of Bessel functions [1]. The unknown coefficients are determined from known conjugation conditions on the boundary of spherical layers. As a result, a matrix system of linear equations is obtained for their determination. For a small number of layers, its solution can be obtained in explicit form. Since Bessel functions of different types tend to zero and infinity rapidly, uncertainty arises in the solution. And the more the radius of the sphere in relative values (wavelengths), the faster it arises. In this situation, computer calculations become unstable. To construct a stable solution, it is proposed to use the classic asymptotic of Bessel functions [2]. In the article [3] it is shown that the classical asymptotic behavior of Bessel functions gives an error in the solution. To construct the solution, we use the new asymptotes of cylindrical functions obtained in the article [3]. This gives a stable analytical solution for wave fields in an inhomogeneous sphere of arbitrary size.

Formulation of the problem:

The mathematical statement of the problem of modeling the P-wave is formulated in a spherical coordinate system $(0 \le r \le R_1,\ 0 \le \theta \le \pi,\ 0 \le \phi \le 2\pi)$ as follows: define a function from equation

$$\frac{1}{v^2(r)}\frac{\partial^2 u}{\partial t^2} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial u}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 u}{\partial\phi^2} + f_r \times f(t) \qquad (1)$$

with initial conditions and boundary data

$$u = \frac{\partial u}{\partial t}\Big|_{t=0} = 0,\ \ \frac{\partial u}{\partial r}\Big|_{r=R_1} = 0. \qquad (2)$$

In (1), (2) R_1 – the radius of the sphere, f_r — is the source function over space, is the source function with respect to time t.

At boundaries $r=R_j$ where the velocity of longitudinal waves v(r) suffers a discontinuity, known conjugation conditions are introduced [1]:

$$[u] = \left[\frac{\partial u}{\partial r}\right]\Big|_{r=R} = 0.$$

Analytical Solution Results:
Figure 1 shows the result of calculating P-waves for a simplified model of the Moon, consisting of a mantle and a low-velocity core. The velocity of longitudinal waves in the mantle is 7.8 km/s, in the low-velocity core is 5.81 km/s. The radius of the Moon is 1,737 km. The radius of the low-velocity core is 380 km. Time source function f(t) is taken in the form of a Gauss-Puzyrev pulse [3]:

To clarify this effect, figure 2 shows the drawing of rays. The rays exit the source at a uniform pitch. Figure 2 clearly shows that the low-velocity core has the properties of a collecting lens. There is a focus area around 180 degrees. We can see a strong expansion of the amplitude around 180 degrees on this figure. This increase in the amplitude is denoted by L (Lens). This fully corresponds to the ray pattern shown in Figure 2. The low-velocity core on the Moon has the properties of a collecting lens. A focus area appears. This leads to the formation of a powerful wave.

Figure 3 shows the result of calculating P-waves for a simplified model of the Moon consisting of a mantle and a high-velocity core. The velocity of longitudinal waves in the mantle is 7.8 km/s, in the high-velocity core — 10.0 km/s. The radius of the high-velocity core is also taken equal to 400 km. Here the arrow indicates the beginning of the shadow zone for the direct wave P propagating in the mantle of the Moon. And Figure 4 shows the drawing of rays. It can be seen from Figures 3 and 4 that the high-velocity core of the Moon does not possess the properties of a collecting lens.

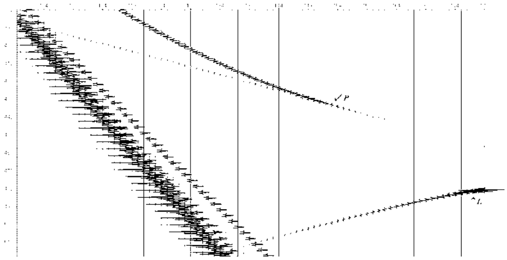

Fig. 1. Fragment of the wave field of P-wave for the Moon model with a low-velocity core. The arrow marks the beginning of the shadow zone for the refracted wave. L - a focusing effect of the low-speed core of the Moon

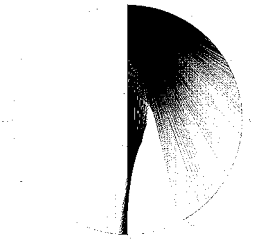

Fig. 2. The pattern rays for the Moon model with a low-velocity core.

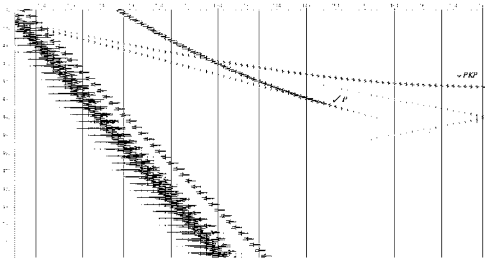

Fig. 3. Fragment of the wave field of P-waves for the Moon model with a high-velocity core with the first arrival of PKP-waves. The arrow marks the beginning of the shadow zone for the refracted wave.

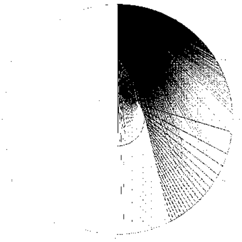

Fig. 4. The pattern rays for the Moon model with a high-velocity core.

Conclusions:

We obtained a stable analytical solution for the wave fields of longitudinal waves in a two- and three-layer sphere of arbitrary size. This made it possible to calculate the total wave fields for simplified models of the Moon and the Earth with real parameters. The results of mathematical modeling and ray pattern showed that the low-velocity cores of the Moon and the Earth possess the properties of collecting lenses.

As a result of focusing, waves of considerable amplitude appear on the surface of the Moon and the Earth. And on the surface of the Earth they come before the first entry of the PKP-wave. These are so-called “precursors” which continue in the subsequent arrivals of waves.

We also note that the detection of the effect of focusing region appearance of the oscillations emerging on the surface in the first arrivals is extremely important in elucidating the question of the state of the core not only for the Earth and the Moon, but also for other planets. If it were possible to detect the above oscillations on lunar seismograms, the question of the state and size of the core, as well as the velocity of longitudinal waves in the lunar core, would be decided unambiguously. For the currently considered models of the inner structure of the Moon, the oscillations generated by focusing inside the core must reach the surface of the Moon at distances of 180–220 degrees, or in the opposite direction in the range of 140–180 degrees. Unfortunately, at present no such oscillations are detected in the indicated range of angles [5].

References:

[1] Tikhonov A.N. and Samarskii A. A. Equations of mathematical physics [in Russian]. Nauka, Moscow, 1997.

[2] Korneev V.A. and Johnson L.R. // Geophys. J. Int 1993. No. 115. P. 230–250.

[3] Fatyanov A.G. The stable analytical solution for the wave fields in the sphere [in Russian], Mat. Zamet. SVFU, 2016, 23, No. 3, P. 91–103.

[4] Burmin V.Yu Seismic wave velocities in the Earth’s core // Izvestiya. Physics of the Solid Earth. 2004. T. 40. No. 6. P. 477–490.

[5] Lognonné P., Gagnepain-Beyneix J., Chenet H. Earth and Planetary Science Letters. 2003. Vol. 211. P. 27–44.