

# Elements of active geophysical monitoring theory

# 1.1

**Anatoly S. Alekseev, Gennady M. Tsibulchik, Valery V. Kovalevsky  
and Andrey S. Belonosov**

*Institute of Computational Mathematics and Mathematical Geophysics SB RAS, Novosibirsk,  
Russia*

## Chapter Outline

1.1.1 Introduction .....	3
1.1.2 Main properties of the integral precursor .....	4
1.1.3 Multidisciplinary model of integral precursor and combined inverse problems.....	9
1.1.4 Methods for vibroseismic monitoring of seismic-prone zones .....	12
1.1.5 Conclusion .....	15
Acknowledgments .....	16
References .....	16

## 1.1.1 Introduction

Different types of geophysical precursors are used in earthquake prediction to reduce the probability of unexpected catastrophic earthquakes. Several years prior to such earthquakes, some anomalies occur within geophysical fields, such as crustal deformation, seismicity, and electric conductivity. Zones that manifest such anomalies will migrate within 200–300 km from the epicenter of a subsequent earthquake. The migration mechanisms for different anomalous zones, and the interrelations among them, reflect the evolution of seismotectonic stress fields, which could contain valuable information about the timing of seismic activity. Over the past 30 years, data have been collected from a number of different scientific disciplines and from several different countries, including China, Japan, the United States, Greece, Turkey, and Russia.

In China, for example, information on hundreds of large earthquakes is now available from databases that are part of a substantial observational network. The accumulated information is used to investigate problems of earthquake prediction, using statistical analyses and various characteristics of geophysical anomalies at different preearthquake stages. Some previously successful earthquake predictions

by Chinese geophysicists were based on synthesized information related to changes in the behavior of anomalies at medium- and short-range preearthquake stages (Ma et al., 1995; Mei, 1992; Zhang and Zhang, 1992). However, the use of multidisciplinary resources in earthquake prediction, and the results from them, raise new, important questions with regard to determining interrelationships among seismicity processes, variations in geophysical fields, and the sources of anomalies (Alekseev, 1993; Keilis-Borok and Molchan, 1968).

---

### 1.1.2 Main properties of the integral precursor

Earthquakes occur through massive rock failure, which begins in a source zone. Therefore study of the prefailure processes and monitoring of these processes are of major importance for earthquake prediction. An investigation into rock failure in samples of various materials in the laboratory, as well as on a larger scale (in particular Earth crust blocks during earthquakes), reveals the general patterns of the rock failure process.

Step-by-step development of this process over time is the most general principle. Some kinetic laws and concepts of rock failure were established by S.N. Zhurkov and his colleagues from the Physical-Technical Institute of the Russian Academy of Sciences (RAS) in St. Petersburg (Zhurkov et al., 1977; Zhurkov, 1968). They are similar to the rock-failure-scheme concepts for large-scale objects in the Earth's crust during earthquakes that were proposed by researchers from the Institute of the Earth's Physics of the RAS (Myachkin et al., 1975, 1974). One of these researchers, G.A. Sobolev, formulated the following three principles, which he described as being of major importance in searching for earthquake precursors and predicting earthquakes (Sobolev, 1978):

1. The development of crack systems in preearthquake zones results from an increase in microcrack volume density, from the stage of increasing crack sizes (and decreasing numbers of cracks) to the formation of large fractures.
2. The relationship describing step-by-step transition from small cracks to larger cracks, when smaller cracks reach some critical value, in accordance with the concentration criterion (Zhurkov et al., 1977; Zhurkov, 1968), has the form

$$K^* = \frac{1}{\sqrt[3]{N} L}, \quad (1.1.1)$$

where  $N$  is the number of cracks of size  $L$  per unit volume,  $K^*$  is the critical average distance between cracks, measured in units of average crack lengths. When the average distance between cracks becomes smaller than a certain critical value, there is an abrupt reorganization of the entire system of cracks, with an increase in the average crack sizes (in some geometrical proportion) and a decrease in the average volume concentration. Cracks tend to localize in

the area of a future macrofracture. These phenomena are typical for any scale and any loading regime.

3. Reorganization of the crack system manifests itself as a change to some of the characteristics of the medium in a developing earthquake source and as the formation of anomalies in some geophysical fields. In particular, concentration of the crack-formation process can be evidence of a change in the seismicity regime for weak earthquakes, and in the appearance of rock anisotropy in a future earthquake source. The appearance of elastic anisotropy is most conspicuous in the formation of anomalies prior to large earthquakes (Nersesov et al., 1971).

Some geophysical fields can be affected by the opening of microcracks. In particular, gas and fluid permeability increases in those areas of the Earth's crust where this process takes place. As a consequence, the groundwater level, the intensity of gas flow, and the electrical resistance can change. The loosening of rock resulting from the increase in total crack volume must also cause local gravity anomalies.

Thus the crack-density function as a measure of rock failure has some advantages, because this function is present in the formulation of all three principles. Another advantage is that the crack-density function can be more accurately and reliably determined from multidisciplinary data, owing to its presence in the models of various geophysical fields—the complementary principle (Alekseev, 1992).

An analysis of the preexisting stress fields at earthquake locations (Miao, 1993; Wang and Liao, 1996) and the results from dilatancy-zone numerical modeling (presented later) suggest that cracks of some scale level can be formed at distances of 200–300 km from the source of a future earthquake.

Although the earthquake development process is “slow,” lasting up to several hundreds of years, it is an energy-intensive process. Considerable rheological change in the medium takes place, and anomalous zones form within different kinds of geophysical fields. Crack openings in zones with increased shearing and tensile stresses are the most basic mechanism of medium change. Such zones are formed near the sources of future earthquakes, if the spatial distribution of forces is nonuniform. Many seismologists consider that the initial stage of crack opening and the subsequent state of the medium when rock failure develops are connected to medium dilatancy (Nikolayevskii, 1982; Nur, 1971; Brace et al., 1966).

*Dilatancy* is the nonlinear loosening of rocks caused by crack formation from shear. This process takes place when tangential stresses exceed a certain threshold. A dilatancy zone includes points within an elastic medium, for which the following condition is satisfied:

$$D_{\tau} \equiv \tau - \alpha(P + \rho gz) - Y \geq 0, \quad (1.1.2)$$

where  $\rho$  is the density of rocks,  $g$  is the gravitational acceleration,  $\alpha$  is the coefficient of internal friction,  $Y$  is the cohesion of rocks,  $z$  is the depth of the point, and  $P$  is the hydrodynamic pressure

$$P = -\frac{1}{3}(\sigma_{11} + \sigma_{22} + \sigma_{33}), \quad (1.1.3)$$

where  $\sigma_{ij}$  is the stresses and  $\tau$  is the intensity of the tangential stresses:

$$\tau = \frac{\sqrt{3}}{2} \left[ (\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2 + 6(\sigma_{12}^2 + \sigma_{13}^2 + \sigma_{23}^2) \right]^{1/2}. \quad (1.1.4)$$

Condition of Eq. (1.1.2) coincides with Schleicher–Nadai’s criterion of rock failure caused by shearing loads and describes the beginning of the rock-failure process. It can also be used at the “rock prefailure” stage (when loading constitutes up to 60%–90% of the critical value) for describing the shape of areas with rapid crack growth.

To demonstrate the complex character of dilatancy zones, we use the simplest model of the Earth’s crust, which is taken as a uniform, isotropically elastic half-space. This complexity manifests itself even when a point force is a source of tectonic stresses. Exact solutions for elastic displacements and stresses, from a point source satisfying the conditions of zero stresses at the surface  $z = 0$ , were used to model the stress field in an elastic half-space (Alekseev et al., 1998b).

The domain surface  $D_\tau = 0$  from Eq. (1.1.2) for the double-force source at a depth of 15 km is shown in Fig. 1.1.1. Here, the parameters of the elastic half-space are as follows:

$$v_p = 6000 \text{ m/s}, \quad v_s = \frac{v_p}{\sqrt{3}}, \quad \lambda = \mu = \rho v_s^2 = 3.48 \times 10^{10} \text{ Pa},$$

$$\rho = 2900 \text{ kg/m}^3, \quad g = 9.9 \text{ m/s}^2, \quad Y = 3 \times 10^6 \text{ Pa}, \quad \alpha = 0.5.$$

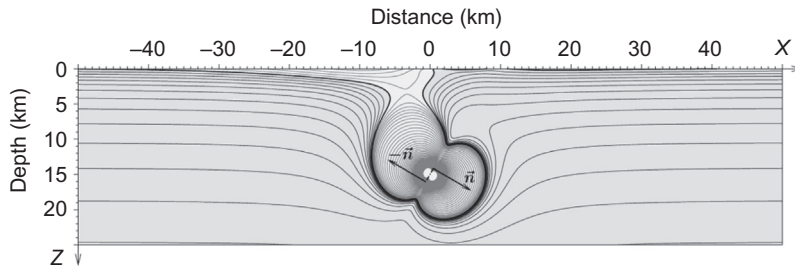


FIGURE 1.1.1

Exact solutions for elastic displacements and stresses from a point source satisfying the conditions of zero stresses at the free surface show that there are two dilatancy zones—the “source” zone in the vicinity of the elastic dipole application point and the “surface” zone in the layer near the free surface. Shapes of “source” and “surface” dilatancy zones in the plane  $y = 0$  (15 km = source depth), double force  $M_0 = 3 \times 10^{20}$  N,  $\mathbf{n} = (\cos \varphi, 0, \sin \varphi)$ ,  $\mathbf{p} = (0, 1, 0)$ ,  $\varphi = 30^\circ$ .

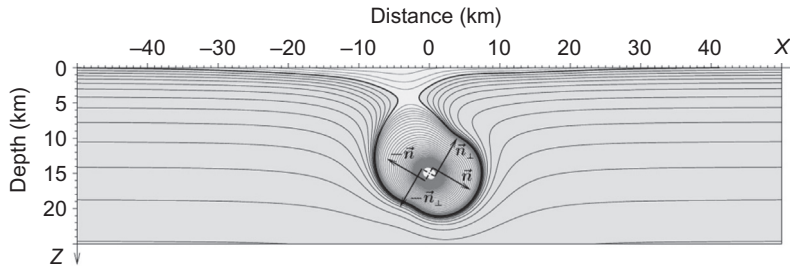
The double force is specified in the form of a moment tensor,  $\mathbf{M} = M_0 \mathbf{n}\mathbf{p}$ , where  $M_0$  is the dipole momentum (scalar), and  $\mathbf{n}\mathbf{p}$  is a dyad characterizing the orientation of the force ( $\mathbf{n}$ ) and the arm of the force ( $\mathbf{p}$ ).

The domain surface  $D_\tau = 0$  from Eq. (1.1.2) for the double-force couple source at a depth of 15 km is shown in Fig. 1.1.2. The double-force couple is set in the form of a moment tensor,  $\mathbf{M} = (1/2)M_0(\mathbf{n}\mathbf{p} + \mathbf{p}\mathbf{n})$ . In this case, the conformable matrix is symmetric.

Tangential stresses inside the domain  $D_\tau \geq 0$  dominate over compressional stresses. The resistance of the medium to shearing forces is overcome due to cohesion. Conditions favorable to crack increases are modeled. Note that the mechanisms of crack opening and the rheological changes to the geological medium in the zone  $D_\tau \geq 0$  are not described by these solutions. The solutions are valid only for determining the transition from the elastic state to the state of nonlinear loosening.

The interesting feature in both Figs. 1.1.1 and 1.1.2 is the formation of two dilatancy zones, which are the “source” zone in the vicinity of the source point and the “surface” zone in the upper part of the model. Here, the stress field from the source mostly affects tangential stresses, while the compressional stresses and the hydrostatic pressure contribute only slightly, due to the proximity to the surface.

The behavior of the surface dilatancy zone varies, depending on the following parameters:  $h$  (the source depth),  $M_0$  (the source intensity), the angle  $\varphi$  (the force orientation in the source), and  $Y$  (the cohesion of the medium’s elements). It can vanish with increasing source depth or merge with the source zone as the source intensity increases. In some cases, the horizontal size of the surface zone is 200 (or more) km, with a complex shape when projected onto the Earth’s surface. It is easy to verify that the pattern of displacement in dilatancy zones along the surface can be complex, particularly when influenced by several sources distributed in space whose intensity varies over time.



**FIGURE 1.1.2**

As in Fig. 1.1.1 except for the double-force couple

$$M_0 = 6.75 \times 10^{20} \text{ N}, \mathbf{n}_\perp = \mathbf{p}, \mathbf{p}_\perp = \mathbf{n}, \mathbf{n} = (\cos \varphi, 0, \sin \varphi), \mathbf{p} = (0, 1, 0), \varphi = 30^\circ.$$

Since anomalous geophysical fields are naturally related to surface dilatancy zones, to effectively investigate the sources of these anomalies, the location of the dilatancy zones must be determined as reliably as possible.

The condition of loosening rocks, taking into account the mechanisms of crack formation by tension of the medium, can be written in the form of a new criterion:

$$0 \leq D_\sigma \equiv \begin{cases} \frac{1}{2}\sigma_1(1 - \sin \varphi) - \frac{1}{2}\sigma_3(1 + \sin \varphi) - Y \cos \varphi, & \text{if } -\sigma_3 \geq \sigma_p, \\ -\sigma_3 - \sigma_p, & \text{if } -\sigma_3 < \sigma_p, \end{cases} \quad (1.1.5)$$

where  $\sigma_1$  and  $\sigma_3$  are the largest and smallest principal stresses, respectively;  $\varphi$  is the angle of internal friction;  $Y$  is the cohesion; and  $\sigma_p$  is the rock strength. This criterion determines dilatancy zones under conditions in which the medium can withstand large shearing stresses, but offers less resistance to tensile forces.

Note that, so far, the existence of surface dilatancy zones under real conditions should be considered a hypothesis. The use of this hypothesis for practical analysis of anomalies within various geophysical fields is an attempt to find reasons for the evolution of these anomalies, and to clarify the mechanisms of their interrelation.

Monitoring of the crack-density variation in the source zone is of special importance for short-range earthquake prediction. This monitoring should not be separated from observations of the surface dilatancy zone and the anomalous fields associated with it. First, the location of the future source is not known exactly, and its determination is closely related to the behavior of surface anomalous zones. Second, the reliability and accuracy of estimating the crack-density function in the source zone depend on the information regarding field anomalies in surface zones.

Before proceeding to the scheme for estimating the integral precursor in the source using multidisciplinary monitoring, it is reasonable to consider the relationship between crack-formation processes in the source and surface dilatancy areas.

Generally, a stress field is an energy-charged medium in which a relationship exists between the source and surface zones. The lines of largest tangential stress (or largest tensile stress) provide an estimate for the predominant orientation of cracks that occur in zones of the source and surface types.

When dilatancy zones of the two types are combined into one zone, there is a joint area of fracturing. This area combines the surface zones of anomalous fields with the source zone, in which the development of the rock failure process can directly influence the change in the geophysical anomalies. When dilatancy areas are separated, the source zone can retain its direct influence on the anomalies of some fields (e.g., on the values of the velocities  $v_p$  and  $v_s$ ) by a joint area of introduced anisotropy within the medium. The anisotropy coefficients can be expected to vary in a special way during the crack growth process, because the

orientation of the axis of symmetry remains the same. This can simplify the problem of estimating the average number of cracks by a high-resolution vibroseismic method (Aleksseev et al., 1998a,b, 1999, 2001).

---

### 1.1.3 Multidisciplinary model of integral precursor and combined inverse problems

An analysis of crack-system development at earthquake sites prior to seismic activity shows that earthquake prediction of rock failure should include a determination of the major space–time characteristics of the crack systems at these sites. Such investigations should be performed in the dilatancy zones where the crack systems are developed.

Observations of geophysical anomalies enable investigators to determine the crack-density function. It was assumed in the previous section that the crack-opening processes in dilatancy zones are related to the mechanisms forming anomalous fields. Qualitatively, the formation of anomalies in gravitational, electrical-conductivity, groundwater-level, and gas- and fluid-permeability fields can be explained by cracks. It is evident that special investigations are needed to obtain quantitative models of geophysical fields in fractured media (Brace et al., 1966).

In the process of deformation (prior to failure), loosening is characterized by the volume expansion (dilatation)  $\Theta = \text{div } \mathbf{U}$ , where the divergence is calculated from the elastic displacement vector. It is assumed here that the vector components are sufficiently smooth (differentiable) functions. If we consider a small volume  $V_0$ , which is  $V_1$  after deformation, then  $V_1 = V_0 (1 + \Theta)$ . If the medium's density is  $\rho_0$ , after deformation it is  $\rho_1 = \rho_0/(1 + \Theta)$ . For large, deformed volumes, this loosening is considerable. It generates an anomaly in the gravitational field  $V(x, y, 0) = V^0(x, y)$ , which we can use to solve the inverse problem

$$\Delta V = -4\pi\rho_1\Theta, \quad V|_{z=0} = V^0(x, y), \quad (1.1.6)$$

to determine density  $\rho_1(x, y, z) = \rho_0/(1 + \Theta)$ , and loosening  $\Theta$  (if this inverse problem can be solved uniquely and the initial density is known). The main difficulty—the one that leads us to consider multidisciplinary (combined) statements of inverse problems—is that the inverse problem posed by Eq. (1.1.6), does not have a unique solution. It is ill-posed, an attempt to find a three-dimensional function  $\rho_1(x, y, z)$  using a known two-dimensional function  $V^0(x, y)$ . This is impossible without additional information. The significance of combining problem statements is in the use of additional information from the solution to state a new inverse problem for the same physical quantity.

The approach for determining the characteristics of cracking (the integral precursor) using data from geophysical anomalies can also utilize the idea of the surface dilatancy zone. Let us introduce a medium's volume expansion

(loosening) function,  $\theta(x, y, z, t)$ . This function can be considered piecewise continuous, and it is assumed to be equal to the total relative volume of cracks in the medium's unit volume. The number of cracks in the unit volume can be determined by the formula  $N = \theta(x, y, z, t) / \theta_L(x, y, z, t)$ , where  $\theta_L(x, y, z, t)$  is the relative average volume of a crack with length  $L$ .

Let us consider a combined inverse problem for gravitational and electric data, groundwater-level evaluation, and the seismic method for measuring the effective anisotropy coefficients of cracked rocks, on the basis of the complementary principle of geophysical methods (Alekseev et al., 1995; Alekseev, 1992)—to obtain reliable estimates of the function. Each of these methods is based on measurements at the surface  $z = 0$  of a corresponding geophysical field

$$U_v(x, y, 0, t_k) = U_v^0(x, y, t_k), \quad (1.1.7)$$

here,  $t_k = kT_v$ , with  $T_v$  being the time interval between the recording times of field values during monitoring.

Methods for solving direct and inverse problems exist for all geophysical fields that are used in the problem of earthquake prediction (Alekseev, 1967; Alekseev et al., 1971, 1958; Alekseev and Mikhailenko, 1977; Alekseev and Tsubulchik, 1996; Alterman and Karal, 1968; Babich et al., 1985; Mikhailenko, 1978; Petrashen, 1978). In direct problems, the equations for the field

$$L_v(U_v, \alpha_v, \beta_v) = f_v(x, y, z, t), \quad (1.1.8)$$

surface conditions

$$l_v(U_v, \alpha_v, \beta_v)|_S = h_v(s, t), \quad (1.1.9)$$

and the initial data

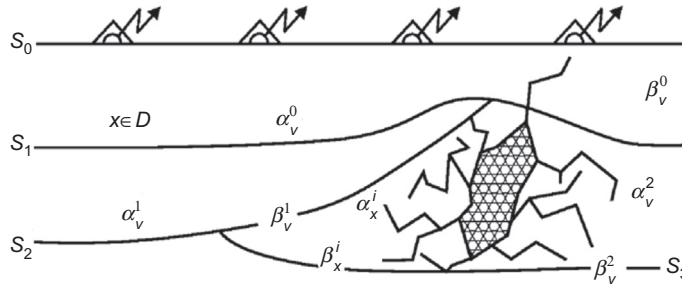
$$U_v(x, y, z, t)|_{t=0} = U_v^0(x, y, z) \quad (1.1.10)$$

are assumed to be given. Here  $\alpha_v(x, y, z)$  and  $\beta_v(x, y, z)$  are the physical and geometrical characteristics of the medium;  $f_v(x, y, z, t)$  is the external volume sources of the field; and  $h_v(s, t)$  is the sources at the surface  $S$ . The statement of the combined inverse problem is illustrated by Fig. 1.1.3.

Numerical methods for solving direct problems exist for many of the above-mentioned fields. These methods use specified geological medium characteristics  $\alpha_v(x, y, z)$ ,  $\beta_v(x, y, z)$ , the field sources, and the surface  $S$ . In inverse problems, the following information is known: the field  $U_v^0(s_i, t)$  at a series of points  $s_i$  at the surface  $S$  and the sought-for characteristics of the medium  $\alpha_v$ ,  $\beta_v$ , or other elements of the problem (the shape of the surface  $S$ , some sources  $f_v$  or  $h_v$ ).

The model of a multidisciplinary (combined) inverse problem determines the integral precursor  $\theta(x, y, z, t)$ , that is, the relative crack-density function. In this case, all geometrical and physical parameters of the medium, with the exception of the function  $\theta(x, y, z, t)$ , are considered to be known, and the function  $\theta(x, y, z, t)$  is considered to be independent of time during each field measurement  $t_k = kT_v$ .





**FIGURE 1.1.3**

Statement of the combined inverse problem. The physical and geometrical characteristics of the medium  $\alpha_v(x, y, z)$ ,  $\beta_v(x, y, z)$ , the field sources, and the surfaces  $S_i$ .

An optimization method can be used to solve the combined inverse problem. Let  $\beta_v(x, y, z, t, \theta)$  represent the operator for calculation of the field  $U_v(x, y, z, t)$  in the direct problem for the method with the number  $v$ . The problem lies in determining  $\theta(x, y, z, t)$  which minimizes the functional:

$$I(\theta) = \min_{\theta \in M_\theta} \sum_{v=1}^m \gamma_v [U_v^0(x, y) - B_v(x, y, 0, t_k, \theta)]^2, \quad (1.1.11)$$

where  $\gamma_v$  is the weight coefficients for individual methods,  $M_\theta$  is an a priori set of possible solutions  $\theta$ ;  $U_v^0(x, y)$  is the measured field, and  $\beta_v(x, y, 0, t_k, \theta)$  is the modeled field. The functional represented in Eq. (1.1.11) assumes no statistical correlation between the measured fields.

Optimization methods traditionally involve considerable computational difficulties. They are associated with simultaneously solving a large number of direct problems for different fields. In addition, the functional being minimized often has many local minima, making searches for the global minimum difficult. To solve such problems successfully, one should use high-performance computers and good initial approximations to the sought-for functions.

The seismic method, using powerful vibroseismic sources, can yield more detailed data about the medium structure, including an evolution of fractured zones. Employing observation systems with multiple overlaps, these sources provide resolution similar to well-known results in seismic prospecting.

Here we will not consider the capabilities of active seismology (Alekseev et al., 1997) using powerful vibrators, signals from which can be recorded at distances of up to 500–1000 km. Rather, we shall consider vibroseismic sounding of dilatancy zones as necessary instruments to increase the reliability and accuracy of the obtained information.

### 1.1.4 Methods for vibroseismic monitoring of seismic-prone zones

The Siberian Branch of the RAS has gathered unique experimental data from field observations (Alekseev et al., 1995, 1996, 2004, 2005). Seismograms have been obtained at distances of up to 400 km, and records of monofrequency signals have been obtained at distances of up to 1000 km, using vibrators with forces of 50, 100, and 250 tons. Among the important problems for active seismology are the methods for vibroseismic monitoring of seismic-prone zones and, in particular, a method for determining the function  $\theta(x, y, z, t_k)$ . To determine  $\theta(x, y, z, t_k)$ , the deep seismic sounding (DSS) scheme can be used, together with the common depth point scheme, at profiles 150–200 km in length over the source of an impending earthquake.

At the stage of long-range prediction, the period between soundings can be from 6 months to 1 year. At the stage of short-range prediction, soundings must be more frequent and observation systems must be more detailed.

We assume that the medium's properties vary only slightly between measurements. These small variations can be made into the major elements of variability in seismic cross-sections with the help of the “interframe correlation” method (i.e., by the subtraction of sequential images of the medium and analysis of increments).

An analysis of experiments on rock failure shows that the variability in crack sizes is greater than the variability in the dominant orientation of cracks (Nur, 1971). Sometimes crack sizes vary abruptly during the transition to the next scale level of rock failure (Zhurkov, 1968). This property enables us to simplify and refine the algorithms for processing of vibroseismic observations.

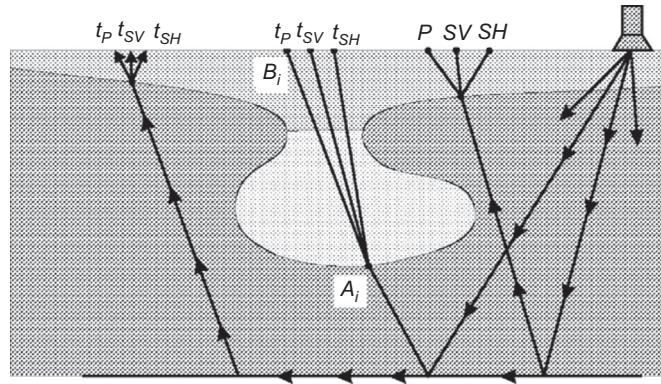
A general monitoring scheme is shown in Fig. 1.1.4. Automatic data processing using this scheme assumes the development of migration methods and the solution of inverse dynamic problems within the total system of equations for dynamic elasticity in an anisotropic medium:

$$\frac{\partial \sigma_{ij}}{\partial x_j} + \rho \frac{\partial \Phi}{\partial x_i} = \rho \frac{\partial^2 U_i}{\partial t^2} \quad (1.1.12)$$

with the generalized Hooke's law

$$\sigma_{ij} = C_{ik}(v_s, K_s, K_f, \theta) \varepsilon_{kj}, \quad (1.1.13)$$

where  $\sigma_{ij}$  is the stresses;  $\varepsilon_{kj}$  is the deformations;  $\Phi$  is the gravitational potential;  $C_{ik}$  is the effective parameters of anisotropy for the fractured medium;  $v_s$  is the Poisson's coefficient for the imbedding (elastic isotropic) medium;  $K_s$  is the modulus of volume deformation in the imbedding medium;  $K_f$  is the modulus of volume deformation for the liquid or gaseous phase in the porous half-space; and  $\theta$  is the volume density of cracks.



**FIGURE 1.1.4**

The general scheme of monitoring of the medium using a vibrational source. Vibroseismic observation profile of  $P$ ,  $SV$ , and  $SH$  waves for the monitoring function  $\theta(x, y, z, t_k)$  in the dilatancy zone. This function characterizes the development of crack systems in the earthquake source and in anomaly zones of geophysical fields. To determine  $\theta(x, y, z, t_k)$ , the deep seismic sounding (DSS) scheme can be used together with the common depth point (CDP) scheme at profiles 150–200 km in length over the source of an incipient earthquake.

Eq. (1.1.12) (often lacking the gravitational potential term) and Hooke's law [Eq. (1.1.13)] are widely used in geophysics to describe seismic waves in fractured media. There are several formulations of the generalized Hooke's law, with the anisotropy coefficients approximating the wave processes in fractured media at low frequencies (Budiansky and O'Connell, 1976; Crampin, 1978, 1984). The density of cracks  $\theta$  is present explicitly in Hooke's law [Eq. (1.1.13)]; depending on the assumed shape of cracks. The following formula is given in Hoenig (1979):

$$e = \frac{2N A^2}{\pi P_e}. \quad (1.1.14)$$

This formula defines the density of cracks  $e$  for the densely packed  $N$  parallel-plane elliptic cracks of the area  $A$  with perimeter  $P_e$ . It is valid for any plane cracks with convex boundary shapes.

Eqs. (1.1.12) and (1.1.13) form the basis for seismic data processing in seismic prospecting and seismology. In practice, simplified kinematic approaches have been used so far.

The observation scheme for waves reflected and refracted from the Moho surface in the Earth's crust, using vibroseismic sounding of the source and surface dilatancy zones, is shown in Fig. 1.1.4. The presence of fractures in these zones, and the changes in their volume density during the periods between monitoring sessions, can be determined from the changes in anisotropy coefficients and wave

propagation velocities. The transverse wave  $S$  splits into  $SV$  and  $SH$  waves at the boundaries of dilatancy zones. The depth and shape of the boundaries, as well as the wave propagation velocities, can be determined (by well-known methods) from the lags  $\Delta t_{SV}$ ,  $\Delta t_{SH}$  in the arrival times of the corresponding waves at the points  $B_{SV}^i$ ,  $B_{SH}^i$ , and the source located at the point  $A^i$ .

For the fractured model of the type (Hoenig, 1979), the velocities of all three wave types  $v_P$ ,  $v_{SV}$ , and  $v_{SH}$  are determined by the formula

$$v = \frac{v_0}{\sqrt{1 + ef(\gamma)}}, \quad (1.1.15)$$

where  $v_0$  is the wave velocity in the medium before the appearance of cracks,  $f(\gamma)$  is one function for all types of waves (Garbin and Knopoff, 1975; Crampin, 1978), and  $\gamma$  is the angle between the direction of wave propagation and the direction normal to the orientation of plane cracks. The quantity  $e$  from Eqs. (1.1.13) and (1.1.14) is the sought-for function  $\theta(x, y, z, t_k)$ . It can be determined not only from seismic monitoring data, but also from routine seismologic observations of the velocities  $v_P(t_k)$ ,  $v_{SV}(t_k)$ , and  $v_{SH}(t_k)$  at seismic stations.

Estimating the sensitivity of active monitoring to changes in the elastic characteristics of the interior zone of the Earth's crust can be made using mathematical modeling (Kovalevsky, 2006). The model of the Earth's crust–mantle system in the form of a layer at a half-space with different velocity values of elastic waves is considered. The mathematical statement of the problem is made by approximating the acoustic wave equation. It is assumed that the vibrational source has a constant oscillation frequency and that the zone of changes in the medium has a spherical shape. The wave field in the medium is calculated using a ray approximation. Wave field variations in the medium and at the free surface are determined for the case of small velocity changes by calculating the beam pattern of a fictitious 3D source in a diffraction approach. As a result of such modeling, we estimated the sensitivity of active monitoring methods to harmonic vibrational signals.

The relationship between the relative variations in velocity within the zone of parameter variation and those of the recorded-signal amplitudes is as follows:

$$\frac{\delta c}{c} = 3 \times 10^{-3} \alpha \frac{\delta u}{u} \left( \frac{R_{V-Z} R_{Z-S}}{r_0 R_{V-S}} \right) \left( \frac{\lambda}{r_0} \right)^2, \quad (1.1.16)$$

where  $\delta c/c$  is the relative variations in wave velocities within the zone of parameter variation,  $\delta u/u$  is the relative variations in the signal amplitudes recorded on the free surface,  $R_{V-Z}$  is the distance between the vibrator and the zone of parameter variation,  $R_{Z-S}$  is the distance from the zone of parameter variation to the recording point (seismometer),  $r_0$  is the radius of the zone,  $\lambda$  is the wavelength, and  $\alpha$  is the reflection coefficient, lying within 0.15–1 for the model and the wave velocities in the core and mantle (Kovalevsky, 2006).

Experience shows that variations in the amplitudes of monofrequency signals at distances of 100–400 km from the vibrator, at the existing microseismic noise

level, can be determined with an accuracy of  $10^{-2}$ . Therefore monitoring at the frequency  $f = 6$  Hz (character wavelength  $\lambda = 1$  km) and at typical source–recorder and source–anomaly distances of 50–100 km (and for the zone of parameter variation with a radius of 1–10 km) gives the following estimates of the relative variations in seismic wave velocities:

$$\begin{aligned} r_0 = 1 \text{ km}, \quad \delta c/c &= 10^{-2} - 10^{-3}, \\ r_0 = 10 \text{ km}, \quad \delta c/c &= 10^{-5} - 10^{-6}. \end{aligned} \quad (1.1.17)$$

These estimates show that the sensitivity of the active monitoring method is somewhat high for seismologic methods. This suggests its potential effectiveness for monitoring changes in the stressed-deformed state in the dilatancy zones of future earthquakes.

---

### 1.1.5 Conclusion

Although the earthquake development process takes a long time (up to several hundred years), it is an energy-saturated process. Substantial rheological changes in the geologic medium take place at imminent-earthquake sites, and the varying sorts of anomalous zones are formed within geophysical fields at such sites. Crack opening in zones of increased shearing and tensile stresses is the most universal mechanism of rock changes. Most seismologists believe that the initial stage of crack opening, and the subsequent state of the medium, when rock failure processes are developing, are associated with a medium's dilatancy.

This chapter describes the formation of surface dilatancy zones, which can cause variations in geophysical fields—and can thus be potential earthquake precursors. This process is illustrated using an example of point sources for double forces and a double pair of forces. The necessary condition for the creation of dilatancy zones is an Earth surface that is free from stresses. In this case, dilatancy zones are formed from any distribution of forces, creating a nonzero component of shearing stresses. Therefore most earthquakes are accompanied by the creation of such zones. Note that the existence of surface dilatancy zones under real conditions at this point should be taken as a hypothesis. Using this hypothesis—for practical analysis of extensive accumulated data on the monitoring of anomalies at various geophysical fields—is an attempt to find the reasons for the evolution of these anomalies and to elucidate the mechanisms of their interrelation.

In this chapter a mathematical model of an integral earthquake precursor is proposed. Physically, it represents the space–time function of crack density in a zone of the highest stresses on the Earth's surface, which manifests itself in anomalous geophysical fields. Mathematically, the integral precursor is determined using the optimization method of a multidisciplinary (combined) statement of the inverse problem for the corresponding geophysical fields (the field of

displacements and deformations on the Earth's surface, the electric conductivity field, anomalies of the gravitational field, the groundwater level, etc.).

A vibroseismic monitoring scheme for dilatancy zones is discussed in detail. Estimates of the accuracy obtained with the use of this scheme, which employs powerful vibrators and recording systems capable of long signal accumulation, are presented. We show that it is possible to detect relative changes in seismic wave velocity of about  $10^{-5}$ – $10^{-6}$  in an internal zone, with radius 10 km, using a vibromonitoring system with a 100-ton vibrator and a recording system with 50–100 km offset. These estimates show the high resolution of active vibroseismic monitoring compared to standard seismological methods for the probing of seismic-prone zones within the Earth's crust.

---

## Acknowledgments

The work has received support from the Russian Foundation for Basic Research, Grants No. 04-05-64177, 05-05-64245, 06-05-64265, 07-05-00858, 07-07-00214; Fundamental Research Program of the RAS No. 16.5 and 16.6; Interdisciplinary Integration Projects of the SB RAS No. 16, 57, and 133.

---

## References

- Alekseev, A.S., 1967. Inverse dynamic problems of seismics. In: Lavrentyev, M.M. (Ed.), *Some Methods and Algorithms for Interpretation of Geophysical Data*. Nauka Publishing House, Moscow, pp. 9–84. (in Russian).
- Alekseev, A.S., 1992. Complementary features of geophysical methods and the computational aspect of joint data inversion. In: *Proceedings of 54th Meeting of European Association of Exploration Geophysics*, Paris, pp. 750–751.
- Alekseev, A.S., 1993. A multidisciplinary mathematical model of combined foreshock for earthquake prediction research. *J. Earthquake Pred. Res.* 2, 137–151.
- Alekseev, A.S., Mikhailenko, B.G., 1977. Numerical modeling of seismic wave propagation in a radially inhomogeneous Earth's model. *Dokladi Akademii Nauk SSSR* 235, 46–49 (in Russian).
- Alekseev, A.S., Tsubulchik, G.M., 1996. Inverse dynamic problems of wave diffraction in the problem of seismic monitoring. *Vibrational Sounding of the Earth Vibrosounding*. Nauka Publishing House, Moscow, pp. 22–25 (in Russian).
- Alekseev, A.S., Babich, V.M., Gelchinsky, B.Ya, 1958. On determination of the intensity of waves in elasticity theory by the ray method. *Dokladi Akademii Nauk SSSR* 1, 661–664 (in Russian).
- Alekseev, A.S., Lavrentyev, M.M., Mukhometov, R.G., Nersesov, I.L., Romanov, V.G., 1971. A numerical method for determining the structure of the Earth's upper mantle, *Mathematical Problems of Geophysics*, vol. 2. CC SB USSR AS Publishing House, Novosibirsk, pp. 143–165 (in Russian).
- Alekseev, A.S., Glinsky, B.M., Kovalevsky, V.V., Mikhailenko, B.G., 1995. A multidisciplinary mathematical model for earthquake prediction studies and vibroseismic

- monitoring of seismic prone zones. In: Proceedings of Second International Conference of Seismology and Earthquake Engineering. Teheran: JJEES 1, pp. 97–104.
- Alekseev, A.S., Glinsky, B.M., Emanov, A.F., Kovalevsky, V.V., Mikhailenko, B.G., Seleznev, V.S., et al., 1996. Investigation of the structure of the vibroseismic field from powerful sources. *Problems of Geoacoustics – Methods and Facilities*. MSMU Publishing House, Moscow, pp. 155–160.
- Alekseev, A.S., Glinsky, B.M., Kovalevsky, V.V., Mikhailenko, B.G., 1997. Problems of active seismology. In: Fuchs, K. (Ed.), *Upper Mantle Heterogeneities From Active and Passive Seismology*. Nato ASI Series, vol. 17. Kluwer Academic Publishers, 123–130.
- Alekseev, A.S., Belonosov, A.S., Petrenko, V.E., 1998a. A mathematical model of determining the stress field and dilatancy zones by geodetic data. *Bull. Novosibirsk Comput. Center, Ser.: Math. Model. Geophys.* 4, 15–22.
- Alekseev, A.S., Belonosov, A.S., Petrenko, V.E., 1998b. On the multidisciplinary approach to determination of an integral earthquake precursor. *Bull. Novosibirsk Comput. Center, Ser.: Math. Model. Geophys.* 4, 23–45.
- Alekseev, A.S., Belonosov, A.S., Petrenko, V.E., 1999. On the multidisciplinary approach to determination of an integral earthquake precursor. *J. Earthquake Pred. Res.* 10, 462–471.
- Alekseev, A.S., Belonosov, A.S., Petrenko, V.E., 2001. On the concept of multidisciplinary earthquake prediction with the use of an integral precursor, *Problems of the Dynamics of Lithosphere and Seismicity, Computational Seismology*, vol. 32. GEOS Publishing House, Moscow, pp. 81–97 (in Russian).
- Alekseev, A.S., Geza, N.I., Glinsky, B.M., Emanov, A.F., Kashun, V.N., Kovalevsky, V.V., et al., 2004. *Active Seismology With Powerful Vibrational Sources*. Novosibirsk, ICM&MG SB RAS, Filial “Geo” Publishing House of the SB RAS (in Russian).
- Alekseev, A.S., Chichinin, I.S., Korneev, V.A., 2005. Powerful low-frequency vibrators for active seismology. *Bull. Seismol. Soc. Am.* 95, 1–17.
- Alterman, Z.S., Karal Jr., F.C., 1968. Propagation of elastic waves in layered media by finite-difference methods. *Bull. Seismol. Soc. Am.* 58, 367–398.
- Babich, V.M., Buldyrev, V.S., Molotkov, I.A., 1985. *The Space-time Ray Method*. LSU Publishing House, Leningrad (in Russian).
- Brace, W.F., Paulding, B.W., Scholz, C., 1966. Dilatancy in the fracture of crystalline rocks. *J. Geophys. Res.* 71, 3939–3952.
- Budiansky, B., O’Connell, R.J., 1976. Elastic moduli of a cracked solid. *Int. J. Solids Struct.* 12, 81–97.
- Crampin, S., 1978. Seismic wave propagation through a cracked solid: polarization as a possible dilatancy diagnostic. *Geophys. J. R. Astron. Soc.* 53, 467–496.
- Crampin, S., 1984. Effective anisotropic elastic constants for wave propagation through cracked solids. *Geophys. J. R. Astron. Soc.* 76, 135–145.
- Garbin, H., Knopoff, L., 1975. Elastic moduli of a medium with liquid-filled cracks. *Q. Appl. Math.* 33, 301–303.
- Hoenig, A., 1979. Elastic moduli of a nonrandomly cracked body. *Int. J. Solids Struct.* 15, 137–154.
- Keilis-Borok, V.I., Molchan, G.M., 1968. *Computational Seismology*. GEOS Publishing House, Moscow (in Russian).
- Kovalevsky, V.V., 2006. Estimation of sensitivity of the active monitoring method by harmonic signals. *Bull. Novosibirsk Comput. Center Ser.: Math. Model. Geophys.* 11, 105–114.
- Ma, L., Chen, J., Chen, Q., Liu, G., 1995. Features of precursor fields before and after the Datong–Yanggao Earthquake swarm. *J. Earthquake Pred. Res.* 4, 1–30.

- Mei, S.-R., 1992. Progress in earthquake prediction in China during the 80ies. *J. Earthquake Pred. Res.* 1, 43–57.
- Miao, L., 1993. Monitoring and prediction of the Datong earthquake. *J. Earthquake Pred. Res.* 2, 299–310.
- Mikhailenko, B.G., 1978. Calculation of theoretical seismograms for multidimensional-inhomogeneous models of media. *Conditionally Well-Posed Problems of Mathematical Physics in the Interpretation of Geophysical Observations*. CC SB USSR AS Publishing House, Novosibirsk, pp. 75–88 (in Russian).
- Myachkin, V.I., Kostrov, B.V., Sobolev, G.A., Shamina, O.G., 1974. Laboratory and theoretical investigations of the earthquake preparation processes. *Izvestiya Phys. Solid Earth* 10, 107–112 (in Russian).
- Myachkin, V.I., Kostrov, B.V., Sobolev, G.A., Shamina, O.G., 1975. Fundamentals of the physics of the source and earthquake precursors. *Earthquake Source Physics*. Nauka Publishing House, Moscow, pp. 6–29 (in Russian).
- Nersesov, I.L., Semenov, A.N., Simbireva, I.G., 1971. Space-time distribution of travel times of p and s waves in the garm region. *Experimental Seismology*. Nauka Publishing House, Moscow, pp. 334–345 (in Russian).
- Nikolayevskii, V.N., 1982. A review: Earth's crust, dilatancy and earthquakes. *Advances of Science and Engineering*. Mir Publishing House, Moscow, pp. 133–215 (in Russian).
- Nur, A., 1971. Effects of stress on velocity anisotropy in rocks with cracks. *J. Geophys. Res.* 78, 2022–2034.
- Petrashen, G.I., 1978. Fundamentals of the mathematical theory of elastic wave propagation, *Issues of the Dynamic Theory of Seismic Wave Propagation*, vol. 18. LSU Publishing House, Leningrad, pp. 1–248 (in Russian).
- Sobolev, G.A., 1978. Investigation of the formation of shear-type fracture precursors in laboratory conditions. *Search for Earthquake Precursors*. Nauka Publishing House, Moscow, pp. 86–99 (in Russian).
- Wang, C., Liao, S., 1996. Experimental study of the preparation and occurrence of strong earthquakes. *J. Earthquake Pred. Res.* 5, 525 p.
- Zhang, G., Zhang, Z., 1992. The study of multidisciplinary earthquake prediction in China. *J. Earthquake Pred. Res.* 1, 71–85.
- Zhurkov, S.N., 1968. Kinetic concept of the strength of solids. *Vestnik Akademii Nauk SSSR* 3, 46–52 (in Russian).
- Zhurkov, S.N., Kuksenko, V.S., Petrov, V.A., 1977. On the question of prediction of rock failure. *Izvestiya Phys. Solid Earth* 8, 11–18 (in Russian).