Mathematics and telecommunication networks: general principles and open issues

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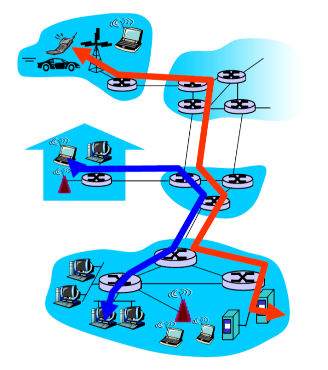
Outline

1. Telephone networks and Classical Teletraffic Theory
   - Public Switched Telephone Network
   - Teletraffic theory
2. Network architecture of the Internet
   - Packet Switching Principles
   - Congestion control
   - TCP Linux
3. Internet traffic features
   - POTS vs. Internet
   - Why fractals?
   - Queueing performance
4. Conclusions

Plain Old Telephone Service (POTS)



- Circuit switching
  - Resources reserved for call
    - link bandwidth
    - switch capacity
  - Dedicated resources: no sharing
  - Call set-up required
- Teletraffic theory and POTS: one of the most successful applications of mathematics in industry

POTS and teletraffic theory

- Highly static nature of PSTN
  - *Homogeneous systems*
  - *Limited variability*

  $\Rightarrow$ Existence of universal laws
- Agner Krarup Erlang (1878-1929) – Danish mathematician, the father of queueing theory, has worked for the Copenhagen Telephone Company (KTAS in Danish) for almost 20 years
  - The Erlang B formula gives the (steady-state) probability that a trunk is not available as a function of the load and the number of trunks in a loss system
  - The Erlang C formula gives the (steady-state) probability that an arrival must wait before beginning service in a delay system

General features of POTS traffic

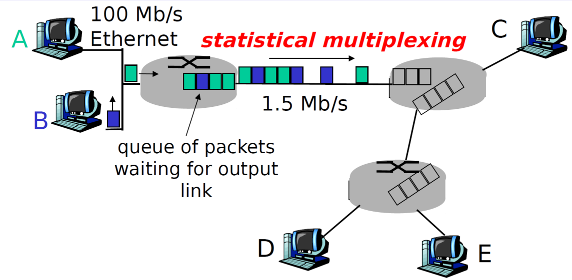
- Poisson nature of call arrivals at links where traffic is highly aggregated
  - *Parsimonious traffic model*

Palm–Khintchine theorem

The superposition of a large number of independent equilibrium renewal processes, each with a small intensity, behaves asymptotically like a Poisson process

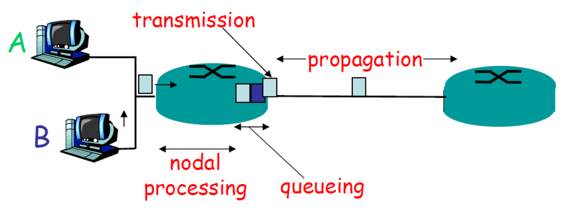
- Call holding times follow more or less an exponential distribution
  - *Insensitivity property of the Erlang B formula (Kosten, 1948)*
- Mathematically tractable models could be used to predict accurately many performance measures of interest

Packet Switching



- Each end-end data stream is divided into *packets*
  - Users share network resources: statistical multiplexing
  - Each packet uses full link bandwidth
  - Resources are used *as needed*
- Resource contention
  - Aggregate resource demand can exceed the amount available
  - Need for end-to-end congestion control mechanisms

Node delay



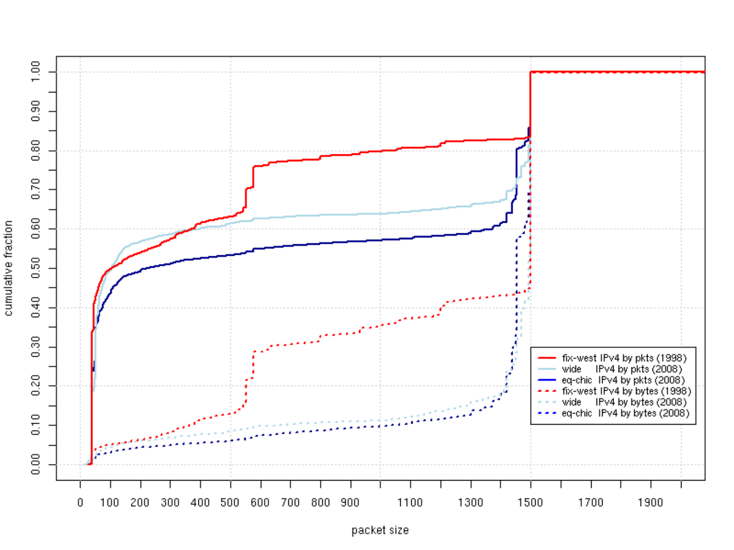
- Processing delay d_{proc}
- Queueing delay d_{queue}
- Transmission delay d_{trans} — the delay between the times that the first and the last bits of the packet are transmitted

$$d_{\text{trans}} = L/R$$

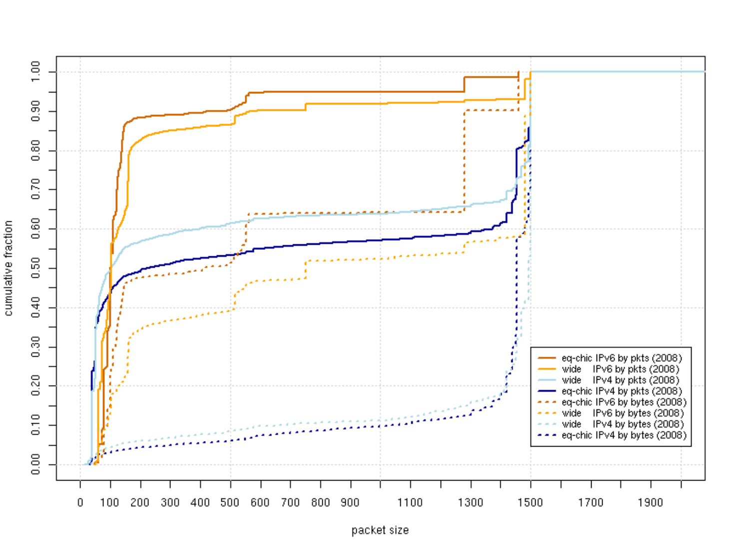
where L is the packet length and R is the transmission rate

- Propagation delay d_{prop}

Packet length distribution

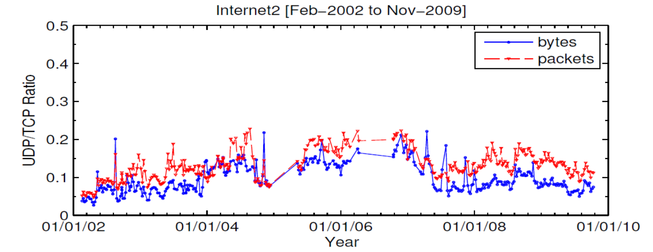


Packet length distribution



Transmission Control Protocol (TCP)

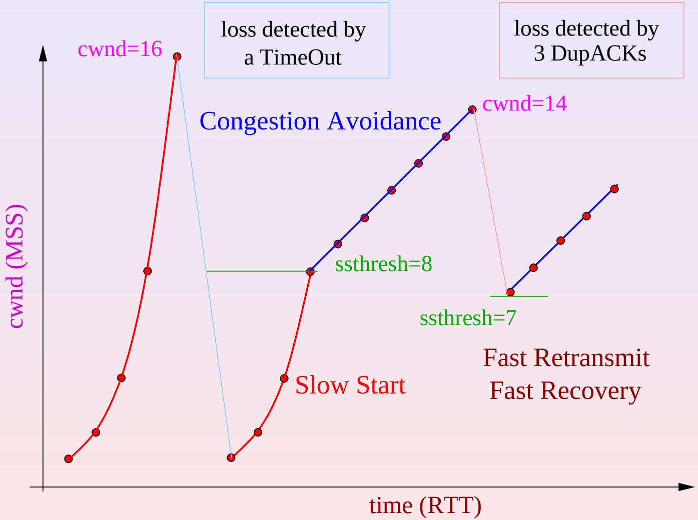
- Connection-oriented transport protocol that provides a reliable byte-stream data transfer service between pairs of processes
- Key features:
  - Connection-oriented
  - Multiplexing/Demultiplexing
  - Reliability
  - Flow Control
  - Congestion Control — TCP sensitive to network conditions



TCP Congestion Control

- TCP seeks to
  - achieve high utilization
  - *control* congestion
  - share bandwidth
- TCP Congestion Control is window-based
  - cwnd – state variable that limits how much data TCP is allowed to have in transit
  - A TCP source calculates cwnd according to the level of congestion *it perceives to exist* in the network
- TCP assumes packet losses are caused by congestion
- Behaviour of cwnd
  - no losses $\Rightarrow$ more bandwidth is available $\Rightarrow$ cwnd $\nearrow$
  - loss of a packet $\Rightarrow$ network congestion $\Rightarrow$ cwnd $\searrow$
- Differentiation between major and minor congestion events
  - Introduction of Fast Recovery mechanism

Classical TCP Congestion Control (TCP Reno)



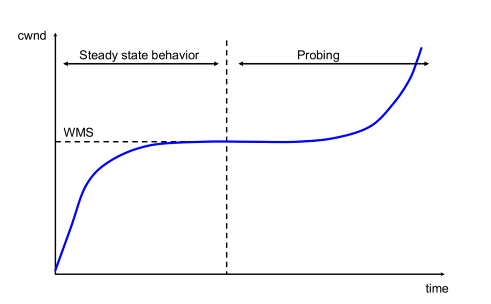
Main TCP Linux Variants

Cubic and Reno (NewReno)

- Loaded into the kernel, via standard kernel module mechanism
- Information available in /proc/sys/net/ipv4
  - tcp_allowed_congestion_control — cubic reno
  - tcp_congestion_control — cubic

Key features of TCP CUBIC

- Cubic growth of cwnd
- Default since 2.6.19 Linux kernel



ls -a /lib/modules/'uname -r'/kernel/net/ipv4/tcp*

kernel 2.6.35-30

- tcp_bic.ko
- tcp_highspeed.ko
- tcp_htcp.ko
- tcp_hybla.ko
- tcp_illinois.ko
- tcp_lp.ko
- tcp_probe.ko
- tcp_scalable.ko
- tcp_vegas.ko
- tcp_veno.ko
- tcp_westwood.ko
- tcp_yeah.ko

kernel 4.4.0-97

- tcp_bic.ko
- tcp_cdg.ko
- tcp_dctcp.ko
- tcp_diag.ko
- tcp_highspeed.ko
- tcp_htcp.ko
- tcp_hybla.ko
- tcp_illinois.ko
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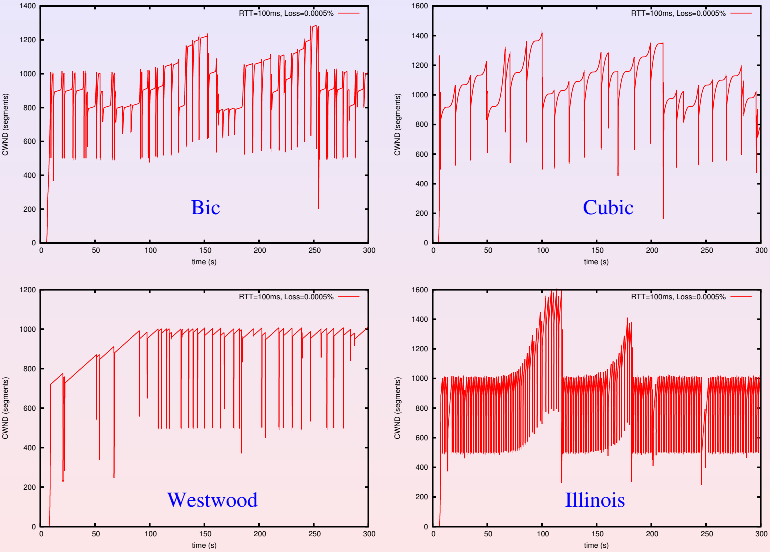
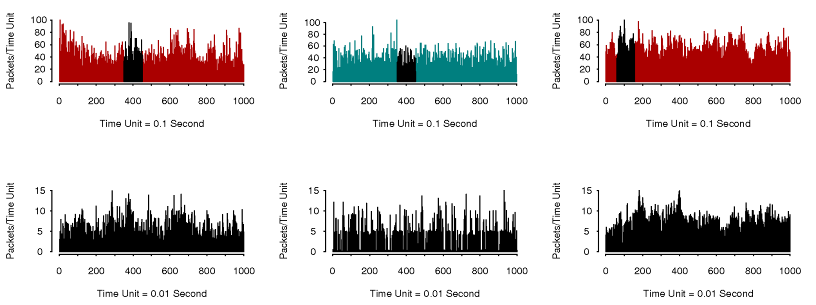
Behaviour of some Linux TCP Variants



Statistical features of data traffic

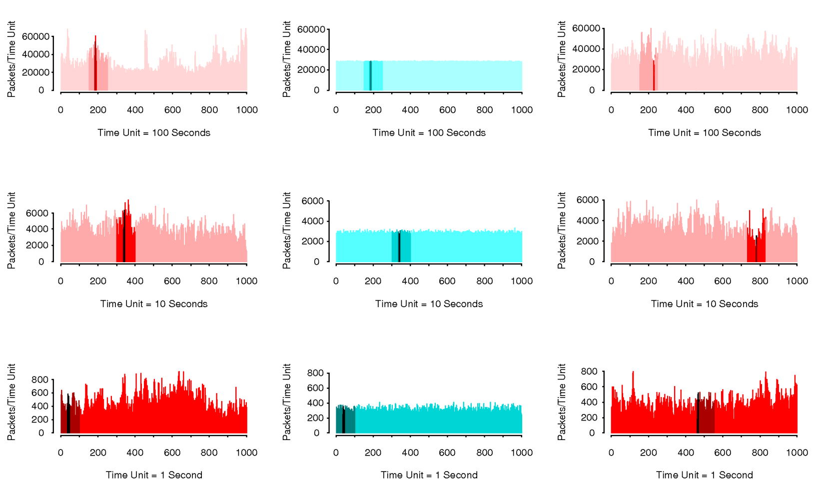
- End-to-end congestion control
  - Traffic is shaped by the conditions each connections has encountered in its past
- Data traffic is much more variable than voice traffic
  - Mice and elephant data flows
  - Highly different rates
  - Heterogeneous applications
  - High burstiness
- Mathematics of high or extreme variabilities
  - Temporal high variability ⇒ Long Range Dependence
  - Spatial high variability ⇒ Heavy-tailed distributions
  - Lack of a dominant *time scale* ⇒ Fractals

Traffic Self-similarity



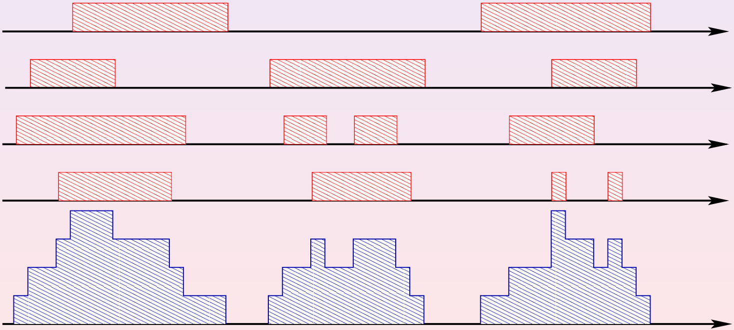
M. S. Taqqu, W. Willinger, R. Sherman *Proof of a fundamental result in self-similar traffic modeling*, Computer communication review, 1997

Traffic Self-similarity



Limit theorems for aggregated traffic

- Let us consider a network with a high number of hosts communicating with each other
- Each source is modeled according to a binary On–Off alternating renewal process



Flow characterization

Fluid on–off model

$$I(t) = \begin{cases} 0 & t \in \text{Off interval} \\ 1 & t \in \text{On interval} \end{cases}$$

- Denote by μ_{on} and μ_{off} the mean sojourn times in On and Off states
- Assume that

$$\overline{F}_{\text{on}} \simeq \ell_{\text{on}}\, x^{-\alpha_{\text{on}}}\, L_{\text{on}}(x) \quad (1 < \alpha_{\text{on}} < 2)$$

$$\text{or} \qquad \sigma^2_{\text{on}} < \infty \;\Rightarrow \alpha_{\text{on}} \triangleq 2$$

and

$$\overline{F}_{\text{off}} \simeq \ell_{\text{off}}\, x^{-\alpha_{\text{off}}}\, L_{\text{off}}(x) \quad (1 < \alpha_{\text{off}} < 2)$$

$$\text{or} \qquad \sigma^2_{\text{off}} < \infty \;\Rightarrow \alpha_{\text{off}} \triangleq 2$$

Superposition of On–Off sources: Key result

- Aggregate cumulative packet counts for N IID sources in the interval $\left[0, tT\right]$

$$A_N(tT) = \int_0^{tT} \left(\sum_{k=1}^{N} I_k(u) \right) du$$

- Convergence to fractional Brownian motion (fBm) $Z_H(t)$ for high values of N and T

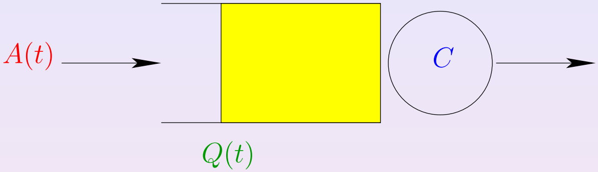
$$\lim_{T\to\infty} \lim_{N\to\infty} \frac{1}{T^H \sqrt{L(T)}\,\sqrt{N}} \left(A_N(tT) - TN\, \frac{\mu_{\text{on}}}{\mu_{\text{on}} + \mu_{\text{off}}}\, t \right) \overset{(d)}{=} \sigma_{\lim} Z_H(t)$$

where

$$H = \frac{3 - \alpha_{\min}}{2} \in (1/2, 1) \quad \text{and} \quad \alpha_{\min} = \min\left(\alpha_{\text{on}}, \alpha_{\text{off}}\right)$$

- Noah Effect at source level (heavy tails) produces aggregate network traffic that exhibits the Joseph Effect (self-similarity)

Single server queue with infinite buffer



- $A(t) = mt + X(t)$ – Gaussian traffic model
- Constant service rate $C > 0$ and $r \triangleq C - m > 0$

Logarithmic large buffer asymptotic (LDT result)

$$\mathbb{P}(Q > b) \asymp \sup_{t \in \mathbb{R}} \exp\left(-\frac{(b+rt)^2}{2v(t)}\right) = \exp\left(-\inf_{t \in \mathbb{R}} \frac{(b+rt)^2}{2v(t)}\right)$$

where $v(t) = \mathbb{D}X(t)$

Large buffer asymptotic ($b \to \infty$)

Exact asymptotic for fBm

$$\mathbb{P}(Q > b) \sim \frac{\alpha(H)}{\sqrt{2\pi}\beta(H)} \cdot b^{\gamma} \exp\left(-\Theta b^{2-2H}\right)$$

where $\gamma = 2H - 3 + \frac{1}{H}$

$$\alpha(H) \triangleq \frac{\mathcal{H}_{2H}\sqrt{\pi}}{2^{(1-H)/2H}\sqrt{H}} \left(\frac{H}{r(1-H)}\right)^{H-1} \left(\frac{1}{1-H}\right)^{(2-H)/H}$$

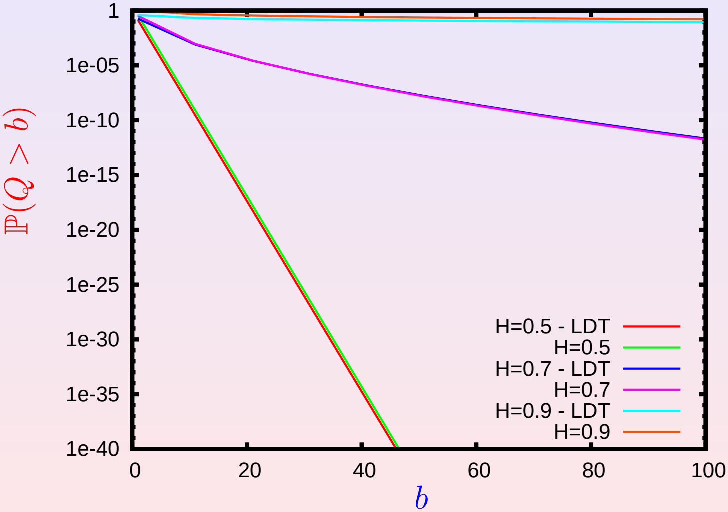
$$\beta(H) \triangleq \left(\frac{r(1-H)}{H}\right)^{H} \frac{1}{1-H}$$

Pickands constants

$$\mathcal{H}_{\alpha} \triangleq \lim_{T\to\infty} \frac{1}{T} \cdot \mathbb{E} \exp\left(\sup_{t\in[0,T]} \left(\sqrt{2}B_{\alpha/2}(t) - t^{\alpha}\right)\right)$$

where $\alpha \in (0,2]$ and $B_{\alpha/2}$ – fBm with Hurst parameter $\alpha/2$

Asymptotic ($b \to \infty$) for fBm



Conclusions

- IETF (Internet Engineering Task Force) motto

  We reject kings, presidents and voting.
  We believe in: rough consensus and running code

- Differences between POTS and Internet
  - Circuit vs. packet switching
  - Homogeneous vs. heterogeneous systems
  - Limited variability vs. burstiness
  - Poisson vs. LRD
  - Exponential distribution vs. heavy tails
- Relevance of LRD in terms of network performances
  - Buffers are not the solution!
- Big challenges for mathematicians and statisticians
  - Parsimonious modelling
  - Parameter estimations
  - Queueing performance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