

Restoration of Borehole Source Coordinates and Parameters of the Near Wellbore Environment

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Abstract – The problem of the location determining of a borehole source, represented as low-power explosions, moving along the depth of the borehole, is considered. The problem is solved by the method of the inverse problem solving, where initial parameters are arrival times of the waves. As a result of the solution are the source spatial coordinates and the speed characteristics of the medium near the well, depending on the source immersion depth. The method of automatic measuring the waves arrival times is considered. Combination of both methods in the model experiment to determine the source location in the borehole and the velocity characteristics of the medium near the well in depth is used. The results of experimental measurements are given.

Index Terms – Borehole explosions, location, inverse problem.

I. INTRODUCTION

THE ACCURACY of determining the seismic parameters of the borehole environment – in-seam seismic velocities and the geometry of the boundaries is mainly determined by the data for the borehole trajectory in the three-dimensional space. Both problems are interrelated: the accuracy of the solution to the latter depends on that of the former. The determination of the borehole trajectory, in particular, the inclinometry of inclined boreholes, is rather difficult. It is well-known that the solution becomes more complex when the problem is solved in real time. To solve this problem, new algorithms and programs for automatic measurement of the arrival times of direct and reflected waves have been developed. This measurement uses the data on the recording of signals from the source by the areal observation system and by solving the inverse problem of reconstructing the parameters of the borehole source: the time in the source and its coordinates, like the seismic velocity in the medium.

The effectiveness of the algorithms and programs created was estimated by the data processing obtained on the basis of a scheme of direct and inverse VSP from ground-based and pulsed borehole sources. The results of numerical and model experiments have shown the accuracy in determining the coordinates within the first meters, which indicates that the methods created to solve the problem are rather effective.

II. PROBLEM DEFINITION

Let the axes x and y in the Cartesian system of coordinates x, y, z be directed along the Earth's surface, and let the axis z

be directed down to the earth's center. Let v denote the average propagation speed of the seismic wave in the vicinity of the borehole. The sensors that record (or radiate) seismic signals are located at the Earth's surface or in small boreholes, at points with coordinates (x_i, y_i, z_i) . Let t_i denote the propagation time of the seismic signal from the source at the borehole bottom (for instance, from the drilling bit) to the i -th point (or vice versa). It is necessary to determine the coordinates (x^*, y^*, z^*) of the borehole bottom and the velocity v . One can also formulate a problem in which it is difficult to fix the radiation time of the seismic signal, and it has to be included in the unknowns to be determined. Then it will be necessary to determine the coordinates (x^*, y^*, z^*) of the borehole bottom, the time in the source t^* , and the velocity v . The minimal number of sensors will increase to five. When estimating the unknown parameters of the borehole bottom, we use a nonlinear system of the so-called conditional equations [1, 2, 3]:

$$\vec{t} = \vec{\eta}(X, \theta) + \vec{\varepsilon} \quad (1)$$

where $\vec{t} = (t_1, t_2, \dots, t_N)$ is the vector of travel times of seismic signals, $\vec{\eta}(X, \theta)$ is the N -dimensional vector of measured travel times (a theoretical hodograph) or the regression function, $\vec{\varepsilon} = (\varepsilon_1, \dots, \varepsilon_N)^T$ is the residual vector, $\vec{\theta} = (x, y, z, v, t)^T$ is the m -dimensional vector of the parameters being estimated, $X = (\vec{x}_1, \vec{x}_2, \dots, \vec{x}_N)$ is the matrix of the coordinates of sensors (or radiation points), and N is the number of sensors (or radiation points). Information about the distribution of errors $\varepsilon_i = t_i(\vec{x}_i, \vec{\theta}) - \eta(\vec{x}_i, \vec{\theta})$ is used to estimate the parameters. From here on we assume that ε_i denotes mutually independent random variables distributed with the zero average and given variances: $E\varepsilon_i = 0$, $E\varepsilon_i \varepsilon_j = \sigma_i^2 \delta_{ij}$, $\sigma_i = \sigma(\vec{x}_i)$, δ_{ij} is the Kronecker delta, $i=1, 2, \dots, N$. In case of difficulties with the specification of variances, they are assumed to be equal, and an unbiased estimate of the observation variance with a unit weight in the problem solution is obtained. The latter approach is used in this paper.

III. COMPUTATIONAL METHODS

The problem of estimating the parameters $\vec{\theta}$ is part of the so-called regression analysis, and estimates of the least-squares method are its solution:

$$\vec{\theta} = \arg \min_{\vec{\theta} \in \Omega} Q(\vec{\theta}), \quad Q(\vec{\theta}) = \sum_{i=1}^N \sigma_i^{-2} (t_i - \eta(\vec{x}_i, \vec{\theta}))^2. \quad (2)$$

To find the minimum of the functional $Q(\vec{\theta})$, the Gauss-Newton iterative method or its modifications based on a linear approximation of the regression function in the vicinity of the point $\vec{\theta}^k$ are used:

$$J(X, \vec{\theta}^k) \Delta \vec{\theta}^k + \vec{\eta}(X, \vec{\theta}^k) - \vec{t} + \vec{\varepsilon} = 0 \quad (3)$$

where

$$J(X, \vec{\theta}) = \left(\frac{\partial \eta(\vec{x}_i, \vec{\theta})}{\partial \theta_1}, \frac{\partial \eta(\vec{x}_i, \vec{\theta})}{\partial \theta_2}, \dots, \frac{\partial \eta(\vec{x}_i, \vec{\theta})}{\partial \theta_m} \right), i=1, 2, \dots, n. \quad (4)$$

Estimates $\vec{\theta}$ are found as a result of the iterative process $\vec{\theta} = \lim_{k \rightarrow \infty} \vec{\theta}^k$:

$$\begin{aligned} \vec{\theta}^{k+1} &= \vec{\theta}^k + \Delta \vec{\theta}^k, \quad [J^T(X, \vec{\theta}^k) J(X, \vec{\theta}^k) + \alpha^2 I] \Delta \vec{\theta}^k = \\ &= J^T(X, \vec{\theta}^k) y(X, \vec{\theta}^k), \quad k = 0, 1, \dots \end{aligned} \quad (5)$$

Here $y(X, \vec{\theta}) = (\vec{t} - \eta(X, \vec{\theta}))^T$, α is the regularization parameter, and I is the unit matrix.

The other approach to solving problem (1)-(4), also used by the authors, is to solve system (3) directly at each step of the iterative process. At the present time, the method of pseudoinversion (or generalized inversion) based on singular decomposition (SVD-decomposition) is most widely used to solve this system [4-7]. Modern versions of the MATLAB system have a built-in function `svd(A)` that realizes this decomposition for an arbitrary matrix A of the order $n \times m$.

IV. PLANNING THE OBSERVATION SYSTEM

No matter how good the methods of solving systems (3) and (5) may be, in practice they are not very effective in the case of bad conditionality of matrix (4). Often, this is caused by poor organization of observations, namely, by inappropriate arrangement of seismic sensors with respect to the borehole bottom. One can conclude that observations should be planned, that is, one should select such an arrangement of sensors within the given territory that could maximally increase the conditionality of matrix (4) and, hence, improve the estimate of the borehole parameters. Specific problem statements of designing seismic observation systems are considered in [1-3]. Special software has been developed to solve these problems.

IV. ALGORITHM FOR AUTOMATIC DETERMINATION OF WAVE ARRIVAL TIMES

To determine the vector of wave arrival times t in the automatic measurement mode, one uses an algorithm of determining the arrival times of a quasi-periodic sequence of pulses at the background of Gaussian noise and estimating their shape [8, 9]. The following expression is taken as a goal function:

$$S_I(t_1, \dots, t_M) = \frac{1}{M} \sum_{i=1}^M \sum_{j=1}^M \sum_{k=0}^{q-1} y_{t_i+k} y_{t_j+k} \rightarrow \max_{\Omega} \quad (6)$$

where t_i, t_j , $i, j = \overline{1, M}$ are the arrival times of first waves;

y_{t_i+k}, y_{t_j+k} , $i, j = \overline{1, M}, k = \overline{0, q-1}$ are the waves of given duration q ;

$\Omega = \{(t_1, \dots, t_M) \mid 0 \leq t_1 \leq T_{\max} - q - 1, q \leq T_{\min} \leq t_i - t_{i-1} \leq T_{\max},$

$N - T_{\max} - q \leq t_M \leq N - q - 1, i = \overline{2, M}\}$

T_{\min}, T_{\max} specify the minimal and maximal values of the quasi-period, and M is the number of seismograms.

The criterion (6) is based on the maximum likelihood method. As a result of some transformations presented in [5], relation (6) is equivalent to the following expression:

$$\tilde{S}(t_1, \dots, t_M) = \sum_{i=1}^M G(t_i) = \sum_{k=0}^{q-1} \tilde{u}_k (\tilde{u}_k - 2y_{t_i+k}) \rightarrow \min_{\Omega} \quad (7)$$

where $\tilde{u}_k = y_{t_i+k}^*$, $k = 0, \dots, q-1$, $t_1^* = \underset{0 \leq t_1 \leq T_{\max}-q-1}{\operatorname{Argmax}} S_1(t_1)$.

An algorithm based on the method of dynamic programming described in (6) is proposed to solve the minimization problem (7). The following recurrence formulas of dynamic programming are valid for the minimization problem (7) on the set Ω :

$$S(n) = 0, \quad n \in [-T_{\max}, T_{\max} - T_{\min} - q - 1],$$

$$S(n) = \min_{n-T_{\max} \leq m \leq n-T_{\min}} \{S(m) + G(m)\},$$

$$t = 0, N - q + T_{\min} - 1,$$

$$S(N) = \min_{N-q \leq n \leq N-q-1+T_{\min}} \{S(n) + G(n)\},$$

$$\operatorname{Ind}(n) = 0, \quad n \in [-T_{\max}, T_{\max} - T_{\min} - q - 1],$$

$$\operatorname{Ind}(n) = \underset{n-T_{\max} \leq m \leq n-T_{\min}}{\operatorname{Argmin}} (S(m) + G(m)),$$

$$n = 0, N - q + T_{\min} - 1.$$

where $S(n)$ and $\operatorname{Ind}(n)$ denote the minimum value of the functional and the minimum indicator at the n -th step. The number of waves and their location in the sequence is determined by the recurrent calculation in the reverse order by using the minimum indicator:

$$\begin{cases} m_0 = \underset{N-q \leq n \leq N-q+T_{\min}-1}{\operatorname{Argmin}} (S(n) + G(n)) \\ m_i = \operatorname{Ind}(m_{i-1}), i = 1, 2, \dots, \end{cases} \quad (8)$$

And the process stops at such step $i=r$ that $\operatorname{Ind}(m_r) = 0$.

As a result of calculation by using formula (8), we obtain a sequence m_r, m_{r-1}, \dots, m_1 such that $(\tilde{t}_1, \tilde{t}_2, \dots, \tilde{t}_{\tilde{M}-1}, \tilde{t}_{\tilde{M}}) = (m_r, m_{r-1}, \dots, m_1)$. The quantity r gives the estimate \tilde{M} of the number of pulses that got in the frame. As a result of solving the minimization problem, we find an optimal set of the times of wave arrivals and their number:

$$(\tilde{t}_1, \tilde{t}_2, \dots, \tilde{t}_{\tilde{M}-1}, \tilde{t}_{\tilde{M}}, \tilde{M}) = \underset{\Omega}{\text{Arg min}} \tilde{S}(t_1, \dots, t_M).$$

Taking into account the estimates of maximum likelihood and the found parameters $\tilde{t}_i = 1, \tilde{M}, \tilde{M}$, one can easily find the sought-for components of the U-wave:

$$\hat{u}_k = \frac{\sum_{i=1}^{\tilde{M}} y_{\tilde{t}_i+k}}{\tilde{M}}, \quad k = 0, \dots, q-1.$$

IV. EXPERIMENTAL RESULTS

Model experiments were made with the help of the method of inverse vertical seismic profiling (IVSP) by using a water-filled borehole 135 m deep. A scheme of the experiments is presented in Fig. 1.

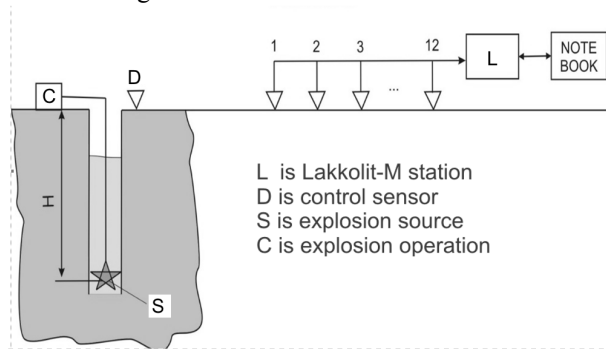


Fig. 1. Scheme of model experiments.

Powder explosions of 12.5 g and 30 g, respectively, were used as a source of seismic oscillations. Blasting control was remote, with electric current passing from a 220 V supply line through a wire in a glass with the explosive. The process of wire burnout initiated the powder blasting. The reference signal was recorded from sensor S_1 located at the borehole head. The signal was initiated by a hydroacoustic wave, which propagated from the source along the liquid column filling the borehole cavity. The reference signal was transmitted via the lines to the recording seismic station. A 12-channel digital seismic station "Lakkolit-M" is used to record seismic signals. For each 12-channel arrangement of seismic sensors (Fig. 1), explosions at depths of up to 120 m were recorded. The arrival times of direct waves were determined automatically with the help of the algorithm (8). The results of determining the arrival times of first waves are illustrated graphically in Fig. 2. The arrival times are denoted by points in each of the 11 seismograms. The measured values were used to solve the inverse problem (1) in order to determine errors in the calculation of the coordinates of the borehole bottom and wave velocities for various source depths. The results of the calculations are presented in Table I. The Table presents source depths, errors in determination

of the borehole bottom by the coordinates x, y, z , velocity values of direct waves, and errors in their determination, successively. The table data illustrate a rather high accuracy in determination of the source coordinates (the error along the coordinate z at maximal depths does not exceed (1-2)%, and the horizontal deviation does not exceed 2 m).

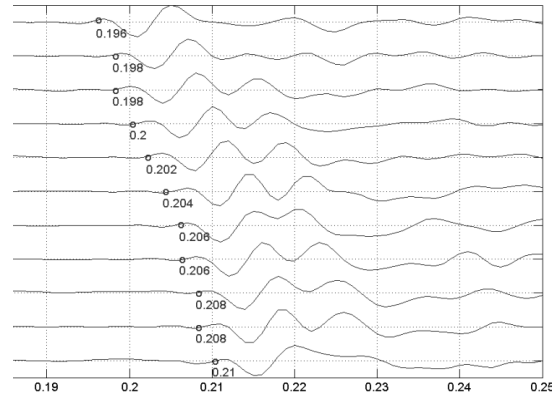


Fig. 2. The results of determining the arrival times of first waves.

TABLE I
COORDINATES OF BOREHOLE BOTTOM AND WAVE
VELOCITIES FOR VARIOUS SOURCE DEPTHS

Source Depth (m)	Error of determination (m)			Speed of seismic waves	Error. speed determination
	x	y	z	V (km/s)	Ev
br1	0.55	0.757	0.08	1.599	0.0058
br5	0.678	0.927	3.116	1.586	0.0061
br10	0.789	1.07	3.783	1.745	0.0077
br25	0.877	1.208	1.985	2.044	0.0099
br100	1.070	1.478	0.887	3.219	0.0186
br120	1.577	2.163	1.014	3.343	0.0276

VI. CONCLUSION

The problem of determining the position of the well source, presented in the form of low-power explosions, moving along the depth of the well, is considered. Its solution is represented by the method of solving an inverse ill-posed problem. The initial parameters for the solution are the wave arrival times. A method for automatically measuring the arrival times of waves is proposed and implemented, based on the discrete optimization of a purposeful search of seismograms. Combination of both methods allows to determine the spatial coordinates of the source and the speed characteristics of the near wellbore environment. The latter, in turn, are related to the depth of immersion of the source. The performed model experiment with charges of low power from the source coordinates in the

range of depths 0-120 m with the help of the created algorithms and programs showed the accuracy of the estimation of the parameters in the region (1-2)%. Thus the high efficiency of the created algorithms and programs is experimentally shown.

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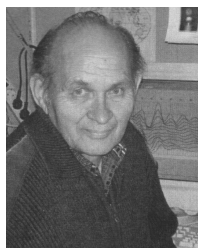
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