Using adaptive nested mesh code HydroBox3D for numerical simulation of Type Ia supernovae: merger of carbon-oxygen white dwarf stars, collapse, and non-central explosion

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Abstract—The paper presents the results of mathematical modeling of the three main stages of the explosion of supernovae of type Ia (SNIa): merging of white dwarfs, collapse and noncentral explosion. In the fusion problem shown the general fall mechanism of the satellite star onto the central star. At the modeling of the collapse process, the behavior of various types of energies is investigated. Also shown is the qualitative dynamics of a noncentral of a star explosion, whose source is the characteristic value of the energy released by the reaction C + C. The simulation was carried out using the original HydroBox3D code, in which the concept of adaptive nested mesh is implemented.

Index Terms—Adaptive nested mesh, Computational Hydrodynamics, Computation Astrophysics, Numerical simulation, SNIa.

1 Introduction

Supernovae are the main source of the elements of "life" from carbon to iron. Supernovae type Ia is unusually bright, so today they are used as "standard candles" to determine distances to galaxies and to determine the speed of expansion of the universe. Mathematical simulation of supernova explosion is the main tool for studying their dynamics and formation scenarios. The complexity of the flows in the explosion of supernovae puts forward stringent requirements for spatial resolution.

The main scenario [1] for the explosion of supernovae is based on the fusion of two degenerate white dwarf stars, followed by the collapse of a new star upon reaching the Chandrasekhar limit, launching of a carbon burning process and the subsequent explosion of a type Ia supernova. In the course of such a scenario, the following processes take place, requiring a separate study: merger, collapse, explosion. It is these steps that will be modeled separately in this paper.

Over the past year, the large number of hydrodynamic simulations of various processes was held at occurred in the process of white dwarfs leading to the formation of SNIa. An undoubted trend is the study of detonation of the helium shell with the subsequent nuclear ignition of the carbon nucleus, the modeling of such processes was carried out in the works [2], [3]. At the same time, the processes of detonation of the O-Ne shell with the subsequent nuclear combustion of the C-O nucleus are actively studied. [4]. In such processes, the non-trivial problem is the construction of an equilibrium profile under various components of a degenerate gas mixture, as well as the reproduction of nuclear reactions. In the following, we shall consider the problem of detonation of a shell with the subsequent ignition of the nucleus in more detail.

Another cause of the high temperature of the shell of a degenerate dwarf satellite of a close double may be the tidal heating of its shell and mantle. Investigation of the tidal heating of the degenerate component [5] has shown that the maximum temperature is reached in the dwarf mantle at a point with a mass of about 90 % of its mass. Taking into account the dissipation of the main energy of the jet of matter of the satellite in the mantle makes its base the hottest region. A simple estimate of the burning of nuclear fuel shows that at a density larger than $\sim 10^5$ g/cm^3, the combustion time of the fuel is less than the dynamic time of the dwarf, which is only a few seconds. That is, at such densities for modeling the combustion of nuclear fuel, it is sufficient to confine ourselves to point-like off-center ignition of the nuclear fuel. The depth of the ignition point remains an important parameter, the determination of which is an independent difficult task. The purpose of this paper is to elucidate the role of the ignition point in the course of the burning process of nuclear fuel and the dynamics of the explosion remainder of a degenerate dwarf.

To simulate supernovae, a fairly large number of codes are used based on both Adaptive Mesh Refinement (AMR) and Smoothed Particle Hydrodynamics (SPH) approaches. The list and analysis of such codes can be found in the works [6], [7], [8]. Especially worth mentioning is the code BETHE-Hydro [9]. This code is based on the Arbitrary

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Lagrangian-Eulerian approach (ALE), which combines the advantages of both Euler and Lagrangian approaches. The equations of hydrodynamics to formulate in a Lagrangian non-conservative form and to solve on an unstructured grid. The numerical method is based on the operator approach, which makes it possible to construct consistent schemes for the approximation of the gradient and divergence operators. To solve the Poisson equation in a one-dimensional formulation, the tridiagonal matrix algorithm (or the Thomas algorithm in foreign literature) is used. In the two-dimensional formulation of the Poisson equation is solved using the conjugate gradients method. Next, the potential is corrected to conserve the total energy (the sum of the kinetic, internal, and potential energies) of the system. It is worth noting that the total energy of the system can not be fully preserved, but the error on the collapse problem is of the order of 10^{-2} percent, which is very insignificant. Unfortunately, the approach was not developed in the three-dimensional case.

The second section will describe the numerical model of degenerate white dwarf stars - a description of each of the stages of the evolution of SNIa, the equation of gravitational hydrodynamics, receiving of the hydrostatic profile and the source when writing the equation for entropy. The third section is devoted to a brief description of the HydroBox3D code, which will be used to model all three steps, which are described in the fourth section. In the fifth section, a conclusion is formulated and further work is determined to investigate the SNIa explosion.

2 THE NUMERICAL MODEL

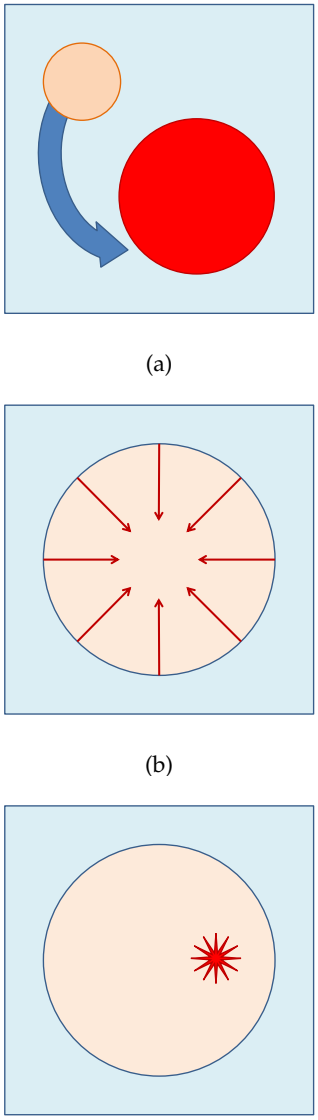
2.1 The description of SNIa explosion stages

In the present study, we shall consider three main stages of Ia type supernova explosion: a confluence of white dwarf stars, collapse and non-central explosion. In the initial time step, two degenerate white dwarfs with masses not exceeding Chandrasekar limit are modeling. For definiteness mode, we shall observe two stars with masses $1.2M_{\odot}$ (a central star) and $0.8M_{\odot}$ (satellite star). During the process of merging the mass of central star exceeds Chandrasekar limit, reaching the value $2M_{\odot}$ and leaves the state of hydrostatic balance due to mass increasing. Therefore the collapse process of a star begins. To model process of collapse, we shall use a hydrostatic balance with a two-time profile of density, thus resulting in starting the compression process. In the result of core-collapse, we achieve the temperature when the process of carbon nuclear burning starts [10], [11], [12]. It does not have to be in the center of the kernel thus resulting in the nonsymmetrical explosion of accuracy 10^{51} Ergs. Schematic description for each stage is presented on figure (1).

2.2 The hydrodynamic equations

In a some papers [13], [14], [15] in the field of computational hydrodynamics an equation for entropy is used, written in a conservative form:

$$\frac{\partial}{\partial t}\begin{pmatrix}\rho\\ \rho\vec{u}\\ \rho S\\ \rho E\end{pmatrix}+\bigtriangledown\cdot\begin{pmatrix}\rho\vec{u}\\ \rho\vec{u}\otimes\vec{u}+p\\ \rho S\vec{u}\\ [\rho E+p]\,\vec{u}\end{pmatrix}=\begin{pmatrix}0\\ -\rho\bigtriangledown\Phi\\ 0\\ -\rho\vec{u}\bigtriangledown\Phi\end{pmatrix} \quad (1)$$



(a)

(b)

(c)

Fig. 1. The stages of SNIa explosion: merger of dwarf stars (a), collapse of star (b), non-central explosion of star (c)

where ρ is the density, $\vec{u}$ is the velocity vector, S is the constant entropy, $p = S\rho^{\gamma}$ is the pressure of degenerate gas, $\rho E = p/(\gamma-1)+\rho\vec{u}^2/2$ is the total mechanical energy, $\gamma = 5/3$ is the adiabatic index, $\bigtriangledown\Phi$ is the gradient of the potential, is determined from the Poisson equation.

$$\triangle\Phi = 4\pi G\rho \quad (2)$$

The advantages and disadvantages of using of entropy equation are discussed in the paper [16].

2.3 The hydrostatic profile

The hydrostatic profile is obtained under suggestion of equality to zero of velocities, its derivate and Poisson equa-

tion for gravitational potential, written in spherical coordinates:

$$-\frac{\partial p}{\partial r} - \rho\frac{\partial \Phi}{\partial r} = 0 \qquad \frac{\partial}{\partial r}\left(r^2\frac{\partial \Phi}{\partial r}\right) = 4\pi r^2 G\rho$$

where r is a spherical radius. Recalling that equation of state is defined in form of

$$p = S\rho^\gamma$$

And after substituting that expression into prevision equations and after replacing partial differentiations with ordinary ones (as we differentiate solutions only by space) we will get Lane-Emden equation:

$$-\frac{d}{dr}\left(\gamma S r^2 \rho^{\gamma-2}\frac{d\rho}{dr}\right) = 4\pi G r^2 \rho$$

such equation is solved in elementary way with any numerical method to solve ordinary differential equations.

2.4 The energy source term

When modeling nuclear combustion of carbon, it is necessary to take into account the cooling and heating functions, which are written in the form of the right-hand side of the energy equation. For definiteness, we shall assume that the term Q – is a source of total energy, then:

$$\frac{dE}{dt} = Q$$

in this case, for the equation of state $p = S\rho^\gamma$ and the low mass on cooling / heating, the source for the entropy equation is written as:

$$\frac{d(\rho S)}{dt} = \frac{\gamma - 1}{\rho^{\gamma-1}}Q$$

The heat source recorded in this form will be used for modeling the chemokinetic processes.

3 The HydroBox3D code

To solve equations (1) and (2) was used the calculation schemes shown in the figure 2. The main idea of the method is to use the vector notation of the equations of hydrodynamics:

$$\frac{\partial v}{\partial t} + \bigtriangledown \cdot f(v) = 0 \qquad (3)$$

where v is a vector of conservative variables. For equation (3) the numerical scheme in one direction is written in the form:

$$\frac{v_i^{n+1} - v_i^n}{\tau} + \frac{F_{i+1/2} - F_{i-1/2}}{h} = 0 \qquad (4)$$

where F is the solution of the Riemann problem:

$$F = \frac{f(v_L) + f(v_R)}{2} + \frac{c + \|\vec{u}\|}{2}(v_L - v_R) \qquad (5)$$

The detailed arrangement of quantities for the organization of calculations is given in the figure 3.

The solution of the equations of hydrodynamics (finding the solution of Riemann's problems) occurs in two stages: solution of Riemann problems on all boundaries of a nested grid, solving Riemann problems on all internal interfaces of a nested grid. If the second part of finding the solution of Riemann's problems is fairly trivial, then the first part

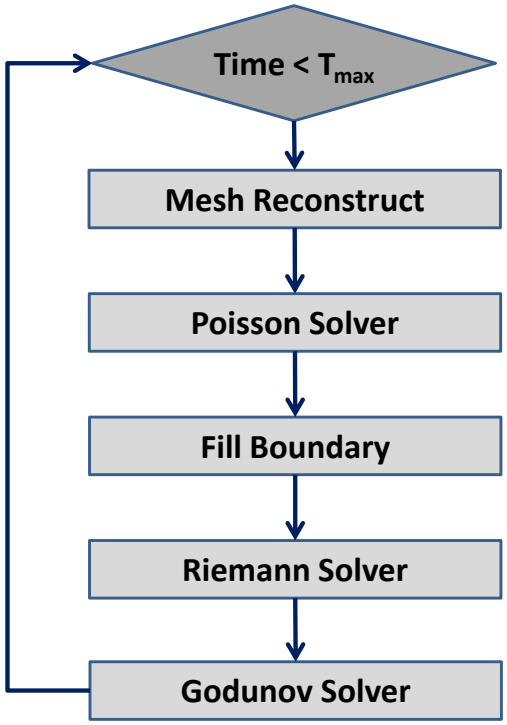


Fig. 2. The scheme of the organization of calculations

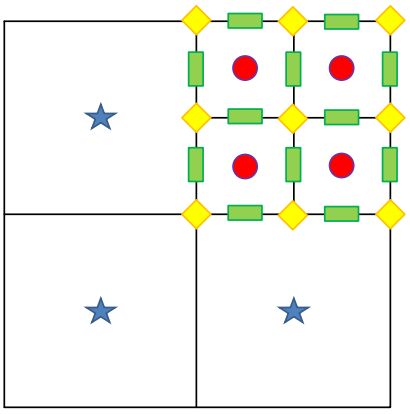


Fig. 3. The location of the hydrodynamic quantities on the root and embedded nets: the hydrodynamic parameters and the potential value on the root grid (blue stars), the hydrodynamic parameters on the embedded meshes (red circles), the potential on the embedded grid (yellow diamonds), the Riemann problem solution on the interfaces between the internal cells of the nested grid and intra-grid cells of the nested grid and the cells of the neighboring root cell

requires a different way of computing, depending on the cell sizes of two adjacent nested meshes. In total, only three methods of the relationship between the cells of neighboring grids are possible.

The solution of the Poisson equation is carried out in two stages: solution of the Poisson equation on the root

grid using the fast Fourier transform, solution of the Poisson equation on an embedded grid using the SOR method. Also, we do not dwell on the solution of the Poisson equation in the first stage (a detailed description of the solution method can be found in [16]). The Successive Over-Relaxation (SOR) method is an iterative process of finding the potential on an embedded grid for given initial and boundary conditions obtained from solving the Poisson equation on the root grid. A similar approach to solving the Poisson equation has proved itself in a number of programming codes, for example, in the code GAMER [17].

Step by time τ is calculated from the Courant condition

$$\frac{\tau \times (c + \|\vec{u}\|)}{h_{min}} = CFL < 1$$

where CFL is Courant number, $c = \sqrt{\gamma p/\rho}$ is the sound speed, h_{min} is the size of the minimal cell of the nested grid.

At the final stage of solving the hydrodynamic equations, the solution is corrected. In the case of a gas-vacuum boundary using the formula:

$$|\vec{u}| = \sqrt{2(E-\epsilon)}, (E - \vec{u}^2/2)/E < 10^{-3} \quad (6)$$

in the rest of the region there is an adjustment, which guarantees nondecreasing entropy:

$$|\rho\epsilon| = \left(\rho E - \frac{\rho\vec{u}^2}{2}\right), (E - \vec{u}^2/2)/E \geq 10^{-3} \quad (7)$$

Such a modification provides a detailed balance of energies and guarantees nondecreasing entropy.

Restructuring of the grid takes place according to the criterion of the cell mass of the root grid. The size of the nested grid is determined from the condition:

$$M = 2^{\mathcal{C}_1 \lfloor log(\rho) \rfloor + \mathcal{C}_2} \quad (8)$$

where $\mathcal{C}_{1,2}$ – scaling constants, which are selected depending on the task (the characteristic density) and the requirements for a minimum resolution [18].

4 The numerical simulation of SNIa

In SNIa explosion problem, we will consider model problems: a merger of white dwarfs using the degenerate gas equation of state, the problem of collapse, and the non-central explosion of a white dwarf with carbon burning energy injected. The stellar equation of state, which must be introduced to model white dwarfs, will be described in subsequent articles.

4.1 Merger of carbon-oxygen white dwarf stars

At the initial moment of time, two degenerate white dwarfs with masses not exceeding the Chandrasekhar limit are modeled. For definiteness, we consider two stars with masses $1.2M_\odot$ (central star) and $0.8M_\odot$ (satellite star) with equilibrium density profiles. The satellite star moves along a parabolic trajectory to the central star. The figure (4) shows the process of mass flow from the satellite star to the central star.

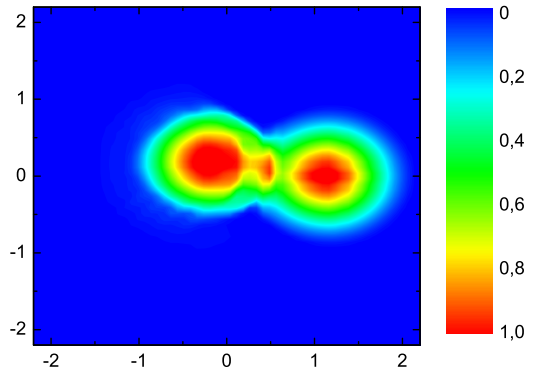


Fig. 4. Mass flow at the confluence of two degenerate white dwarfs

4.2 Collapse problem

During the confluence, the mass of the central star exceeds the Chandrasekhar limit, reaching $2M_\odot$ and leaves the state of hydrostatic equilibrium due to the increase in mass. Thus, the process of collapse of the star begins. To simulate the collapse process, we will use a hydrostatic equilibrium with a double density profile given by the corresponding value of the constant entropy. The figure (5)shows the behavior

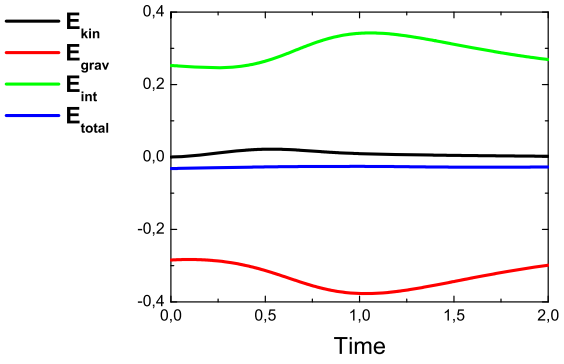


Fig. 5. Behavior of various types of energy at the collapse of the star's core

of various types of energy at star core-collapse. We note a small error in the law of conservation of the total (kinetic + internal + gravitational) energy.

4.3 Non-central Explosion

As a result core-collapse, a temperature is reached at which the process of nuclear burning of carbon begins, which will occur at the periphery of a star with an explosion energy of the order of $Q = 10^{51}$ Ergs. The hydrostatic profile will be used as the initial one. The figure (6) shows the column of density of a star at a noncentral explosion. Note that the solution over time is similar to solving the Sedov problem about a point central explosion, which probably explains the limitation for type Ia supernovae to determine the point of nuclear burning of carbon - the cause of the explosion.

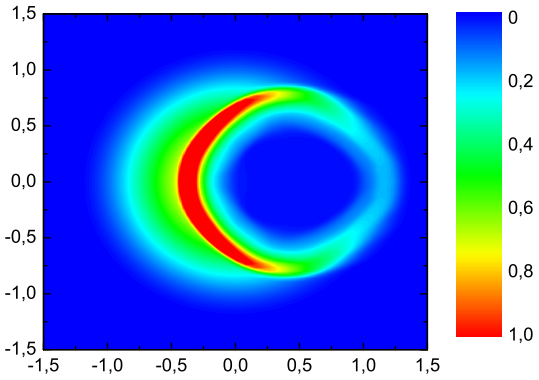


Fig. 6. The column of density of the star at noncentral star explosion

5 Conclusion and future work

The paper presented the results of mathematical modeling of the three main stages of the explosion for type Ia supernovae: the fusion of white dwarfs, collapse, and noncentral explosion. The mathematical formalization of the problem is described in detail. In the fusion problem, a general fall mechanism for the satellite star onto a central star was shown, which leads to a non-central mass distribution leading to a non-central collapse. In the modeling of the collapse process, the behavior of various types of energies was investigated, the characteristic behavior of each of the energy types is explained. The paper shows the qualitative dynamics a star noncentral explosion , whose source is the characteristic value of the energy released by the reaction C + C. The article describes the original HydroBox3D code, in which the concept of an adaptive nested mesh is implemented, a brief description of the numerical methods implemented in the code is given.

The next stage in the investigation of the problem of the explosion of supernovae based on the merging of white dwarf stars will be a large-scale simulation, which includes all three stages realized as one scenario.

The adaptive nested mesh HydroBox3D code is available on site https://github.com/IgorKulikov/HydroBox3D.

Acknowledgments

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