MCQMC 2018

13th International Conference in Monte Carlo &
Quasi-Monte Carlo Methods in Scientific Computing

Rennes, France, July 1-6, 2018

Conference Program and Information

Contents

Welcome to Rennes for MCQMC 2018

We are pleased and excited to host you in Rennes for the *13th Conference on Monte Carlo and Quasi-Monte Carlo Methods in Scientific Computing*, MCQMC 2018. We hope you will have a memorable week here, enjoying both the scientific program and the vibrant life of the city. The MCQMC conference series is the major event for researchers investigating and developing Monte Carlo and quasi-Monte Carlo methods. We are very pleased and excited about the overall quality and variety of the proposed sessions and presentations. We hope you will have the opportunity to learn about new ideas, and to start new collaborations. Our scientific program features eight one-hour invited plenary talks from top contributors in our field. We also have around 190 regular talks of 30 minutes each (including questions).

Rennes is the capital of Brittany and is ideally situated at the heart of a dual carriageway network linking Brittany to Paris and Normandy and to the areas south of the Loire divide. The city of Rennes is very pleasant with its parks and gardens and the delightful old town with its faithfully restored streets and squares, colorful traditional timber-framed houses and outstanding buildings by famous architects. A visit to the Musée de Bretagne is recommended to discover what life in Brittany was like in ancient times as well as the present day ? there?s also a fine art museum in the same building. You?ll find a great choice of restaurants, bars and pavement cafés in the old town and one of the largest markets in France is held on Saturday mornings. There you can find all kinds of French delicacies such as cheese, sea food, crêpes and galettes and everything else from flowers to fresh meet. From Rennes, you can also easily reach the beautiful coast of Brittany. A few (among many) other interesting places to see from Rennes: the world famous Mont Saint Michel, the old cities of Saint Malo or Dinan, all reachable by direct coaches, or by train for Saint Malo.

All registered participants are invited to a welcome reception at the City Hall on Monday, a wine and cheese party on Tuesday, and to the conference banquet at the Château d'Apigné in the suburb of Rennes (be present to take the reserved coaches) on Wednesday evening.

Following the tradition, a post-conference volume will be published in *the book series Springer Proceedings in Mathematics & Statistics*. It will be devoted to a selection of articles based on presentations made at this conference. Instructions on how and when to submit your manuscripts will be provided on the conference web site, soon after the conference: http://mcqmc2018.inria.fr/.

We hope that you will enjoy your time in Rennes and have a productive and interesting week at MCQMC 2018.

If you have any problems or special requests during the conference, do not hesitate to ask the organizers.

The organizers,

Bruno Tuffin, Pierre L'Ecuyer, Gerardo Rubino

The MCQMC Conference Series

The MCQMC conference series is a biennial meeting focused on Monte Carlo (MC) and quasi-Monte Carlo (QMC) methods in scientific computing. The conference attracts between 150 and 200 participants. Its aim is to provide a forum where leading researchers and users can exchange information on the latest theoretical developments and important applications of these methods. Recent conferences have attracted researchers in Markov chain Monte Carlo (MCMC). In a nutshell, MC methods study complex systems by simulations fed by computer-generated pseudorandom numbers. QMC methods replace these random numbers by more evenly distributed (carefully selected) numbers to improve their effectiveness. A large variety of special techniques are developed and used to make these methods more effective in terms of speed and accuracy. The conference focuses primarily on the mathematical study of these techniques, their implementation and adaptation for concrete applications, and their empirical assessment.

The conference was initiated by Harald Niederreiter, who co-chaired the first seven conferences. In 2006 Harald Niederreiter announced his wish to step down from the organizational role, and a Steering Committee was formed to ensure and oversee the continuation of the conference series. The locations of the 13 first conferences are set out below.

Year	Location
1994	Las Vegas, NV USA
1996	Salzburg, Austria
1998	Claremont, CA USA
2000	Hong Kong
2002	Singapore
2004	Juan-Les-Pins, France
2006	Ulm, Germany
2008	Montreal, Canada
2010	Warsaw, Poland
2012	Sydney, Australia
2014	KU Leuven, Belgium
2016	Stanford, CA USA
2018	Rennes, France

Committees

Organizing Committee

Bruno Tuffin	(France, Inria), Chair
Pierre L'Ecuyer	(Canada, U. Montreal)
Gerardo Rubino	(France, Inria)
Edith Blin	(France, Inria)
Fabienne Cuyollaa	(France, Inria)

Steering Committee

Alexander Keller	(Germany, NVIDIA) , Chair
Josef Dick	(Australia, U. New South Wales)
Fred J. Hickernell	(USA, Illinois Institute of Technology)
Aicke Hinrichs	(Austria, JKU Linz)
Pierre L'Ecuyer	(Canada, U. Montréal)
Art Owen	(USA, Stanford University)
Friedrich	Pillichshammer (Austria, JKU Linz)

Scientific Committee

Zdravko Botev	(Australia, U.New South Wales)
Hector Cancela	(Uruguay, U. of the Republic)
Frédéric Cérou	(France, Inria)
Nicolas Chopin	(France, ENSAE)
Ronald Cools	(Belgium, KU Leuven)
Josef Dick	(Australia, U.New South Wales)
Arnaud Doucet	(UK, Oxford)
Paul Dupuis	(USA, Brown University)
Mike Giles	(UK, Oxford)
Mark Girolami	(UK, U. of Warwick)
Paul Glasserman	(USA, Columbia)
Peter Glynn	(USA, Stanford University)
Michael Gnewuch	(Germany, Kiel)
Emmanuel Gobbet	(France, Ecole Polytechnique)
Takashi Goda	(Japan, The University of Tokyo)
Arnaud Guyader	(France, U. Pierre et Marie Curie)
Stefan Heinrich	(Germany, U. Kaiserslautern)
Fred J. Hickernell	(USA, IIT)
Aicke Hinrichs	(Austria, JKU Linz)
Wenzel Jakob	(Switzerland, ETH and Disney)
Alexander Keller	(Germany, NVIDIA)
Dirk Kroese	(Australia, U Queensland)
Frances Kuo	(Australia, U. New South Wales)
Gerhard Larcher	(Austria, JKU Linz)
Christian Lécot	(France, U. Savoie)
Pierre L'Ecuyer	(Canada, U. Montréal)
Christiane Lemieux	(Canada, Waterloo)
Faming Liang	(USA, U. Florida, Gainesville)
Makoto Matsumoto	(Japan, Hiroshima)
Eric Moulines	(France, Ecole Polytechnique)
Thomas Mueller-Gronbach	(Germany, Passau)
Harald Niederreiter	(Austria, Academy of Sciences)
Erich Novak	(Germany, FSU, Jena)
Dirk Nuyens	(Belgium, KU Leuven)
Art Owen	(USA, Stanford University)
Gareth Peters	(UK, University College London)
Friedrich Pillichshammer	(Austria, JKU Linz)
Klaus Ritter	(Germany, Kaiserslauten)
Gerardo Rubino	(France, Inria)
Wolfgang Schmid	(Austria, Salzburg)
Ian Sloan	(Australia, U.New South Wales)
Raul Tempone	(Saudi Arabia, KAUST)
Xiaoqun Wang	(China, Tsinghua)
Grzegorz Wasilkowski	(USA, U Kentucky)
Henryk Wozniakowski	(USA, Columbia U.)

Sponsors

We are very grateful for the financial support from the following sponsors

- Inria
- The University of Rennes 1
- Région Bretagne
- Rennes Métropole

Practical information

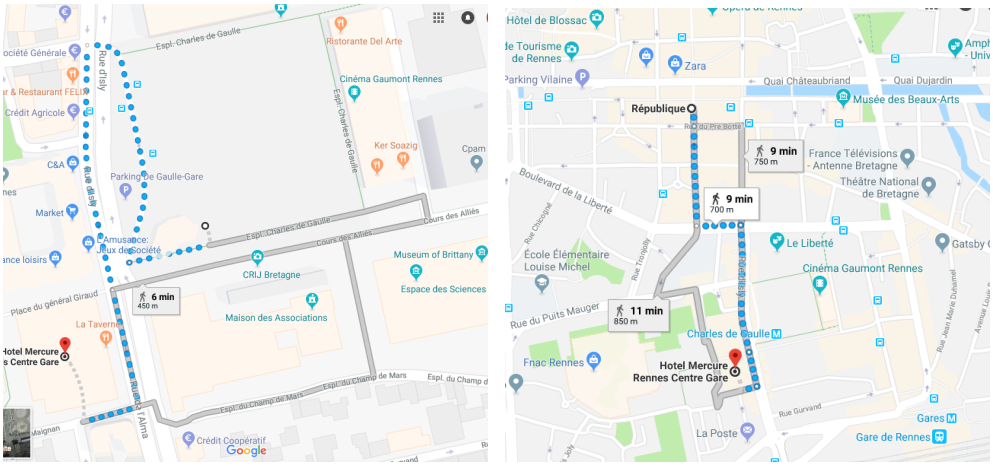
5.1 Conference Venue

The 13th International Conference in Monte Carlo & Quasi-Monte Carlo in Scientific Computing will be held in downtown Rennes, at the Faculty of Economics of the University of Rennes 1, except the Sunday tutorials which will take place at the hotel Mercure Rennes centre gare.

General maps of the conference venues are located on the back cover of this document or on the conference web site. More information can be found on how to reach them by foot from the closest metro stations.

Tutorials

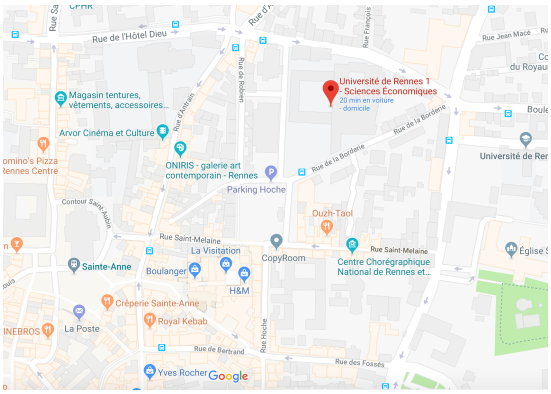
The tutorials will take place at the **Hotel Mercure Rennes Centre Gare (Address: 1 Rue du Capitaine Maignan, 35000 Rennes)** on Sunday, 1st, from 14:00 to 17:30. You can find below a Google Map description about how to reach by feet the location from Charles de Gaulle and République metro stations.

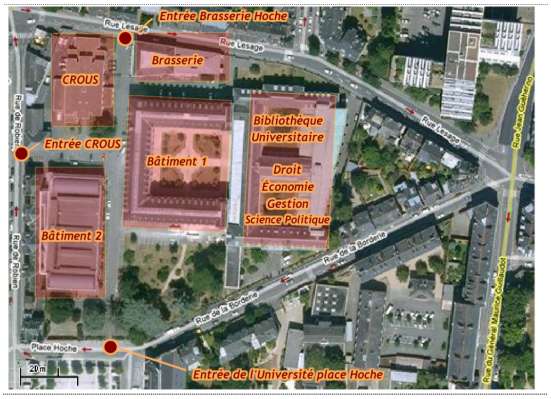


Main conference

From Monday to Friday, the conference is held at the **Faculty of Economics of the University of Rennes 1. Exact address: 7, place Hoche, Rennes** (see the map on the back cover or on the conference web site for general maps).

We provide here two other pictures showing the conference venue location and how to reach it from the closest subway station, “Sainte Anne". The conference will be located in Building (“Bâtiment") 2.





If you come by car, there is an underground (pay) parking close by, Place Hoche, open from 7:30 to 0:00, at 0.40€ per 15 minutes for the first hour, and then 0.30€ per 15 minutes. Bicycles can park for free.

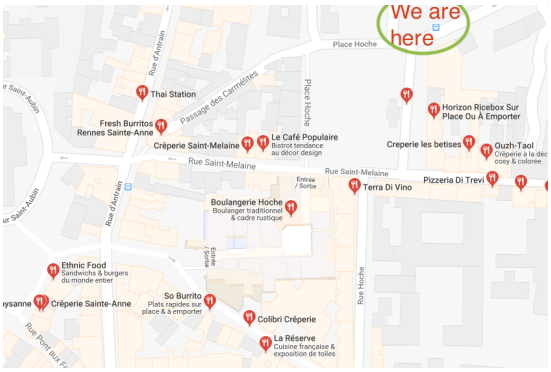
Registration, information desk, coffee breaks, and conference rooms

The registration and information desk will be located at the entrance of Bâtiment 2 of the Faculty of Economics of the University of Rennes 1, where the event takes place. On Sunday, you will be able to register during tutorials in front of the conference room at the Hotel Mercure, starting from 13:30. The coffee breaks will be located outside, at the entrance of Bâtiment 2, except in case of extreme weather. The parallel sessions and the tutorials will be in Rooms 5 to 8, ground floor of Bâtiment 2, and Amphi 3, one level up in the same building.

Lunch breaks

The participants are on their own for lunches. There is plenty of choice when it comes to food in Rennes, many located nearby. Lunch break is 1:45 and should be sufficient. You can find suggestions:

- From TripAdvisor close to Place Hoche
  https://www.tripadvisor.fr/Restaurants-g187103-zfn17971-Rennes_Ille_et_Vilaine_Brittany.html
- From TripAdvisor for Rennes downtown
  https://www.tripadvisor.fr/Restaurants-g187103-Rennes_Ille_et_Vilaine_Brittany.html
- From Google



- If you are short in time you can also buy a sandwich at the well-known Boulangerie Hoche (17 Rue Hoche) or one of the many places selling sandwiches (ex: chain Brioche Dorée)
- Walking a little bit more (less than 10 minutes), you can find many other places around Place des Lices or Rue Saint-Georges.

Wireless

If your home institution has *eduroam* then you will be able to connect freely to the internet at the conference (wherever there is wifi available). Please note that you need to set up *eduroam* at your home institution before you will be able to use it on our campus.

Alternatively, we have set up individual accounts for *eduspot* network. Login and password are available at the registration desk and/or in your conference package. To use it, connect to *eduspot* and open a browser. You then just have to use the provided login and password.

During tutorials, a specific wireless access will be provided by Mercure Hotel.

Links for tourist information

- More about Brittany at http://www.brittanytourism.com/
- Rennes tourist office web site (getting around, eating out, hotels, etc.) at https://www.tourisme-rennes.com/en

Emergency numbers

When in doubt, dial the **European Emergency Number at 112**. Operators responding to this number will be able to communicate with you in French or in English, and can redirect your call to the appropriate emergency service where necessary.

French emergency numbers are 18 for fire brigade, 17 for police, and 15 for ambulance and emergency services.

Public transportation

Detailed information on Rennes' public transportation system can be found in english at https://www.tourisme-rennes.com/en/organize-my-trip/the-practical-informations/travelling-by-bus-and-metro-in-rennes. The price for a 1 hour ticket is : €1.50. Tickets can be bought in buses or at (metro) stations.

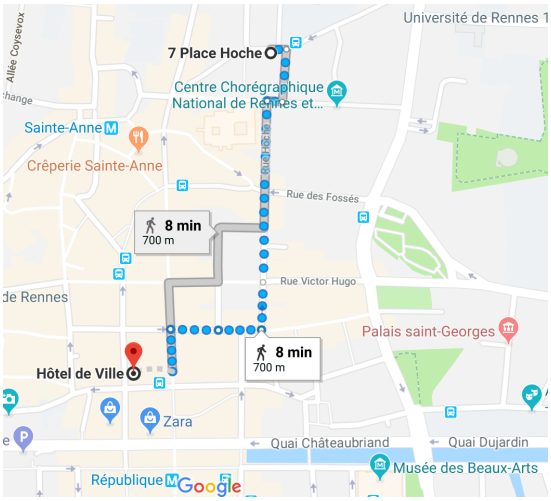
You can also use Rennes' bike sharing system, *LE vélo STAR*. You can buy a 1-day access (https://www.levelostar.fr/fr/tarifs/24-heures.html) or 1-week access (https://www.levelostar.fr/fr/tarifs/7-jours.html). Bikes are available in more than 90 stations all over the city. -day access or one-way trip. Rides of 30 minutes or less have no additional fees, and a limited costs for more time (rates are explained on the above web links).

5.2 Social Events

We have planned four social events: a welcome reception on Monday evening, a wine & cheese on Tuesday evening, a guided tour of Rennes downtown on Wednesday afternoon, and the conference banquet on Wednesday evening.

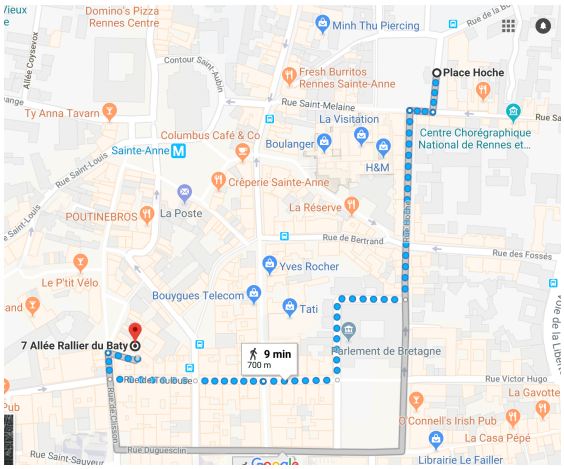
Welcome reception

A Welcome reception, sponsored by the city of Rennes, is held in the nice City Hall of Rennes. The reception starts at 18:30 and a cocktail will be served.
Venue: Hôtel de Ville, Place de la Mairie, 35000 Rennes. It is an at most 8 minutes walk from the conference venue.



Wine & Cheese

A wine and cheese reception will take place on Tuesday July 3, at 18:30. Venue: Bar l'Aventure. Cour de la prison Saint-Michel. 7, allée Rallier du Baty - 35000 Rennes. Not more than 9 minutes walk from the conference venue.



Please wear your badge.

Guided tour of Rennes

On Wednesday afternoon, at 16:00 and before the banquet, there will be a guided tour in Rennes downtown. This will be a great opportunity to visit the historical heart of the capital of Brittany. In particular, we will have access to a guided tour in the Historical Breton Parliament, which has a central place in the Breton history, and which is, in the meantime, a testimony of the royal ornaments of the XVII-th century.

Conference banquet

The conference banquet will be held on Wednesday evening at the *Château d'Apigné* in the surburb of Rennes. See https://www.chateau-apigne.fr/?lang=en

We will go there by coach. Please be sure to be on time at the meeting point. Should you come by car, the address is Château d'Apigné, 35650 Le Rheu.

Please bring your name badge to the dinner. The choice to participate in the conference dinner was already made on your registration page. (A similar remark holds if you are bringing your partner or family to the dinner.)

5.3 Presentations

Instructions for speakers

Plenary talks are 50 minutes plus 10 minutes for questions and discussion. All other talks are 25 minutes, plus 5 minutes for questions and discussion.

Please make sure that you do not exceed your time. Focus on the essential of your message. Given the short time allowed to each speaker, it is generally not possible to give the full details of your work. You should concentrate on providing a clear explanation of your main results and their significance.

instructions for session chairs

Session chairs have the responsibility to make sure the speakers adhere tightly to the schedule. Some participants might want to switch between parallel sessions to attend specific talks. To make sure that this can be done smoothly, session chairs should enforce strict adherence to the schedule. We will provide cards to be shown to the speaker for indicating 5, 3, and 1 remaining minutes of speaking time.

Session chairs should also contact their session speakers ahead of time to verify their presence and inform the organizers of any potential no-shows.

Equipment

You can bring and use your own laptop for your presentation. VGA cables are available for projection, as well as an HDMI one in Amphitheater 3. If you do not bring one, each lecture room will be equipped with a computer and a projector for displaying computer output.

5.4 Conference Proceedings

There is a long standing tradition that a selection of strictly refereed papers are published after the conference as a Springer-Verlag book in the book series *Springer Proceedings in Mathematics & Statistics*. Every speaker can submit a paper on the topic of the presented talk. The book entitled Monte Carlo and Quasi-Monte Carlo Methods 2018, edited by Bruno Tuffin and Pierre L'Ecuyer.

The deadline for submission will be Dec. 15, 2018. Detailed instructions will be made available through the conference website.

Schedule

Special sessions are in green and contributed sessions are in blue.

The abstracts can be found at the page numbers shown in red.

Sunday afternoon, July 1

13^{30}	**Registration opens**
$14^{00}-15^{30}$	**Tutorial** Room: Hotel Mercure Centre Gare *Alexander Keller* **Monte Carlo and Quasi-Monte Carlo Methods in Machine Learning** Chair: *Pierre L'Ecuyer* *p. 32*
$15^{30}-16^{00}$	Coffee break
$16^{00}-17^{30}$	**Tutorial** Room: Hotel Mercure Centre Gare *Gerardo Rubino* **Introduction to rare event analysis using Monte Carlo** Chair: *Bruno Tuffin* *p. 33*

Monday morning, July 2

8^{00}	**Registration opens**				
$8^{20} - 8^{30}$	**Opening Session** Room: Amphi 3 *Bruno Tuffin*				
$8^{30} - 9^{30}$	**Invited Plenary Talk** Room: Amphi 3 *Christophe Andrieu* **Some theoretical results concerning nonreversible Markov chain and Markov process Monte Carlo algorithms** Chair: *Christian Robert* *p. 36*				
$9^{30} - 10^{00}$	Coffee break				
	Room: Amphi 3 **Construction of QMC point sets and sequences** Chair: *Josef Dick* *p. 62*	Room: Salle 5 **Numerical approximation of SDEs under non-standard assumptions (1)** Chair: *Mario Hefter* *p. 50*	Room: Salle 6 **Non-Reversible Markov Chain Monte Carlo (1)** Chair: *Alex Thiery* *p. 46*	Room: Salle 7 **Nuclear applications** Chair: *Gerardo Rubino* *p. 116*	Room: Salle 8 **Forward and inverse UQ with hierarchical models (1)** Chair: *Abdul-Lateef Haji-Ali* *p. 55*
$10^{00} - 10^{30}$	*Frances Kuo* Discrete least squares approximation on multivariate polynomial spaces using lattice points	*Larisa Yaroslavtseva* On loss of regularity in the initial value for SDEs with non-globally Lipschitz continuous coefficients	*Joris Bierkens* Reflections on the bouncy particle sampler and Zig-Zag sampler	*Bert Mortier* Study of Source Term Estimators in Coupled Finite-Volume/Monte-Carlo Methods for Plasma Edge Simulations in Nuclear Fusion	*Benjamin Peherstorfer* Multifidelity Monte Carlo estimation with adaptive low-fidelity models
$10^{30} - 11^{00}$	*Kosuke Suzuki* Lattice Rules in Non-periodic Subspaces of Sobolev Spaces	*Diyora Salimova* Numerical approximations of nonlinear stochastic differential equations	*Alexandre Thiery* Discrete-Time Bouncy Particle Samplers and Generalisations	*Dmitry Savin* Monte Carlo simulation of multiple particle spectra with energy and momentum conservation	*Ahmed Kebaier* Adaptive Importance Sampling for Multilevel Monte Carlo Euler method
$11^{00} - 11^{30}$	*Ralph Kritzinger* Haar Analysis of Digital Nets and Sequences	*Michaela Szölgyenyi* Euler-type schemes for SDEs with discontinuous drift	*Alain Durmus* On the convergence of Hamiltonian Monte Carlo	*Zhicheng Ji* A Batch on Patch Parallel Scheme in Monte Carlo Particle Transport Program	*Sebastian Krumscheid* Multilevel Monte Carlo Approximation of Functions
$11^{30} - 12^{00}$	*Dirk Nuyens* Lattice rules with random number of points and near $O(n^{-\alpha-1/2})$ convergence	*Holger Stroot* Strong Approximation of Stochastic Mechanical Systems with Nonlinear Holonomic Constraints	*Arnak Dalalyan* User-friendly guarantees for the Langevin Monte Carlo	*Gang Li* High Precision Shielding Calculation For Qinshan-I Reactor Model With Monte Carlo Particle Transport Code JMCT	*Abdul-Lateef Haji-Ali* Multilevel Nested Simulation for Efficient Risk Estimation
$12^{00} - 13^{45}$	Lunch break				

Monday afternoon, July 2

$13^{45}-14^{45}$ **Invited Plenary Talk** Room: Amphi 3

Christoph Schwab

Highdimensional Quadrature in UQ for PDEs

Chair: *Josef Dick* *p. 37*

	Room: Amphi 3	Room: Salle 5	Room: Salle 6	Room: Salle 7
	Improving MC and QMC integration	**Points on the Sphere and Other Manifolds: New Frontiers and Recent Progress (1)**	**Practice of QMC methods (1)**	**Feynman-Kac**
	Chair: *Sergei Kucherenko* *p. 118*	Chair: *Johann Brauchart* *p. 67*	Chair: *Friedrich Pillichsammer* *p. 119*	Chair: *Arnaud Guyader* *p. 120*
$14^{45}-15^{15}$	*Florian Puchhammer* Density estimation by randomized quasi-Monte Carlo	*Johann S. Brauchart* Overview / Hyperuniformity in the Compact Setting: Deterministic and Random Aspects	*Adrian Ebert* Efficient usage and construction of QMC methods	*Arnaud Lionnet* The Numerical Approximation Of Polynomial-Growth Backward Stochastic Differential Equations
$15^{15}-15^{45}$	*Yuji Nakatsukasa* Variance reduction in Monte Carlo integration via function approximation	*Jordi Marzo* Determinantal Point Processes and Optimality	*Michael Gnewuch* Probabilistic discrepancy bounds for Latin hypercube sampling with and without padding	*Marie Billaud-Friess* Stochastic methods for solving partial differential equations in high dimension
$15^{45}-16^{15}$	Coffee break			
	Room: Amphi 3	Room: Salle 5	Room: Salle 6	Room: Salle 7
	Markov Chain QMC	**Rare event simulation**	**Simulation in finance and operation management**	**MCMC and large size**
	Chair: *Bruno Tuffin* *p. 121*	Chair: *Chang-Han Rhee* *p. 88*	Chair: *Guangxin Jiang* *p. 90*	Chair: *Alain Durmus* *p. 123*
$16^{15}-16^{45}$	*Shin Harase* An Implementation of Short-Period Tausworthe Generators for Markov Chain quasi-Monte Carlo Methods	*Pierre Nyquist* Rare-event simulation in machine learning: Infinite swapping and restricted Boltzmann machines	*Jun Luo* Speeding Up Ranking and Selection Procedures for Large Scale Problems Using Cloud Computing	*Paulo Orenstein* Scalable MCMC for Bayes Shrinkage Priors
$16^{45}-17^{15}$	*Tobias Schwedes* Adaptive Importance Sampling for Markov Chain Quasi-Monte Carlo	*Art B. Owen* ALOE importance sampler for the union of rare events	*Guangxin Jiang* Constructing Surface for Derivative Pricing and Sensitivity Analysis	*James Johndrow* Scaling MCMC to Large Problem Sizes
$17^{15}-17^{45}$	*Rami El Haddad* Sudoku Sampling For Markov Chains Simulation	*Chang-Han Rhee* Efficient Rare-Event Simulation for Multiple Jump Events in Regularly Varying Random Walks and Compound Poisson Processes		

18^{30} Welcome Reception City Hall

Tuesday morning, July 3

$8^{30} - 9^{30}$	**Invited Plenary Talk** Room: Amphi 3 *Barry Nelson* **Selecting the Best Simulated System: Thinking Differently About an Old Problem** Chair: *Pierre L'Ecuyer* *p. 38*				
$9^{30} - 10^{00}$	Coffee break				
	Room: Amphi 3 **Algorithms for High-Dimensional Approximation (and Integration) Problems** Chair: *Michael Gnewuch* *p. 91*	Room: Salle 5 **Numerical approximation of SDEs under non-standard assumptions (2)** Chair: *Larisa Yaroslavtseva* *p. 52*	Room: Salle 6 **Non-Reversible Markov Chain Monte Carlo (2)** Chair: *Joris Bierkens* *p. 48*	Room: Salle 7 **QMC and applications** Chair: *Takashi Goda* *p. 64*	Room: Salle 8 **Uncertainty Quantification and Sensitivity Analysis in Computational Finance** Chair: *Giray Okten* *p. 93*
$10^{00} - 10^{30}$	*Yuhan Ding* An Optimal Automatic Algorithm Employing Continuous Linear Functionals	*Mario Hefter* Lower Error Bounds for Strong Approximation of Scalar SDEs with non-Lipschitzian Coefficients	*Christian Robert* Generalized Bouncy Particle Sampler	*Ian Sloan* On the generation of random fields	*Sergei Kucherenko* Application of QMC and Global Sensitivity Analysis to Option Pricing and Greeks
$10^{30} - 11^{00}$	*Aicke Hinrichs* How good is random information? - Approximation in the Hilbert space setting	*Mike Giles* Multilevel Monte Carlo Method for Ergodic SDEs without Contractivity	*Pierre Monmarché* Geometric ergodicity for the Bouncy Particle Sampler	*Roswitha Hofer* Kronecker–Halton sequences in $\mathbb{F}_p((X^{-1}))$	*Emanouil Atanassov* Sensitivity Analysis of Quasi-Monte Carlo methods for the Heston Model
$11^{00} - 11^{30}$	*Klaus Ritter* Integration and L_2-Approximation on Hermite Spaces of Functions of Infinitely-Many Variables	*Dai Taguchi* Semi-implicit Euler-Maruyama scheme for non-colliding particle systems	*Chris Sherlock* Explicit, non-reversible, contour-hugging MCMC moves	*Gunther Leobacher* ON HIGHER-ORDER INTEGRATION ALGORITHMS IN (WEIGHTED) HERMITE SPACES	*Giray Ökten* Sensitivity and Robustness of Financial Models
$11^{30} - 12^{00}$	*Henryk Woźniakowski* Tractability of Multivariate Approximation over Weighted Standard Sobolev Spaces	*Sotirios Sabanis* MCMC algorithms based on numerical approximations of SDEs with locally Lipschitz coefficients	*Michela Ottobre* Sampling and irreversibility	*Mario Neumüller* Asymptotic Behaviour of the Sudler Product of Sines for Quadratic Irrationals	*Alexander Kreinin* Sensitivities of Exotic Portfolios
$12^{00} - 13^{45}$	Lunch break				

Tuesday afternoon, July 3

$13^{45}-14^{45}$ — **Invited Plenary Talk** Room: Amphi 3

Pierre Henry Labordère

Branching diffusion representation for nonlinear Cauchy problems and Monte Carlo approximation

Chair: *Jean-François Chassagneux* p. 39

	Room: Amphi 3	Room: Salle 5	Room: Salle 6	Room: Salle 7	
	Acceleration of MCMC	**Points on the Sphere and Other Manifolds: New Frontiers and Recent Progress (2)**	**MC in finance**	**Practice of QMC methods (2)**	**Multilevel Monte Carlo methods**
	Chair: *Hector Cancela* p. 124	Chair: *Johann Brauchart* p. 68	Chair: *Gilles Pagès* p. 126	Chair: *Gunther Leobacher* p. 127	Chair: *Giovanni Samaey* p. 128
$14^{45}-15^{15}$	*Thomas Catanach* Sequential Tempered Markov Chain Monte Carlo for Bayesian Inference	*Damir Ferizović* Bounds for the Green Energy on SO(3)	*Daniel Roth* MONTE CARLO PATHWISE SENSITIVITIES FOR BARRIER OPTIONS	*Yuya Suzuki* Rank-1 lattices and higher-order exponential splitting for the multi-dimensional time-dependent Schrödinger equation	*Emil Løvbak* Multilevel Monte Carlo for Asymptotic-Preserving Particle Schemes
$15^{15}-15^{45}$	*Maksym Byshkin* Fast Maximum Likelihood estimation via Equilibrium Expectation for large network data	*Ujué Etayo* t-Designs on Manifolds: an Asymptotic Bound on the Number of Points.	*Warren Volk-Makarewicz* Detecting the Presence of Jumps in Option Prices	*Ana I. Gomez* Generation of True Random Numbers using quasi-Montecarlo Methods	*Andreas Van Barel* Robust Optimization of PDE Constrained Systems
$15^{45}-16^{15}$	Coffee break				
	Room: Amphi 3	Room: Salle 5	Room: Salle 6	Room: Salle 7	Room: Salle 8
	Rare events	**Points on the Sphere and Other Manifolds: New Frontiers and Recent Progress (3)**	**Design and testing of random number generators**	**Computational challenges in finance**	**New applications of QMC in physics, energy and environment (1)**
	Chair: *Art Owen* p. 129	Chair: *Johann Brauchart* p. 69	Chair: *Pierre L'Ecuyer* p. 94	Chair: *Jérôme Lelong* p. 96	Chair: *Frances Kuo* p. 71
$16^{15}-16^{45}$	*Jere Koskela* Sequential Monte Carlo for efficient sampling of rare trajectories in reverse time	*Peter D. Dragnev* Universal Bounds on Energy of Codes and Designs in Various Settings	*Sebastiano Vigna* `xoshiro`/`xoroshiro`: new families of high-quality, high-speed PRNGs	*Christian Bayer* Smoothing the payoff for computation of basket options	*Chieh-Han Lee* A DATA FUSION APPROACH FOR SPATIO-TEMPORAL PM2.5 ESTIMATION WITH QMC METHOD
$16^{45}-17^{15}$	*Ad Ridder* Monte Carlo Methods for Insurance Risk Computation	*Tetiana Stepaniuk* Estimates for numerical integration errors on unit spheres of arbitrary dimension	*Hiroshi Haramoto* Testing the Reliability of Statistical Tests for Pseudorandom Number Generators	*Jean-François Chassagneux* Cubature method to solve BSDEs: error expansion and complexity control	*Werner Römisch* Randomized QMC methods for two-stage stochastic optimization problems: Recent progress
$17^{15}-17^{45}$			*Pierre L'Ecuyer* On the Lattice Structure of MIXMAX Random Number Generators	*Gilles Pagès* The Parareal Algorithm for American Options	*Naser Vosoughi* Neutron Source Localization by Analyzing the Detector responses and Markov Chain Monte Carlo (MCMC) Method

18^{30} — Wine & Cheese — Bar l'Aventure

Wednesday morning, July 4

$8^{30} - 9^{30}$	**Invited Plenary Talk** Room: Amphi 3 *Eric Moulines* **Langevin MCMC : theory and methods** Chair: *Pierre Del Moral* *p. 40*				
$9^{30} - 10^{00}$	Coffee break				
	Room: Amphi 3 **Stochastic Computation and Complexity (1)** Chair: *Stefan Heinrich* *p. 74*	Room: Salle 5 **Forward and inverse UQ with hierarchical models (2)** Chair: *Fabio Nobile* *p. 57*	Room: Salle 6 **Simulation of mean-field stochastic differential equations** Chair: *Benjamin Jourdain* *p. 97*	Room: Salle 7 **Importance Splitting for Rare Event Simulation** Chair: *Frédéric Cérou* *p. 99*	Room: Salle 8 **Dispersion and Applications** Chair: *Aicke Hinrichs* *p. 101*
$10^{00} - 10^{30}$	*Thomas Müller-Gronbach* On the error rate of the Euler for SDEs with piecewise Lipschitz drift coefficient	*Jonas Latz* Multilevel Sequential^2 Monte Carlo for Bayesian Inverse Problems	*Mireille Bossy* Particle algorithm for McKean SDE: rate of convergence for some non-smooth drift interaction kernel	*Charles-Edouard Bréhier* New results concerning Adaptive Multilevel Splitting algorithms	*Mario Ullrich* The inverse of the dispersion depends logarithmically on the dimension
$10^{30} - 11^{00}$	*Andreas Rößler* Algorithms for the Approximation of Iterated Stochastic Integrals in Infinite Dimensions	*Håkon Hoel* Multilevel ensemble Kalman filtering for spatio-temporal processes	*Denis Belomestny* Variance reduction for mean-field stochastic differential equations	*Gregoire Ferré* Numerical analysis and long time stability of Feynman-Kac dynamics	*David Krieg* On the Dispersion of Sparse Grids
$11^{00} - 11^{30}$	*Timo Welti* Deep optimal stopping: Solving high-dimensional optimal stopping problems with deep learning	*Joakim Beck* Hierarchical sampling methods for Bayesian experimental design	*Lukasz Szpruch* Weak error expansion for mean-field SDEs	*Nicolas Champagnat* Convergence of Fleming-Viot particle systems to the minimal quasi-stationary distribution	*Daniel Rudolf* Recovery algorithms for high-dimensional rank one tensors
$11^{30} - 12^{00}$	*Monika Eisenmann* A Randomized Time-Stepping Method for Differential Equations with Time-Irregular Coefficients	*Kody Law* Inference using Multilevel Monte Carlo	*Alexandre Zhou* Numerical Analysis of a Particle Calibration Procedure for Local and Stochastic Volatility Models	*Henri Louvin* Application of an importance splitting method to radiation shielding simulations	*Jan Vybiral* On further aspects of dispersion
$12^{00} - 13^{45}$	Lunch break				

Wednesday afternoon, July 4

	Room: Amphi 3 **Stochastic Computation and Complexity (2)** Chair: *Thomas Mueller-Gronbach* *p. 76*	Room: Salle 5 **Forward and inverse UQ with hierarchical models (3)** Chair: *Raul Tempone* *p. 60*	Room: Salle 6 **When to stop a simulation** Chair: *Robert Kunsch* *p. 102*	Room: Salle 7 **Recent advances in particle filtering** Chair: *François Le Gland* *p. 104*	Room: Salle 8 **Variance reduction/estimator efficiency/rare-event probability** Chair: *Zdravko Botev* *p. 105*
$13^{45}-14^{15}$	*Steffen Dereich* Central limit theorems for multilevel stochastic approximation algorithms	*Matteo Croci* Efficient white noise sampling and coupling for multilevel Monte Carlo	*Fred J. Hickernell* Fast Adaptive Bayesian Cubature Using Low Discrepancy Sampling	*Mathieu Gerber* Interacting Particles for Online Inference on Static Parameters Using Streaming Data	*Nadhir Ben Rached* Variance Reduction Techniques for the Accurate Computation of the Distribution of the sum of Ordered Random Variables.
$14^{15}-14^{45}$	*Raphael Kruse* On two quadrature rules for stochastic integrals with fractional Sobolev regularity	*Lukas Mayer* Multilevel Monte Carlo for the Quadrature of SDEs Based on Random Bits	*Robert J. Kunsch* Solvable Integration Problems and Optimal Sample Size Selection	*Anna Wigren* Improving the particle filter in high dimensions using conjugate artificial process noise	*Guo-Jhen Wu* Infinite swapping using iid samples
$14^{45}-15^{15}$	*Sonja Cox* Stochastic integration in quasi-Banach spaces: what Besov regularity does the stochastic heat equation posess?	*Soeren Wolfers* Multilevel weighted least squares polynomial approximation	*Mark Huber* Improved Light Tailed Sample Averages for Robust Estimation of the Mean	*Pierre Del Moral* On the stability and the uniform propagation of chaos properties of ensemble Kalman–Bucy filters	*Thomas Taimre* Exploiting Asymptotics and Polar Coordinates for Rare Tail Estimation
$15^{15}-15^{45}$	*Mihály Kovács* Weak and strong approximation of fractional order elliptic equations with spatial white noise	*Andreas Stein* An adaptive Multilevel Monte Carlo algorithm for advection-diffusion PDEs with random discontinuous coefficients		*Nicolas Chopin* Convergence of resampling algorithms	*Fan Zhang* Data-Driven Distributionally Robust Optimization via Optimal Transport: Algorithms and Applications
16^{00}	Guided Tour of Rennes				
18^{30}	Coach to the conference banquet	Château d'Apigné			

Thursday morning, July 5

$8^{25} - 8^{30}$	Journal of Computing Young Researcher Award 2018				
$8^{30} - 9^{30}$	**Invited Plenary Talk** Room: Amphi 3				
	Friedrich Pillichshammer				
	Discrepancy of digital sequences: new results on a classical QMC topic				
	Chair: *Alexander Keller* *p. 41*				
$9^{30} - 10^{00}$	Coffee break				
	Room: Amphi 3	Room: Salle 5	Room: Salle 6	Room: Salle 7	Room: Salle 8
	Low discrepancy sequences and point sets - devoted to the 80th birthday of Henri Faure (1)	**Stochastic Computation and Complexity (3)**	**Non-uniform Random Variate Generation (1)**	**Approximating Markov chain Monte Carlo**	**Analysis of low-discrepancy sequences**
	Chair: *Wolfgang Ch. Schmid* *p. 80*	Chair: *Raphael Kruse* *p. 78*	Chair: *Josef Leydold* *p. 83*	Chair: *Daniel Rudolf* *p. 107*	Chair: *Kosuke Suzuki* *p. 131*
$10^{00} - 10^{30}$	*Christiane Lemieux* Counting Points in Boxes with Henri Faure: From Discrepancy Bounds to Dependence Structures	*Alvin Tse* Multilevel Monte Carlo for McKean-Vlasov SDEs	*Josef Leydold* Optimal Importance Sampling Density 1: Approximation Methods	*Krys Latuszynski* Barkera's algorithm for Bayesian inference with intractable likelihoods	*Lisa Kaltenböck* On Bounded Remainder Sets for Sequences $(\{a_n\alpha\})_{n\geq 1}$ with $(a_n)_{n\geq 1}$ a Lacunary Integer Sequence
$10^{30} - 11^{00}$	*Peter Kritzer* Discrepancy Bounds for Nets and Sequences	*Martin Redmann* Solving parabolic rough partial differential equations using regression	*Wolfgang Hörmann* Optimal Importance Sampling Density 2: Evaluating CDF and PDF of the Sum of Lognormals	*Błażej Miasojedow* On a new approach of the Unadjusted Langevin Algorithm via convex optimization	*Hiroki Kajiura* Characterization of Matrices B such that (I, B, B^2) Generates a Digital Net with t-value Zero
$11^{00} - 11^{30}$	*Takashi Goda* Quasi-Monte Carlo integration over a triangle	*Yue Wu* Randomized Numerical Schemes for SDE/SPDEs	*Luca Martino* Parsimonious Adaptive Rejection Sampling Schemes	*Nikolaus Schweizer* Approximation of geometrically ergodic Metropolis-Hastings algorithms	*Wolfgang Stockinger* SOME NEGATIVE RESULTS RELATED TO POISSONIAN PAIR CORRELATION PROBLEMS
$11^{30} - 12^{00}$	*Florian Pausinger* On the intriguing search for good permutations	*Stefan Heinrich* Lower bounds for stochastic integration in fractional Sobolev classes	*Efraim Shmerling* Acceptance Tail Sampling Method	*Matti Vihola* Importance Sampling Type Estimators based on Approximate Marginal MCMC	*Raffaello Seri* The Asymptotic Distribution of Riesz' Energy
$12^{00} - 13^{45}$	Lunch break				

Thursday afternoon, July 5

$13^{45}-14^{45}$	**Invited Plenary Talk** Room: Amphi 3 *Clémentine Prieur* **Dimension reduction of the input parameter space for multivariate potentially vector-valued functions** Chair: *Art Owen* *p. 42*			
	Room: Salle 5 **Low discrepancy sequences and point sets–devoted to the 80th birthday of Henri Faure (2)** Chair: *Friedrich Pillichshammer* *p. 81*	Room: Salle 6 **Non-uniform Random Variate Generation (2)** Chair: *Josef Leydold* *p. 85*	Room: Salle 7 **Jittered sampling** Chair: *Bruno Tuffin* *p. 132*	Room: Salle 8 **New applications of QMC in physics, energy and environment (2)** Chair: *Karl Jansen* *p. 73*
$14^{45}-15^{15}$	*Gerhard Larcher* On discrepancy and pair correlation of sequences in the unit interval	*Moran Peri* A Table Method for Sampling from Multivariate Distrbutions with Unbounded Support	*Benjamin Doerr* A Sharp Discrepancy Bound for Jittered Sampling	*Karl Jansen* Lattice Field Theory: a physics case for high dimensional integration
$15^{15}-15^{45}$	*Josef Dick* Richardson Extrapolation of Polynomial Lattice Rules	*Yael Hagbi* Generation of Waiting Time in a Markovian Trial Sequence	*Masatake Hirao* On p-frame potential of random point configurations on the sphere	*Julia Volmer* Improving Monte Carlo integration by symmetrization
$15^{45}-16^{15}$	Coffee break			
	Room: Amphi 3 **Efficient Sampling** Chair: *Ad Ridder* *p. 133*	Room: Salle 5 **Stochastic Differential Equations** Chair: *Mireille Bossy* *p. 135*	Room: Salle 6 **Sequential methods and efficiency** Chair: *François Le Gland* *p. 136*	Room: Salle 7 **Monte Carlo in physics (1)** Chair: *DanHua ShangGuan* *p. 138*
$16^{15}-16^{45}$	*Benjamin Jourdain* Sampling of probability measures in the convex order and approximation of Martingale Optimal Transport problems	*Ankush Agarwal* FINITE VARIANCE UNBIASED ESTIMATION OF STOCHASTIC DIFFERENTIAL EQUATIONS	*Andrea Arnold* Sequential Monte Carlo Methods for Time-Varying Parameter Estimation	*Matthias Baeten* Convergence Analysis of a Coupled Monte-Carlo/Pseudo-Timestepping Scheme Arising in Plasma Edge Simulations
$16^{45}-17^{15}$	*Daniele Bigoni* Adaptive Construction of Transport Maps for Efficient Sampling	*Przemysław Zieliński* Micro-macro acceleration method with relative entropy moment matching for scale-separated SDEs	*Christopher Drovandi* New Insights into History Matching via Sequential Monte Carlo	*DanHua ShangGuan* Efficient Strategy for Global Tallying in the Monte Carlo Criticality Calculation
$17^{15}-17^{45}$	*Andrés F. López-Lopera* Efficiently approximating Gaussian Process Emulators with Inequality Constraints using MC/MCMC	*Andreas Petersson* Rapid covariance based sampling of finite element approximations of linear SPDE in MLMC	*Víctor Elvira* Rethinking the Effective Sample Size in Importance Sampling	*Natalya Tracheva* Monte Carlo method projective estimators for angular and temporal characteristics evaluation of polarized radiation

Friday morning, July 6

$8^{30} - 9^{30}$	**Invited Plenary Talk** Room: Amphi 3 *Marvin Nakayama* **Quantile Estimation via a Combination of Conditional Monte Carlo and Latin Hypercube Sampling** Chair: *Bruno Tuffin* *p. 43*			
$9^{30} - 10^{00}$	Coffee break			
	Room: Amphi 3 **QMC and quadrature strategies for integration** Chair: *Fred J. Hickernell* *p. 139*	Room:Salle 5 **MCQMC in Computer Graphics** Chair: *Alexander Keller* *p. 109*	Room: Salle 6 **Monte Carlo for rare events** Chair: *Gerardo Rubino* *p. 110*	Room: Salle 7 **MCMC : Model selection and convergence** Chair: *Krys Latuszynski* *p. 141*
$10^{00} - 10^{30}$	*Pieterjan Robbe* A Multigrid Multilevel Quasi-Monte Carlo Method with Sample Reuse	*Nikolaus Binder* Fragmented Radix Trees for Efficient Sampling of Discrete Probability Distributions	*Javiera Barrera* Sharp Bounds for the Reliability of a k-out-of-n System Under Dependent Failures Using Cutoff Phenomenon Techniques	*Faming Liang* Average (E)BIC-like Criteria for Bayesian Model Selection
$10^{30} - 11^{00}$	*Lutz Kämmerer* Combining Multiple Rank-1 Lattice Rules for Approximation	*Christophe Hery* On the Usage of Control Variates for Monte Carlo Direct Illumination in Movie Rendering	*Gerardo Rubino* The Multi–Level Creation Process in Flow Network Reliability Estimation	*Georgy Sofronov* Spatial Segmentation via the Generalized Gibbs Sampler
$11^{00} - 11^{30}$	*Mutsuo Saito* Experimental Comparison of Higher-Order Digital Nets for QMC	*Wenzel Jacob* Reversible Jump Metropolis Light Transport using Inverse Mappings	*Hector Cancela* Studying Metabolic Networks Through Monte Carlo Simulations	*Dootika Vats* MCMC for Bayesian penalized regression
$11^{30} - 12^{00}$	*Matthias Sachs* Quadrature Points via Heat Kernel Repulsion		*Ajit Rai* Availability Estimation of Markovian Reliability Systems with Logistics via Cross-Entropy	*Marie Vialaret* On the convergence time of some non-reversible Markov chain Monte-Carlo methods
$12^{00} - 13^{45}$	Lunch break			

Friday afternoon, July 6

	Room: Salle 5 **Handling Discontinuities in QMC with Applications to Computational Finance** Chair: *Xiaoqun Wang* *p. 112*	Room: Salle 6 **Monte Carlo in physics (2)** Chair: *Gerardo Rubino* *p. 143*	Room: Salle 7 **SDE, solutions and convergence rate** Chair: *Przemyslaw Zielinski* *p. 145*	Room: Salle 8 **Applications of MC** Chair: *Lingbin Bian* *p. 146*
$13^{45}-14^{15}$	*Xiaoqun Wang* OVERCOMING THE CHALLENGES IN QMC METHODS FOR COMPUTATIONAL FINANCE	*Anna Korda* Monte-Carlo methods for reconstructing the aerosol scattering matrix	*Abir Ghannoum* Mean Reflected Stochastic Differential Equations with jumps : Simulation by using Particle Systems	*Lingbin Bian* Network structure change point detection by posterior predictive discrepancy
$14^{15}-14^{45}$	*Fei Xie* AN IMPORTANCE SAMPLING-BASED SMOOTHING APPROACH FOR QUASI-MONTE CARLO SIMULATION OF DISCRETE BARRIER OPTIONS	*Mariya Korotchenko* Some Applications of Dynamics Simulation for Multi-Particle Systems in the Kinetic Model Framework	*Flavius Guiaş* High precision solvers for autonomous systems of differential equations based on Markov jump processes	*Julien Roussel* A Perturbative Approach to Control Variates in Molecular Dynamics
$15^{15}-15^{45}$	*Zhijian He* ON THE ERROR RATE OF CONDITIONAL QUASI-MONTE CARLO FOR DISCONTINUOUS FUNCTIONS		*Celine Labart* Approximation rate of BSDEs using random walk	*Chi-Ok Hwang* Laplace Surface Green's Function on a Spherical Surface for Last-passage Monte Carlo Methods
$15^{15}-15^{55}$	Goodbye coffee			

Abstracts of Tutorials

Sunday 14:00–15:30 **Room: Hotel Mercure Centre Gare**

Monte Carlo and Quasi-Monte Carlo Methods in Machine Learning

Alexander Keller
NVDIA, Germany, keller.alexander@gmail.com

The recent progress in computer architecture enabled disruptive advances in artificial intelligence, which in turn created an overwhelming industrial and academic interest, especially in deep learning.

The tutorial will survey some principles of machine learning with a focus on deep neural networks and reinforcement learning. The relations between the field and high dimensional function approximation, information based complexity theory, and Monte Carlo and quasi-Monte Carlo methods offer important research opportunities for the Monte Carlo and quasi-Monte Carlo community.

Sunday 16:00–17:30 **Room: Hotel Mercure Centre Gare**

Introduction to rare event analysis using Monte Carlo

Gerardo Rubino
Inria, France, gerardo.rubino@inria.fr

The analysis of rare events is one of the main difficulties that we encounter with Monte Carlo techniques: the standard (or crude, or naive) approach, while general, fails when addressing this task. More powerful methods (but less general ones) must then be employed. In this tutorial, we will introduce some fundamental families of techniques specialized in dealing with rare events. Since this is a broad field, we will use dependability evaluation as the main application area, but we will at least mention other important types of problems where rareness appear, and the associated available approaches to attack them. Dependability evaluation is itself a large domain, and several important classes of Monte Carlo techniques have proven successful with problems involving rare events, some of them very useful in other areas as well. In the presentation, we will illustrate the kind of efficiency that can be achieved using numerical examples coming from realistic models.

Abstracts of Plenary Talks

Monday 8:30–9:30 **Room: Amphithéâtre 3**

Some theoretical results concerning nonreversible Markov chain and Markov process Monte Carlo algorithms

Christophe Andrieu
School of Mathematics, University of Bristol, UK , C.Andrieu@bristol.ac.uk

Nonreversible processes have recently attracted renewed interest in the context of Monte Carlo simulation. Departure from reversibility offers new methodological opportunities and have shown great promise in some contexts but, in contrast with the reversible scenario, the theory behind these algorithms is largely underdeveloped. In this talk we will review recent results which provide some insights into what can be expected of their behaviour.

Monday 13:45–14:45 **Room: Amphithéâtre 3**

Highdimensional Quadrature in UQ for PDEs

Christoph Schwab
SAM, ETH Zürich, HG G57.1, CH 8092 Zurich, Switzerland, schwab@math.ethz.ch

We review recent results on high-dimensional numerical integration by QMC methods applied to direct [1,2,3,4] and inverse uncertainty quantification [6] for PDEs. Particular attention will be paid to the interplay between sparsity of (generalized) polynomial chaos expansions of integrand functions and dimension-independent convergence rates afforded by higher-order QMC integration. Furthermore, we carefully expound the impact of local support structure of representation systems upon the algorithmic complexity of QMC rule construction [1,2,5]. A number of techniques such as polynomial lattice rules and their scrambled versions [5], and extrapolated lattice rules affording fast matrix-vector multiplication, will be considered. We compare (heretically, perhaps, at MCQMC...) dimension-independent QMC convergence rates with rates afforded by integrand-adapted Smolyak type constructions. Applications to direct and Bayesian inverse UQ for PDEs [6] will be considered. Numerical examples will comprise PDEs with log-gaussian inputs, and Bayesian shape identification problems in nondestructive testing. This work was supported in part by the Swiss National Science Foundation (SNSF) under grant SNF 200021 159940/1.

[1] L. Herrmann and Ch. Schwab Multilevel quasi-Monte Carlo integration with product weights for elliptic PDEs with lognormal coefficients, Technical Report 2017-19, SAM, ETH Zürich (in review).

[2] L. Herrmann and Ch. Schwab QMC integration for lognormal-parametric, elliptic PDEs: local supports and product weights (to appear in Numer. Math. 2018).

[3] R. N. Gantner and L. Herrmann and Ch. Schwab Multilevel QMC with Product Weights for Affine-Parametric, Elliptic PDEs. Contemporary Computational Mathematics - a celebration of the 80th birthday of Ian Sloan Springer (to appear 2018)

[4] R. N. Gantner and L. Herrmann and Ch. Schwab Quasi-Monte Carlo integration for affine-parametric, elliptic PDEs: local supports and product weights, SIAM J. Numer. Analysis, 56/1 (2018), pp. 111-135,

[5] J. Dick, T. Goda and K. Suzuki Construction of interlaced polynomial lattice rules for infinitely differentiable functions. Numer. Math. **137**(2017), 257-288.

[6] J. Dick, R. N. Gantner, Q. T. Le Gia and Ch. Schwab Multilevel higher-order quasi-Monte Carlo Bayesian estimation. Math. Mod. Meth. Appl. Sci. **27/5** (2017), pp. 953-995.

[7] J. Dick, F.Y. Kuo, Q. T. Le Gia and Ch. Schwab Fast QMC matrix-vector multiplication. SIAM J. Scient. Computing (2016)

Tuesday 8:30–9:30 **Room: Amphithéâtre 3**

Selecting the Best Simulated System: Thinking Differently About an Old Problem

Barry L. Nelson
Walter P. Murphy Professor, Department of Industrial Engineering and Management Sciences,
Northwestern University, USA , nelsonb@northwestern.edu

Discrete-event, stochastic simulation is typically employed to evaluate the *feasibility* of a system design; to assess a design's *sensitivity* to unknowns; or to *optimize* system performance. When the speaker was taking his first simulation course in 1980, academic thinking about simulation optimization was just beginning to be influenced by the statistical methodologies of *ranking & selection* (R&S), which was pioneered for an entirely different setting by Shanti Gupta at Purdue and Bob Bechhofer at Cornell. Then Ph.D. student Dave Goldsman recognized the relevance to simulation, and nearly 40 years and hundreds of published papers later, R&S is a standard simulation tool for practitioners, a feature of many simulation languages, and still generating papers. After some background on R&S, I attempt to answer these questions: Is the R&S problem still relevant? Are the usual goals of R&S sensible when the number of system designs becomes extremely large? Are the methods used to build R&S procedures incompatible with high-performance computing? In brief, should we think differently about selecting the best system?

Tuesday 13:45–14:45 **Room: Amphithéâtre 3**

Branching diffusion representation for nonlinear Cauchy problems and Monte Carlo approximation

Pierre Henry Labordere
Global Markets Quantitative Research team, Société Générale, France,
pierre.henry-labordere@sgcib.com

We provide a probabilistic representations of the solution of some semilinear hyperbolic and high-order PDEs based on branching diffusions. These representations pave the way for a Monte-Carlo approximation of the solution, thus bypassing the curse of dimensionality. We illustrate the numerical implications in the context of some popular PDEs in physics such as nonlinear Klein-Gordon equation, a simplified scalar version of the Yang-Mills equation, a fourth-order nonlinear beam equation and the Gross-Pitaevskii PDE as an example of nonlinear Schrodinger equations.

Joint work with Nizar Touzi (CMAP, Ecole Polytechnique).

Wednesday 8:30–9:30 **Room: Amphithéâtre 3**

Langevin MCMC : theory and methods

Eric Moulines
Ecole Polytechnique & Académie des Sciences , eric.moulines@polytechnique.edu

In machine learning literature, a large number of problems amount to simulate a density which is log-concave (at least in the tails) but perhaps non smooth. Most of the research efforts so far has been devoted to the Maximum A posteriori problem, which amounts to solve a high-dimensional convex (perhaps non smooth) program.

The purpose of this talk is to understand how we can use ideas which have proven very useful in machine learning community to solve large scale optimization problems to design efficient sampling algorithms, with « useful » convergence bounds.

In high dimension, first order methods (exploiting exclusively gradient information of the log-posterior) are a must. Most of the efficient algorithms know so far may be seen as variants of the gradient descent algorithms. A natural candidate is the "Unadjusted Langevin Algorithm", which is derived from the Euler discretization of the Langevin diffusion, and may be seen as a noisy version of the gradient descent. By replacing the full gradient by stochastic approximation of the gradient evaluated over mini-batches, computationally efficient version may be derived (which have the same cost than a stochastic gradient algorithm). This algorithm may be generalized in the non-smooth case by "regularizing" the objective function. The Moreau-Yosida inf-convolution algorithm is an appropriate candidate in such case, because it does not modify the minimum value of the criterion while transforming a non smooth optimization problem in a smooth one. We will prove convergence results for these algorithms with explicit convergence bounds both in Wasserstein distance and in total variation. We will discuss in particular how these methods can be adapted to sample over a compact domain and for the computation of normalizing constants.

Thursday 8:30–9:30 **Room: Amphithéâtre 3**

Discrepancy of digital sequences: new results on a classical QMC topic

Friedrich Pillichshammer
Department of Financial Mathematics and Applied Number Theory,
Johannes Kepler University Linz, Austria, friedrich.pillichshammer@jku.at

Digital sequences are a topic that belongs to the foundations of QMC theory. Beside the Halton sequence these are prototypes of sequences with low discrepancy. First examples where given by Il'ya Meerovich Sobol' [3] and Henri Faure [1] with their famous constructions. But the unifying theory was developed later by Harald Niederreiter (see [2] for an overview). Nowadays there are a magnitude of examples of digital sequences and it is classical knowledge, that the star discrepancy of the initial N elements of such sequences can achieve a convergence rate of order of magnitude $(\log N)^s/N$, where s denotes the dimension. On the other hand, very little has been known about other norms of the discrepancy function of digital sequences, beside evident estimates in terms of star discrepancy.

In this talk we present some recent results about various types of discrepancy of digital sequences. This comprises: star discrepancy and weighted star discrepancy, L_p-discrepancy, discrepancy with respect to bounded mean oscillation and exponential Orlicz norms, as well as Sobolev, Besov and Triebel-Lizorkin norms with dominating mixed smoothness.

[1] H. Faure: Discrépances de suites associées a un système de numération (en dimension s). *Acta Arith.* 41: 337–351, 1982. (French)

[2] H. Niederreiter: *Random Number Generation and Quasi-Monte Carlo Methods*. SIAM, Philadelphia, 1992.

[3] I.M. Sobol': Distribution of points in a cube and approximate evaluation of integrals. *Ž Vyčisl. Mat. i Mat. Fiz.* 7: 784–802, 1967.

Thursday 13:45–14:45 **Room: Amphithéâtre 3**

Dimension reduction of the input parameter space for multivariate potentially vector-valued functions

Clémentine Prieur
Grenoble Alpes University, Jean Kuntzmann Lab., Inria project/team AIRSEA, France,
clementine.prieur@univ-grenoble-alpes.fr

Many problems that arise in uncertainty quantification, e.g., integrating or approximating multivariate functions, suffer from the curse of dimensionality. The cost of computing a sufficiently accurate approximation grows indeed dramatically with the dimension of the input parameter space. It thus seems important to identify and exploit some notion of low-dimensional structure as, e.g., the intrinsic dimension of the model. A function varying primarily along a few directions of the input parameter space is said of low intrinsic dimension. In that setting, algorithms for quantifying uncertainty focusing on these important directions are expected to reduce the overall cost.

A common approach to reducing a function's input dimension is the truncated Karhunen-Loève decomposition [1], which exploits the correlation structure of the function's input space. In the present talk, we propose to exploit not only input correlations but also the structure of the input-output map itself.

We will first focus the presentation on approaches based on global sensitivity analysis. The main drawback of global sensitivity analysis is the cost required to estimate sensitivity indices such as Sobol' indices [2]. It is the main reason why we turn to the notion of active subspaces [3, 4] defined as eigenspaces of the average outer product of the function's gradient with itself. They capture the directions along which the function varies the most, in the sense of its output responding most strongly to input perturbations, in expectation over the input measure. In particular, we will present recent results stated in [5] dealing with the framework of multivariate vector-valued functions.

[1] C. Schwab, R. A. Todor, Karhunen–Loève approximation of random fields by generalized fast multipole methods, Journal of Computational Physics 217 (1) (2006) 100–122.

[2] I. Sobol, Sensitivity estimates for non linear mathematical models, Mathematical Modelling and Computational Experiments 1 (1993) 407–414.

[3] P. G. Constantine, E. Dow, Q. Wang, Active subspace methods in theory and practice: Applications to kriging surfaces, SIAM Journal on Scientific Computing 36 (4) (2014) A1500–A1524.

[4] P. G. Constantine, Active Subspaces: Emerging Ideas for Dimension Reduction in Parameter Studies, Society for Industrial and Applied Mathematics, Philadelphia, 2015.

[5] O. Zahm, P. Constantine, C. Prieur, Y. Marzouk, Gradient-based dimension reduction of multivariate vector-valued functions (2018).
URL https://hal.inria.fr/hal-01701425

Friday 8:30–9:30 **Room: Amphithéâtre 3**

Quantile Estimation via a Combination of Conditional Monte Carlo and Latin Hypercube Sampling

Marvin Nakayama
Computer Science Department, New Jersey Institute of Technology, USA , nmarvin@njit.edu

Quantiles or percentiles are often used to assess risk in a variety of application areas. For example, financial analysts frequently measure risk of a portfolio through a 0.99-quantile, which is known as a value-at-risk. For complex stochastic models, analytically computing a quantile typically is not possible, so Monte Carlo simulation is employed. In addition to providing a point estimate for a quantile, we also want to measure the simulation estimate's sampling error, which is typically done by giving a confidence interval (CI) for the quantile. Indeed, the U.S. Nuclear Regulatory Commission (NRC) requires a licensee of a nuclear power plant to account for sampling error when performing a probabilistic safety assessment. A licensee can demonstrate compliance with federal regulations using a "95/95 criterion," which entails establishing, with 95% confidence, that a 0.95-quantile lies below a mandated limit.

Simple random sampling may result in a quantile estimator with an unusably wide CI, especially for an extreme quantile. Moreover, each simulation run of the model may require substantial computation time, which motivates the use of variance-reduction techniques (VRTs). We discuss combining two well-known VRTs, conditional Monte Carlo and Latin hypercube sampling, to estimate a quantile. The combination of the methods can work synergistically together for quantile estimation, greatly reducing the variance obtained by either method by itself.

In addition to devising a point estimator for the quantile when applying the combined approaches, we also describe how to construct asymptotically valid CIs for the quantile. Numerical results demonstrate the effectiveness of the methods.

This is joint work with Hui Dong.

Abstracts in mini-symposia

Monday (10:00–12:00), Tuesday (10:00–12:00)

Non-Reversible Markov Chain Monte Carlo

Organizer(s): Joris Bierkens, Sam Livingstone, Alex Thiery

It has long been known that non-reversible Markov chains can mix faster than their reversible counterparts. Until recently, though, only few general purpose methods existed for designing such non-reversible Markov chains. In the last few years, an explosion of activity has rectified this, greatly advancing both our ability to produce nonreversible chains and our theoretical understanding of them. The aim of this minisymposium is to showcase the state of the art in the field, highlight the recent theoretical and methodological developments, and also discuss the many open questions that persist. The session will cover recent results on "kinetic" MCMC methods such as second order Langevin, Hamiltonian Monte Carlo and the exciting recently introduced Piecewise Deterministic Monte Carlo methods such as the Zig-Zag and the Bouncy Particle Sampler.

Monday 10:00–11:30, **Room: Salle 6**

Non-Reversible Markov Chain Monte Carlo (1)

Chair: Alex Thiery

Reflections on the bouncy particle sampler and Zig-Zag sampler

Joris Bierkens
Delft University of Technology, joris.bierkens@tudelft.nl

Kengo Kamatani
Osaka University, kamatani@sigmath.es.osaka-u.ac.jp

Gareth O. Roberts
University of Warwick, Gareth.O.Roberts@warwick.ac.uk

Pierre-André Zitt
Université Paris-Est, pierre-andre.zitt@u-pem.fr

In recent years piecewise deterministic Markov processes (PDMPs) have emerged as a promising alternative to classical MCMC algorithms. In particular these PDMP based algorithms have good convergence properties and allow for efficient subsampling.

Although many different PDMP based algorithms can be designed, two algorithms play fundamental roles: the Bouncy Particle sampler and the Zig-Zag sampler.

In this talk both algorithms will be introduced and a comparison of properties of these algorithms will be presented, including recent results on ergodicity and on scaling with respect to dimension.

Discrete-Time Bouncy Particle Samplers and Generalisations

Chris Sherlock
Mathematics and Statistics, Lancaster University, UK, c.sherlock@lancaster.ac.uk

Alexandre Thiery
Department of Statistics, National University of Singapore, a.h.thiery@nus.edu.sg

In this talk we describe a general mechanism to incorporate a "momentum" in a standard Metropolis-Hastings algorithm. When this idea is applied to the usual Random Walk Metropolis scheme, it naturally leads to discrete time versions of several variants of the bouncy particle sampler. This construction can easily be modified so that, if the original algorithm is variance bounding, then so is the version with momentum – this implies, in particular, that the discrete-time bouncy particle sampler can inherit the strong asymptotic guaranties that are known for the Random Walk Metropolis algorithm. Finally, we discuss the high-dimensional asymptotic properties of the bouncy particle sampler through the lens of diffusion limits.

On the convergence of Hamiltonian Monte Carlo

Alain Durmus
CMLA, ENS Paris-Saclay, France, alain.durmus@cmla.ens-cachan.fr

Éric Moulines
CMAP, Ecole polytechnique, France, eric.moulines@polytechnique.edu

Eero Saksman
Department of Mathematics and Statistics, University of Helsinki, eero.saksman@helsinki.fi

Hamiltonian Monte Carlo is very popular MCMC method amongst Bayesian statisticians to get samples from a posterior distribution. This algorithm relies on the discretization of Hamiltonian dynamics which leave the target density invariant combined with a Metropolis step. In this talk, we will discuss convergence properties of this method to sample from a positive target density π on $\mathbb{R}^d$, with either a fixed or a random number of integration steps. More precisely, we will present some mild conditions on π to ensure ϕ-irreducibility and ergodicity of the associated chain. Under additional assumption, Harris recurrence can be shown as well. We will conclude on verifiable conditions on π which imply geometric convergence.

User-friendly guarantees for the Langevin Monte Carlo

Arnak Dalalyan
CREST, ENSAE, FRANCE, arnak.dalalyan@ensae.fr

In this talk, I will revisit the recently established theoretical guarantees for the convergence of the Langevin Monte Carlo algorithm of sampling from a smooth and (strongly) log-concave density. I will discuss the existing results when the accuracy of sampling is measured in the Wasserstein distance and provide further insights on relations between, on the one hand, the Langevin Monte Carlo for sampling and, on the other hand, the gradient descent for optimization. I will also present non-asymptotic guarantees for the accuracy of a version of the Langevin Monte Carlo algorithm that is based on inaccurate evaluations of the gradient. Finally, I will propose a variable-step version of the Langevin Monte Carlo algorithm that has two advantages. First, its step-sizes are independent of the target accuracy and, second, its rate provides a logarithmic improvement over the constant-step Langevin Monte Carlo algorithm. More details on the presented results can be found in [1] and [2].

[1] A. Dalalyan. Theoretical guarantees for approximate sampling from a smooth and log-concave density. *J. R. Stat. Soc. B*, 79:651-676, 2017.

[2] A. Dalalyan and A. Karagulyan. User-friendly guarantees for the Langevin Monte Carlo with inaccurate gradient. *arXiv:1710.00095*

Tuesday 10:00–12:00, **Room: Salle 6**

Non-Reversible Markov Chain Monte Carlo (2)

Chair: Joris Bierkens

Generalized Bouncy Particle Sampler

Changye Wu
Université Paris-Dauphine, PSL, France, changye.wu@dauphine.eu

Christian Robert
Université Paris-Dauphine, PSL, France, Christian.Robert@ceremade.dauphine.fr

The concept of piecewise deterministic Markov process has recently emerged as a solution worth investigating for exploring complex distributions, along non-reversible alternatives like Hamiltonian Monte-Carlo and several versions are currently proposed in the literature. Following a survey of this field, we will describe a generalisation of the bouncy particle sampler. As a special example of piecewise deterministic Markov process, the bouncy particle sampler is a rejection-free, irreversible, Markov chain Monte Carlo algorithm which can draw samples from target distribution in a most efficient manner. We generalise here the bouncy particle sampler through its transition dynamics, by a randomisation of the moves that guarantees irreducibility.

Geometric ergodicity for the Bouncy Particle Sampler

Alain Durmus
CMLA, ENS Saclay, France, alain.durmus@cmla.ens-cachan.fr

Arnaud Guillin
Université Blaise Pascal, France, guillin@math.univ-bpclermont.fr

Pierre Monmarché
LJLL, Sorbonne Université, France, pierre.monmarche@upmc.fr

The Bouncy Particle Sampler is a piecewise deterministic Markov process obtained as the limit of persistent walk and used in MCMC algorithms. It converges exponentially fast toward its equilibrium, under quite general assumptions on the log-density of the target distribution. This is a joint work with Alain Durmus and Arnaud Guillin.

Explicit, non-reversible, contour-hugging MCMC moves

Matthew Ludkin
Mathematics and Statistics, Lancaster University, UK, m.ludkin1@lancaster.ac.uk

Chris Sherlock
Mathematics and Statistics, Lancaster University, UK, c.sherlock@lancaster.ac.uk

Both the Bouncy Particle Sampler (BPS; [1]) and Discrete Bouncy Particle Samplers (DBPSs; [2, 3]) are non-reversible Markov chain Monte Carlo algorithms whose action can be visualised in terms of a particle moving with a fixed-magnitude velocity. Both algorithm types include an occasional step where the particle 'bounces' off a hyperplane which is tangent to the gradient of the target density, making the BPS rejection-free and allowing the DBPS to make relatively large jumps whilst maintaining a high

acceptance rate. Analogously to the concatenation of leapfrog steps in HMC, we describe an algorithm which omits the straight-line movement of the BPS and DBPS and, instead, at each iteration concatenates several discrete 'bounces' to provide a proposal which is on almost the same target contour as the starting point, producing a a large proposed move with a high acceptance probability. Combined with a separate kernel designed for moving between contours, an explicit bouncing scheme which takes account of the local Hessian at each bounce point leads to an efficient, non-reversible MCMC algorithm.

[1] E. Peters and G. deWith. Rejection-free Monte Carlo sampling for general potentials. *Physical Review E*, 85 (2), 2012.

[2] C. Sherlock and A. Thiery. A discrete bouncy particle sampler. *ArXiv e-prints*, 1707.05200, 2017.

[3] P. Vanetti, A. Bouchard-Côté, G. Deligiannidis and A. Doucet. Piecewise-deterministic Markov Chain Monte Carlo *ArXiv e-prints*, 1707.05296, 2017.

Sampling and irreversibility

Michela Ottobre
Maxwell Institute, Edinburgh, UK, `m.ottobre@hw.ac.uk`

In recent years the observation that "irreversible processes converge to equilibrium faster than their reversible counterparts" has sparked a significant amount of research to exploit irreversibility within sampling schemes, thereby accelerating convergence of the resulting Markov Chains. It is now understood how to design irreversible continuous time dynamics with prescribed invariant measure. However, for sampling/simulation purposes, such dynamics still need to undergo discretization and, as it is well known, naive discretizations can completely destroy all the good properties of the continuous-time process. In this talk we will i) give some background on irreversibility and review the progress made so far on the study of non-reversible processes, both by an algorithmic and by a theoretical point of view; ii) make further considerations on how to use (or not to use) irreversibility for algorithmic purposes

Monday (10:00–12:00), Tuesday (10:00–12:00)

Numerical approximation of SDEs under non-standard assumptions

Organizer(s): Larisa Yaroslavtseva & Mario Hefter

This symposium is devoted to numerical methods for stochastic differential equations (SDEs). While the majority of results in this area deals with globally Lipschitz continuous coefficient functions, such assumptions are typically not met for real world applications from biology, chemistry or computational finance. Systematic investigations on numerics for equations with non-standard coefficients have started only recently and the construction of algorithms and the corresponding error analysis is still at its infancy. The special session will bring together leading experts on numerical methods for SDEs under non-standard assumptions. Topics include the construction of efficient algorithms as well as the error and complexity analysis of such problems.

Monday 10:00–12:00, **Room: Salle 5**

Numerical approximation of SDEs under non-standard assumptions (1)

Chair: Mario Hefter

On loss of regularity in the initial value for SDEs with non-globally Lipschitz continuous coefficients

Arnulf Jentzen
ETH Zurich, Switzerland, arnulf.jentzen@sam.math.ethz.ch

Thomas Müller-Gronbach
University of Passau, Germany, thomas.mueller-gronbach@uni-passau.de

Larisa Yaroslavtseva
University of Passau, Germany and ETH Zurich, Switzerland, larisa.yaroslavtseva@uni-passau.de

Let $d, m \in \mathbb{N}$, let W be an m-dimensional Brownian motion on a probability space $(\Omega, \mathcal{A}, \mathbb{P})$, let $\mu\colon \mathbb{R}^d \to \mathbb{R}^d$ and $\sigma\colon \mathbb{R}^d \to \mathbb{R}^{d\times m}$ be continuous functions and for $x \in \mathbb{R}^d$ consider the stochastic differential equation (SDE)

$$dX^x(t) = \mu(X^x(t))\,dt + \sigma(X^x(t))\,dW(t), \quad t \in [0,1],$$
$$X^x(0) = x.$$

We assume that for every $x \in \mathbb{R}^d$ this SDE has a unique strong solution X^x and that $X^x(t) \in L^p(\Omega, \mathbb{R}^d)$ for every $t \in [0,1]$ and every $p \in [1,\infty)$. We study the regularity of X^x with respect to the initial value x. It is well-known that if the coefficients μ and σ are globally Lipschitz continuous then for every $t \in [0,1]$ and every $p \in [1,\infty)$ the mapping $\mathbb{R}^d \ni x \mapsto X^x(t) \in L^1(\Omega, \mathbb{R}^d)$ is globally Lipschitz continuous as well. However, many SDEs arising in applications have polynomially growing coefficients, which are not globally Lipschitz continuous. Recently, it has been shown in [1] that solutions of such SDEs may fail to be locally Lipschitz continuous with respect to the initial value. In the actual talk we show that solutions of such SDEs may even fail to be locally γ-Hölder continuous with respect to the initial value for every $\gamma \in (0,1]$.

[1] M. Hairer, M. Hutzenthaler, A. Jentzen. Loss of regularity for Kolmogorov equations. *Ann. Probab.*, 43 (2): 468–527, 2015.

Numerical approximations of nonlinear stochastic differential equations

Arnulf Jentzen
Department of Mathematics, ETH Zurich, Switzerland, arnulf.jentzen@sam.math.ethz.ch

Diyora Salimova
Department of Mathematics, ETH Zurich, Switzerland, diyora.salimova@sam.math.ethz.ch

Timo Welti
Department of Mathematics, ETH Zurich, Switzerland, timo.welti@sam.math.ethz.ch

In this talk we present a few recent results on regularity properties and numerical approximation for stochastic ordinary and partial differential equations. We propose an explicit and easily implementable full-discrete numerical approximation scheme and prove that the suggested scheme converges both in the strong and numerically weak sense for a large class of additive noise driven stochastic evolution equations with superlinearly growing nonlinearities. In particular, we establish strong and numerically weak convergence of the proposed scheme in the case of stochastic Kuramoto-Sivashinsky equations, stochastic Burgers equations, and stochastic Allen-Cahn equations.

Euler-type schemes for SDEs with discontinuous drift

Andreas Neuenkirch
Institut für Mathematik, Universität Mannheim, Germany, neuenkirch@math.uni-mannheim.de

Michaela Szölgyenyi
Seminar for Applied Mathematics and RiskLab Switzerland, ETH Zürich, Switzerland, michaela.szoelgyenyi@sam.math.ethz.ch

Lukasz Szpruch
School of Mathematics, University of Edinburgh, Great Britain, L.Szpruch@ed.ac.uk

Stochastic differential equations with irregular (non-globally Lipschitz) coefficients are a very active topic of research. We study equations, where we relax the global Lipschitz condition on the drift coefficient to allow for discontinuities on a set of positive reach. We study strong convergence of an Euler-type scheme, which uses adaptive step-sizing for a better resolution close to the discontinuity. We obtain a numerical method which has – up to logarithmic terms – strong convergence order 1/2 with respect to the average computational cost.

Strong Approximation of Stochastic Mechanical Systems with Nonlinear Holonomic Constraints

Holger Stroot
Mathematics, University of Kaiserslautern, Germany, stroot@mathematik.uni-kl.de

In this talk we introduce a strongly convergent symplectic algorithm for the approximation of stochastic mechanical systems with nonlinear holonomic constraints. Those types of systems occur among others in molecular dynamics as well as in the context of fiber dynamics in turbulent airflows. The corresponding dynamical equations can be interpreted as either a stochastic differential-algebraic equations of constrained stochastic differential equations. Even regarded as SDEs the approximation of those equations faces two major issues. First from an applicational point of view one is mainly interested in algorithms

that are symplectic with respect to the constraints. Secondly, even in the most basic examples the nonlinearity of the constraints generates underlying SDEs with drift coefficients that do not fulfill the one-sided Lipschitz conditions. Up to this point we know of no rigorous proof of a strongly convergent sympletic algorithm applied to stochastic mechanical systems with nonlinear constraints. Further, certain standard algorithms which are currently used in practice are typically only well-defined with probability close to one. We introduce the half-explicit drift-truncated Euler scheme which is sympletic and prove rigorously its strong convergence towards the solution of the stochastic differential-algebraic equation.

This talk is based on joint work with Prof. Felix Lindner, University of Kassel, and Dr. Raimund Wegener, Fraunhofer ITWM.

Tuesday 10:00–12:00, **Room: Salle 5**

Numerical approximation of SDEs under non-standard assumptions (2)

Chair: Larisa Yaroslavtseva

Lower Error Bounds for Strong Approximation of Scalar SDEs with non-Lipschitzian Coefficients

Mario Hefter
Department of Mathematics, TU Kaiserslautern, Germany, hefter@mathematik.uni-kl.de

André Herzwurm
Department of Mathematics, TU Kaiserslautern, Germany, herzwurm@mathematik.uni-kl.de

Thomas Müller-Gronbach
Faculty of Computer Science and Mathematics, University of Passau, Germany,
thomas.mueller-gronbach@uni-passau.de

We study pathwise approximation of scalar stochastic differential equations at a single time point or globally in time by means of methods that are based on finitely many observations of the driving Brownian motion. We prove lower error bounds in terms of the average number of evaluations of the driving Brownian motion that hold for every such method under rather mild assumptions on the coefficients of the equation. The underlying simple idea of our analysis is as follows: the lower error bounds known for equations with coefficients that have sufficient regularity globally in space should still apply in the case of coefficients that have this regularity in space only locally, in a small neighborhood of the initial value. Our results apply to a huge variety of equations with coefficients that are not globally Lipschitz continuous in space including Cox-Ingersoll-Ross processes, equations with superlinearly growing coefficients, and equations with discontinuous coefficients. In many of these cases the resulting lower error bounds even turn out to be sharp.

Multilevel Monte Carlo Method for Ergodic SDEs without Contractivity

Wei Fang
Mathematical Institute, University of Oxford, UK, wei.fang@maths.ox.ac.uk

Mike Giles
Mathematical Institute, University of Oxford, UK, mike.giles@maths.ox.ac.uk

In this talk, based on [1], we propose a new multilevel Monte Carlo (MLMC) method for the ergodic SDEs which do not satisfy the contractivity condition. By employing a change of measure technique, we introduce a contractive coupling between coarse and fine path simulations. The change of measure adds a Radon-Nikodym derivative to the MLMC estimator. The variance of the new level estimator increases linearly in T, which is a great reduction compared with the exponential increase in standard MLMC. The total computational cost is reduced to $O(\varepsilon^{-2}|\log\varepsilon|^2)$ from $O(\varepsilon^{-3}|\log\varepsilon|)$ for the standard Monte Carlo method. Numerical experiments support our analysis.

[1] W. Fang and M.B. Giles. Multilevel Monte Carlo Method for Ergodic SDEs without Contractivity *Working paper in preparation*, 2018.

Semi-implicit Euler-Maruyama scheme for non-colliding particle systems

Hoang-Long Ngo
Hanoi National University of Education, Vietnam, ngolong@hnue.edu.vn

Dai Taguchi
Graduate School of Engineering Science, Osaka University, Japan, dai.taguchi.dai@gmail.com

Let $X = (X(t) = (X_1(t), \dots, X_d(t))^*)_{t\geq 0}$ be a solution of the following system of stochastic differential equation (SDE)

$$\mathrm{d}X_i(t) = \left\{ \sum_{j\neq i} \frac{\gamma_{i,j}}{X_i(t) - X_j(t)} + b_i(X_i(t)) \right\} \mathrm{d}t + \sum_{j=1}^{d} \sigma_{i,j}(X(t))\mathrm{d}W_j(t), \tag{9.1}$$

with $X(0) \in \Delta_d = \{x = (x_1, \dots, x_d)^* \in \mathbb{R}^d : x_1 < x_2 < \cdots < x_d\}$, $\gamma_{i,j} = \gamma_{j,i} \geq 0$, and $W = (W(t) = (W_1(t), \dots, W_d(t))^*)_{t\geq 0}$ a d-dimensional standard Brownian motion. A solution X is rarely analytically tractable, so one often approximates X by using the (explicit) Euler-Maruyama scheme. However, unfortunately the explicit scheme does not preserve the non-colliding property of a solution (9.1).

Recently, many authors study numerical approximation for one-dimensional SDEs with boundary (e.g. Bessel process $\mathrm{d}X_t = dt/X_t + \mathrm{d}W_t, X_t > 0$ and CIR process $\mathrm{d}X_t = (a - bX_t)dt + X_t^{1/2}\mathrm{d}W_t, X_t > 0$). Dereich, Neuenkirch and Szpruch [2] introduced an implicit Euler-Maruyama scheme for CIR process and showed that the rate of convergence is $1/2$, and extended to one-dimensional SDEs with boundary condition by Alfonsi [1] and Neuenkirch and Szpruch [3].

Inspired by [1, 2, 3], we define a semi-implicit Euler-Maruyama scheme for a solution of non-colliding SDE (9.1) as follows: $X^{(n)}(0) := X(0)$ and for each $k = 0, \dots, n-1$, $X^{(n)}(t_{k+1}^{(n)})$ is the unique solution in Δ_d of the following equation:

$$\begin{aligned} X_i^{(n)}(t_{k+1}^{(n)}) = X_i^{(n)}(t_k^{(n)}) &+ \left\{ \sum_{j\neq i} \frac{\gamma_{i,j}}{X_i^{(n)}(t_{k+1}^{(n)}) - X_j^{(n)}(t_{k+1}^{(n)})} + b_i\left(X_i^{(n)}(t_k^{(n)})\right) \right\} \frac{T}{n} \\ &+ \sum_{j=1}^{d} \sigma_{i,j}\left(X^{(n)}(t_k^{(n)})\right) \left\{ W_j(t_{k+1}^{(n)}) - W_j(t_k^{(n)}) \right\}. \end{aligned}$$

Since the equation

$$\xi_i = a_i + \sum_{j\neq i} \frac{c_{i,j}}{\xi_i - \xi_j}, \quad i = 1, \dots, d,$$

has a unique solution in Δ_d for each $a_i \in \mathbb{R}$ and $c_{i,j} \geq 0$ with $c_{i,i+1} > 0$ (see Proposition 2.2 in [4]), thus $X^{(n)} = (X^{(n)}(t_k^{(n)}))_{k=0,\dots,n}$ is well-defined for each $n \in \mathbb{N}$.

In this talk, under some assumptions on the constants $\gamma_{i,j}$ and the coefficients b_i, $\sigma_{i,j}$, we will show that the SDE (9.1) has a unique global strong solution on Δ_d and the semi-implicit Euler-Maruyama approximation $X^{(n)}$ converges to the unique solution to the non-colliding SDE (9.1) in L^p-sense for some $p \geq 1$ or 2 with convergence rate $n^{1/2}$ or n. More preciously, we will show that there exists $C > 0$ such that,

$$\mathbb{E}\left[\sup_{k=1,\dots,n} |X(t_k^{(n)}) - X^{(n)}(t_k^{(n)})|^p\right]^{1/p} \leq \begin{cases} Cn^{-1/2}, & \text{if } b_i \text{ are Lipschitz continuos and } p \geq 1, \\ Cn^{-1}, & \text{if } b_i \in C_b^2(\mathbb{R};\mathbb{R}) \text{ and } p \geq 2, \end{cases}$$

(see Theorem 2.8 and 2.9 in [4]). Note that the singular coefficients $\frac{1}{x_i - x_j}$ make the system difficult to deal with. In order to overcome this obstacle, we need an upper bound for both moments and inverse moments of $X_i(t) - X_j(t)$.

[1] A. Alfonsi. Strong order one convergence of a drift implicit Euler scheme: Application to the CIR process. *Statist. Probab. Lett.* 83(2): 602–607, 2013.

[2] S. Dereich, A. Neuenkirch and L. Szpruch. An Euler-type method for the strong approximation for the Cox-Ingersoll-Ross process. *Proc. R. Soc. A* 468: 1105–1115, 2012.

[3] A. Neuenkirch and L. Szpruch. First order strong approximations of scalar SDEs defied in a domain. *Numer. Math.* 128: 103–136, 2014.

[4] H.-L. Ngo and D. Taguchi. Semi-implicit Euler-Maruyama approximation for non-colliding particle systems. preprint, arXiv:1706.10119.

MCMC algorithms based on numerical approximations of SDEs with locally Lipschitz coefficients

Nicolas Brosse
Ecole Polytechnique, France, `nicolas.brosse@polytechnique.edu`

Alain Durmus
Ecole Normale Supérieure, France, `alain.durmus@cmla.ens-cachan.fr`

Eric Moulines
Ecole Polytechnique, France, `eric.moulines@polytechnique.edu`

Sotirios Sabanis
University of Edinburgh & Alan Turing Institute, U.K., `s.sabanis@ed.ac.uk`

The problem of sampling from a high-dimensional distribution π having a density on $\mathbb{R}^d$ (which is known up to a normalizing constant), $x \mapsto \mathrm{e}^{-U(x)} / \int_{\mathbb{R}^d} \mathrm{e}^{-U(y)} \mathrm{d}y$, will be considered. The main tool for achieving this aim (sampling) typically comes from a numerical approximation of the associated Langevin stochastic differential equation. Key points of this methodology will be discussed in the context of SDEs with superlinear coefficients.

Monday (10:00–12:00), Wednesday (10:00–12:00, 13:45–15:45)

Forward and inverse UQ with hierarchical models

Organizer(s): Abdul-Lateef Haji-Ali, Fabio Nobile & Raul Tempone

Monte Carlo methods are general, flexible sampling methods for the computation of expected values of observables arising in stochastic systems. Monte Carlo methods are desirable since they are simple to implement and their rate of convergence is very robust. Still, in the context of random evolution of large systems arising from the discretization of differential equations subject to randomness, their cost can be too large for practical purposes.

Fortunately, recent years have witnessed an explosion of activity in the development of multi-level, multi-index and multi-fidelity Monte Carlo methods. These methods reduce the computational cost by using multiple coarser numerical approximations and low fidelity models with simplified physics as control variates. As a consequence, they lessen the number of samples in the estimation of output expected values for a given high fidelity model (with

ne discretization and correct physics). These hierarchical techniques have been proven to be extremely useful in many application areas and have also been generalized to Bayesian inverse problems and data assimilation techniques.

In this minisymposium we intend to present the latest algorithmic and theoretical contributions to multi-level and multi-fidelity Monte Carlo methods, in the context of forward uncertainty analysis, rare event simulations and risk analysis, Bayesian inverse problems, data assimilation, and optimization under uncertainty.

Monday 10:00–12:00, **Room: Salle 8**

Forward and inverse UQ with hierarchical models (1)

Chair: Abdul-Lateef Haji-Ali

Multifidelity Monte Carlo estimation with adaptive low-fidelity models

Benjamin Peherstorfer
Department of Mechanical Engineering and Department of Mathematics, University of Wisconsin-Madison, USA, peherstorfer@wisc.edu

Control variates provide a variance reduction framework that can combine samples from low- and high-fidelity models to speed up the estimation of statistics of the computationally expensive high-fidelity model outputs. Optimally trading off the sampling of the low- and high-fidelity models to minimize the mean-squared error of the estimator for a given computational budget has been widely studied. We consider the situation where the low-fidelity models can be adapted to improve their approximation quality with respect to the high-fidelity model. We then derive a quasi-optimal trade-off between adaptation and sampling in the sense that our approach minimizes an upper bound of the mean-squared error of our estimator, instead of the error directly. We show that our quasi-optimal number of adaptations of the low-fidelity models is bounded even in the limit case that an infinite budget is available. Numerical results demonstrate that our adaptive approach can achieve orders of magnitude speedups compared to control variate estimators with static low-fidelity models and compared to Monte Carlo estimators that use the high-fidelity model alone.

Adaptive Importance Sampling for Multilevel Monte Carlo Euler method

Ahmed Kebaier
LAGA, Université Paris 13, France, kebaier@math.univ-paris13.fr

This paper focuses on the study of an original combination of the Multilevel Monte Carlo method introduced by Giles [2] and the popular importance sampling technique. To compute the optimal choice of the parameter involved in the importance sampling method, we rely on Robbins-Monro type stochastic algorithms. On the one hand, we extend our previous work [1] to the Multilevel Monte Carlo setting. On the other hand, we improve [1] by providing a new adaptive algorithm avoiding the discretization of any additional process. Furthermore, from a technical point of view, the use of the same stochastic algorithms as in [1] appears to be problematic. To overcome this issue, we employ an alternative version of stochastic algorithms with projection (see e.g. Laruelle, Lehalle and Pagès [3]). In this setting, we show innovative limit theorems for a doubly indexed stochastic algorithm which appear to be crucial to study the asymptotic behavior of the new adaptive Multilevel Monte Carlo estimator. Finally, we illustrate the efficiency of our method through applications from quantitative finance.

[1] M. Ben Alaya, K. Hajji, and A. Kebaier. Importance sampling and statistical Romberg method. *Bernoulli*, 21(4):1947–1983, 2015.

[2] M. B. Giles. Multilevel Monte Carlo path simulation. *Oper. Res.*, 56(3):607–617, 2008.

[3] S. Laruelle, C. Lehalle, and G. Pagès. Optimal posting price of limit orders: learning by trading. *Math. Financ. Econ.*, 7(3):359–403, 2013.

Multilevel Monte Carlo Approximation of Functions

Sebastian Krumscheid
Institute of Mathematics, EPFL, Switzerland, sebastian.krumscheid@epfl.ch

Fabio Nobile
Institute of Mathematics, EPFL, Switzerland, fabio.nobile@epfl.ch

Many applications across sciences and technologies require a careful quantification of non-deterministic effects to a system output, for example when evaluating the system's reliability or when gearing it towards more robust operation conditions. At the heart of these considerations lies an accurate yet efficient characterisation of uncertain system outputs. For the approximation of moments of said outputs, the multilevel Monte Carlo method has been established as a computationally efficient sampling method that is applicable to a wide range of applications. While a characterisation of the uncertain output in terms of a few moments may be sufficient in some applications, many applications would, however, require many (possibly infinitely many) moments for an accurate approximation of an output's distribution. That is, a moment-based characterisation of an uncertain output (e.g. via a truncated Edgeworth series expansion) is often unfeasible. As a matter of fact, in some practically relevant cases, for example when the system output follows a Lévy distribution (also known as a van der Waals profile), moments do not even exist, so that a moment-based characterisation is even impossible here.

In this talk we will introduce novel multilevel Monte Carlo techniques for an efficient characterisation of an uncertain system output's distribution. These techniques rely on accurately approximating general parametric expectations, i.e. expectations that depend on a parameter, uniformly on some interval. The resulting multilevel Monte Carlo estimators of such functions enable to derive efficient approximations of various means to characterise a system output's distribution, for example an approximation to the characteristic function or to the cumulative distribution function. A further important consequence of these results is that they allow to construct multilevel Monte Carlo estimators for various robustness indicators, such as for quantiles (also known as value-at-risk) or for the conditional value-at-risk. It is

noteworthy that these robustness indicators cannot be expressed as moments. Consequently, they are out of reach for an efficient treatment via standard multilevel Monte Carlo methods, although first specialised multilevel techniques for some robustness indicators have been introduced recently. Here, we will present the construction and the analysis of general multilevel Monte Carlo methods for functions and discuss their applications to the estimation of various robustness indicators. Moreover, we will illustrate the performance of the developed multilevel methodologies using different benchmark examples, before, if time permits, addressing problems arising in the context of robust design in aeronautics.

Multilevel Nested Simulation for Efficient Risk Estimation

Michael B. Giles
Mathematical Institute, Oxford University, UK, mike.giles@maths.ox.ac.uk

Abdul-Lateef Haji-Ali
Mathematical Institute, Oxford University, UK, hajiali@maths.ox.ac.uk

We investigate the problem of computing a nested expectation of the form $P\left[E\left[X|Y\right]\geq 0\right]=E\left[H\left(E\left[X|Y\right]\right)\right]$ where H is the Heaviside function. This nested expectation appears, for example, when estimating the probability of a large loss from a financial portfolio. We present a method that combines the idea of using Multilevel Monte Carlo (MLMC) for nested expectationswith the idea of adaptively selecting the number of samples in the approximation of the inner expectation, as proposed in [1]. We propose and analyse an algorithm that adaptively selects the number of inner samples on each MLMC level and prove that the resulting MLMC method with adaptive sampling has an $\mathcal{O}\left(\varepsilon^{-2}\left|\log(\varepsilon)\right|\right)$ complexity to achieve a root mean-squared error ε. The theoretical analysis is verified by numerical experiments on a simple model problem. We also present a stochastic root-finding algorithm that, combined with our adaptive methods, can be used to compute other risk measures such as Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR), with the latter being achieved with $\mathcal{O}\left(\varepsilon^{-2}\right)$ complexity.

[1] M. Broadie, Y. Du, C. C. Moallemi Efficient risk estimation via nested sequential simulation. *Management Science*, 57 (6): 1172–1194, 2011.

[2] A-L. Haji-Ali, M. Giles Multilevel nested simulation for efficient risk estimation. *arXiv*, 1802.05016, 2018.

Wednesday 10:00–12:00, **Room: Salle 5**

Forward and inverse UQ with hierarchical models (2)

Chair: Fabio Nobile

Multilevel Sequential^2 Monte Carlo for Bayesian Inverse Problems

Jonas Latz
Department of Mathematics, Technische Universität München, Germany, jlatz@ma.tum.de

Iason Papaioannou
Department of Civil, Geo and Environmental Engineering, Technische Universität München, Germany, iason.papaioannou@tum.de

Elisabeth Ullmann
Department of Mathematics, Technische Universität München, Germany, elisabeth.ullmann@ma.tum.de

The identification of parameters in mathematical models using noisy observations is a common task in uncertainty quantification. We employ the framework of Bayesian inversion: we combine monitoring and observational data with prior information to estimate the posterior distribution of a parameter. Specifically, we are interested in the distribution of a diffusion coefficient of an elliptic PDE. In this setting, the sample space is high-dimensional, and each sample of the PDE solution is expensive. To address these issues we propose and analyse a novel Sequential Monte Carlo (SMC) sampler for the approximation of the posterior distribution. Classical, single-level SMC constructs a sequence of measures, starting with the prior distribution, and finishing with the posterior distribution. The intermediate measures arise from a tempering of the likelihood, or, equivalently, a rescaling of the noise. The resolution of the PDE discretisation is fixed. In contrast, our estimator employs a hierarchy of PDE discretisations to decrease the computational cost. We construct a sequence of intermediate measures by decreasing the temperature or by increasing the discretisation level at the same time. We introduce this method in [2]. This idea builds on and generalises the multi-resolution sampler proposed in [1] where a bridging scheme is used to transfer samples from coarse to fine discretisation levels. Importantly, our choice between tempering and bridging is fully adaptive. We present numerical experiments in 2D space, comparing our estimator to single-level SMC and the multi-resolution sampler.

[1] P.S. Koutsourelakis. A multi-resolution, non-parametric, Bayesian framework for identification of spatially-varying model parameters *Journal of Computational Physics*, 228: 6184-6211, 2009.

[2] J. Latz, I. Papaioannou, E. Ullmann. Multilevel Sequential^2 Monte Carlo for Bayesian Inverse Problems *arXiv:* 1709.09763, 2017.

Multilevel ensemble Kalman filtering for spatio-temporal processes

Chernov Alexey
Institute for Mathematics, Carl von Ossietzky University Oldenburg, Germany,
alexey.chernov@uni-oldenburg.de

Hoel Håkon
Department of Mathematical Sciences, Chalmers University of Technology and University of Gothenburg, Sweden, hhakon@chalmers.se

Law Kody
School of Mathematics, University of Manchester, England, kody.law@manchester.ac.uk

Nobile Fabio
Mathematics Institute of Computational Science and Engineering, École polytechnique fédérale de Lausanne, Switzerland, fabio.nobile@epfl.ch

Tempone Raúl
Applied Mathematics and Computational Sciences, KAUST, Thuwal, Saudi Arabia,
raul.tempone@kaust.edu.sa

The ensemble Kalman filter (EnKF) is a sequential filtering method that uses an ensemble of particle paths to estimate the means and covariances required by the Kalman filter by the use of sample moments, i.e., the Monte Carlo method. EnKF is often both robust and efficient, but its performance may suffer in settings where the computational cost of accurate simulations of particles is high. The multilevel Monte Carlo method (MLMC) is an extension of the classical Monte Carlo method, which by sampling stochastic realizations on a hierarchy of resolutions may reduce the computational cost of moment approximations by orders of magnitude. In this talk I will present ideas on combining MLMC and EnKF to construct the multilevel ensemble Kalman filter (MLEnKF) for settings of either finite [1] or infinite dimensional [2] state space. Theoretical results and numerical studies of the performance gain of MLEnKF over EnKF will also be presented.

[1] Hoel H, Law KJ, Tempone R. Multilevel ensemble Kalman filtering. *SIAM Journal on Numerical Analysis*, 54(3):1813–39, 2016.

[2] Chernov A, Hoel H, Law KJ, Nobile F, Tempone R. Multilevel ensemble Kalman filtering for spatio-temporal processes. *arXiv preprint*, arXiv:1710.07282, 2017.

Hierarchical sampling methods for Bayesian experimental design

Joakim Beck
Computer, Electrical and Mathematical Science and Engineering Division, KAUST, Saudi Arabia, joakim.beck@kaust.edu.sa

Ben Mansour Dia
College of Petroleum Engineering and Geosciences, KFUPM, Saudi Arabia, mansourben2002@yahoo.fr

Luis Espath
Computer, Electrical and Mathematical Science and Engineering Division, KAUST, Saudi Arabia, luis.espath@kaust.edu.sa

Raul Tempone
Computer, Electrical and Mathematical Science and Engineering Division, KAUST, Saudi Arabia, raul.tempone@kaust.edu.sa

In this talk, the focus is on hierarchical sampling methods for the efficient computation of the expected information gain in Bayesian optimal experimental design. We propose the use of the Laplace approximation as an effective means of importance sampling, leading to a substantial reduction in computational work, in combination with hierarchical sampling in the context of PDEs. Optimal values for the method parameters are computed, where the average computational cost is minimized subject to a desired error tolerance. We demonstrate the computational efficiency of our methods for optimal electrode placement in electrical impedance tomography to recover the fiber orientation in laminate composites.

Inference using Multilevel Monte Carlo

Kody Law
School of Mathematics, University of Manchester, UK, kody.law@manchester.ac.uk

Ajay Jasra
Department of Statistics and Applied Probability, staja@nus.edu.sg

This talk will concern the recent development of some multilevel Monte Carlo methods for inverse problems and data assimilation in the context of complex engineering applications. In particular, Markov chain Monte Carlo and sequential Monte Carlo methods will be considered. I will discuss some strategies which can be employed to obtain algorithms in this context with provably optimal convergence rates [1, 2].

[1] Beskos, A., Jasra, A., Law, K. J. H., Tempone, R., and Zhou, Y. Multilevel sequential Monte Carlo samplers. *Stochastic Processes and their Applications*, 127(5), 1417-1440, 2017.

[2] Jasra, A., Kamatani, K., Law, K. J. H., and Zhou, Y. Bayesian Static Parameter Estimation for Partially Observed Diffusions via Multilevel Monte Carlo. *arXiv preprint arXiv:1701.05892*, 2017.

Wednesday 13:45–15:45, **Room: Salle 5**

Forward and inverse UQ with hierarchical models (3)

Chair: Raul Tempone

Efficient white noise sampling and coupling for multilevel Monte Carlo

Matteo Croci
Mathematical Institute, University of Oxford, UK, matteo.croci@maths.ox.ac.uk

Michael B. Giles
Mathematical Institute, University of Oxford, UK, mike.giles@maths.ox.ac.uk

Marie E. Rognes
Department for Numerical Analysis and Scientific Computing, Simula Research Laboratory, Norway, meg@simula.no

Patrick E. Farrell
Mathematical Institute, University of Oxford, UK, patrick.farrell@maths.ox.ac.uk

When solving stochastic partial differential equations (SPDEs) driven by additive spatial white noise the efficient sampling of white noise realizations can be challenging. In this talk we present a novel sampling technique that can be used to efficiently compute white noise samples in a finite element and multilevel Monte Carlo (MLMC) setting.

After discretization, the action of white noise on a test function yields a Gaussian vector with the FEM mass matrix as covariance. Sampling such a vector requires an expensive Cholesky factorization and for this reason P0 representations, for which the mass matrix is diagonal, are generally preferred in the literature. This however has other disadvantages. In this talk we introduce an alternative factorization that is naturally parallelizable and has linear cost and memory complexity (in the number of mesh elements).

Moreover, in a MLMC framework the white noise samples must be coupled between subsequent levels so as to respect the telescoping sum. We show how our technique can be used to enforce this coupling even in the case in which the hierarchy is non-nested via a supermesh construction.

We conclude the talk with numerical experiments that demonstrate the efficacy of our method. We observe optimal convergence rates for the finite element solution of the elliptic SPDEs of interest. In a MLMC setting, a good coupling is enforced and the telescoping sum is respected.

Multilevel Monte Carlo for the Quadrature of SDEs Based on Random Bits

Lukas Mayer
Department of Mathematics, University of Kaiserslautern, Germany, lmayer@mathematik.uni-kl.de

We study the approximation of expectations $E(f(X))$ for X being the solution of a system of SDEs, under appropriate regularity assumptions, and for Lipschitz-continuous functionals f. We consider restricted Monte Carlo algorithms which may only use random bits instead of random numbers from $[0, 1]$. This restriction corresponds to the only source of randomness accessible on state of the art hardware accelerators like FPGAs (Field Programmable Gate Arrays). We construct and analyze a multilevel Monte Carlo algorithm for which we establish an upper bound for the computational cost in terms of the error. Up to logarithmic factors we achieve the same order of convergence as in the case of random numbers.

Furthermore, we present numerical results also beyond the setting and algorithm that has been analyzed, e.g., for the Heston model.

This is joint work with Mike Giles, Mario Hefter, and Klaus Ritter.

[1] Michael B Giles, Mario Hefter, Lukas Mayer, and Klaus Ritter. Random Bit Quadrature and Approximation of Distributions on Hilbert Spaces. *Foundations of Computational Mathematics*, To appear.

Rev

Multilevel weighted least squares polynomial approximation

Soeren Wolfers
Computer, Electrical and Mathematical Sciences & Engineering (CEMSE), King Abdullah University of Science and Technology (KAUST), Saudi Arabia, soeren.wolfers@gmail.com

Weighted least squares polynomial approximation uses random samples to determine projections of functions onto spaces of polynomials. It has been shown that, using an optimal distribution of sample locations, the number of samples required to achieve quasi-optimal approximation in a given polynomial subspace scales, up to a logarithmic factor, linearly in the dimension of this space. However, in many applications, the computation of samples includes a numerical discretization error. Thus, obtaining polynomial approximations with a single level method can become prohibitively expensive, as it requires a sufficiently large number of samples, each computed with a sufficiently small discretization error. As a solution to this problem, we propose a multilevel method that utilizes samples computed with different accuracies and is able to match the accuracy of single-level approximations with reduced computational cost. We derive complexity bounds under certain assumptions about polynomial approximability and sample work. Furthermore, we propose an adaptive algorithm for situations where such assumptions cannot be verified a priori. Finally, we provide an efficient algorithm for the sampling from optimal distributions and an analysis of computationally favorable alternative distributions. Numerical experiments underscore the practical applicability of our method.

An adaptive Multilevel Monte Carlo algorithm for advection-diffusion PDEs with random discontinuous coefficients

Andreas Stein
SimTech, University of Stuttgart, Germany, andreas.stein@mathematik.uni-stuttgart.de

Advection-diffusion equations arise in various applications, for instance, in the modeling of subsurface flows. We consider advection-diffusion equations with random coefficients, which may account for insufficient measurements or uncertain material procurement. To represent, for example, transitions in heterogeneous\fractured\porous media, we include spatial discontinuities in the parameters of the equation. More precisely, we consider a second order parabolic problem where the random advection and diffusion coefficients are given by sums of (continuous) Gaussian random fields and (discontinuous) jump parts. We introduce an adaptive, pathwise finite element algorithm which accounts for the varying discontinuities in each sample of the advection resp. diffusion coefficient. Then, a time stepping scheme is chosen accordingly. By combining the adaptive finite element method with multilevel Monte Carlo sampling techniques, we are able to estimate moments of the solution to the random partial differential equation. As the adaptive spatial discretization leads to path-dependent triangulations, it is not possible to create a nested sequence of grids which is suitable for each sample path a-priori. We address this issue by an adaptive multilevel (or multifidelity) algorithm, where the discretization on each level is sample-dependent.

This is joint work with Andrea Barth (SimTech, University of Stuttgart)

Monday (10:00–12:00), Tuesday (10:00–12:00)

Construction and application of QMC point sets and sequences

Organizer(s): Josef Dick & Takashi Goda

This special symposium is devoted to constructions of good quasi-Monte Carlo (QMC) point sets and sequences. Classical uniform distribution theory has long provided a motivation for constructing low-discrepancy point sets and sequences. Recent applications of QMC, such as PDEs with random coefficients, however, have witnessed a need for constructing good quadrature rules over $\mathbb{R}^d$ or constructing quadrature rules over the unit cube which achieve a high order of convergence. So far, randomly shifted lattice rules and higher order nets (in particular, interlaced polynomial lattice rules) are mainly used for these purposes, respectively. The first session will present recent developments on various alternative constructions and their implementations. Quasi-Monte Carlo methods have found important applications for instance in finance, uncertainty quantification and discrepancy theory. The second session brings together expertise in some of these areas to discuss the newest research results on QMC and its applications.

Monday 10:00–12:00, **Room: Amphi 3**

Construction of QMC point sets and sequences

Chair: Josef Dick

Construction of QMC point sets and sequences

Discrete least squares approximation on multivariate polynomial spaces using lattice points

Frances Kuo
School of Mathematics and Statistics, UNSW Sydney, Australia, f.kuo@unsw.edu.au

Giovanni Migliorati
Université Pierre et Marie Curie, France, giovanni.migliorati@gmail.com

Fabio Nobile
EPFL, Switzerland, fabio.nobile@epfl.ch

Dirk Nuyens
NUMA, KU Leuven, Belgium, dirk.nuyens@cs.kuleuven.be

We analyze the stability and accuracy of discrete least squares on certain multivariate polynomial spaces to approximate a given function based on evaluations at one family of quasi-Monte Carlo points, namely, lattice points. We show that it is possible to construct suitable generating vectors for lattice points such that the discrete least squares approximation is both stable and accurate, with the required number of evaluations proportional to the square of the size of the polynomial space index set. This improves upon previous results by removing the exponential dependence on dimension.

Lattice Rules in Non-periodic Subspaces of Sobolev Spaces

Takashi Goda
School of Engineering, University of Tokyo, Japan, goda@frcer.t.u-tokyo.ac.jp

Kosuke Suzuki
Graduate School of Science, Hiroshima University, Japan, kosuke-suzuki@hiroshima-u.ac.jp

Takehito Yoshiki
Graduate School of Informatics, Kyoto University, Japan, yoyoy@live.jp

We investigate quasi-Monte Carlo (QMC) integration based on rank-1 lattice point sets in weighted non-periodic Sobolev spaces and their subspaces of high order smoothness. A recent paper by Dick, Nuyens and Pillichshammer [1] has studied QMC integration in half-period cosine spaces with smoothness parameter $\alpha > 1/2$ consisting of non-periodic smooth functions and also in the sum of half-period cosine spaces and Korobov spaces with common parameter α. Motivated by the results shown there, we first study embeddings and norm equivalences on those function spaces. In particular, for an integer $\alpha \geq 2$, we provide their corresponding norm-equivalent proper subspaces of a non-periodic Sobolev space of dominating mixed smoothness α, which solves an open problem in [1]. Then we study the worst-case error of symmetrized lattice rules in an intermediate space between the Sobolev space and the sum space and also that of tent-transformed lattice rules in the non-periodic Sobolev space of second order smoothness. We show that the almost optimal rate of convergence can be achieved for both cases, while a good dependence of the worst-case error bound on the dimension can be obtained for the latter case.

[1] J. Dick, D. Nuyens and F. Pillichshammer, Lattice rules for nonperiodic smooth integrands, Numer. Math. 126, 259–291, 2014.

Haar Analysis of Digital Nets and Sequences

Ralph Kritzinger
Dept. of Financial Mathematics, JKU Linz, Austria, ralph.kritzinger@jku.at

Friedrich Pillichshammer
Dept. of Financial Mathematics, JKU, Austria, friedrich.pillichshammer@jku.at

We study the discrepancy function of digital $(0, n, 2)$-nets and digital $(0, 1)$-sequences with the aid of Haar functions. The aims are two-fold:

- We show that for certain nets the Haar coefficients of the discrepancy function can be computed precisely in order to obtain exact formulas for the L_2 discrepancy of these nets. The method also works for shifted and symmetrized versions of these digital nets.
- For a larger class of digital nets and sequences, we use the Haar function system in order to find necessary and sufficient conditions on the generating matrices to obtain the optimal order of L_p discrepancy for all $p \in [1, \infty)$.

We will observe that the zeroth Haar (or Fourier) coefficient, which is the integral of the discrepancy function, is not always the one causing a large L_2 discrepancy. In some cases, other Haar coefficients destroy the optimal order of L_2 discrepancy for certain digital nets.

Lattice rules with random number of points and near $O(n^{-\alpha-1/2})$ convergence

Peter Kritzer
RICAM, Austrian Academy of Sciences, Austria, peter.kritzer@oeaw.ac.at

Frances Y. Kuo
School of Mathematics and Statistics, UNSW Sydney, Australia, f.kuo@unsw.edu.au

Dirk Nuyens
NUMA, KU Leuven, Belgium, dirk.nuyens@cs.kuleuven.be

Mario Ullrich
Institute for Analysis, JKU Linz, Austria, mario.ullrich@jku.at

The traditional way to randomize lattice rules is to use a random shift modulo 1, but such a randomization does not improve the convergence rate. We show that by first selecting uniformly a random (prime) number of points $p \sim U(n/2, n]$ and then drawing uniformly one of the "good" generating vectors for this number of points p, we obtain a randomized worst-case error bound arbitrarily close to $n^{-\alpha-1/2}$ where α is the smoothness of the usual Korobov space. Moreover, this bound can be made independent of the number of dimensions by assuming weighted spaces.

[1] Peter Kritzer, Frances Y. Kuo, Dirk Nuyens, Mario Ullrich. Lattice rules with random n achieve nearly the optimal $O(n^{-\alpha-1/2})$ error independently of the dimension. https://arxiv.org/abs/1706.04502, 2017.

Tuesday 10:00–12:00, **Room: Salle 7**

QMC and applications

Chair: Takashi Goda

On the generation of random fields

Ian Sloan
School of Mathematics, University of New South Wales, Australia, i.sloan@unsw.edu.au

The generation of Gaussian random fields is a challenging problem in computational mathematics, especially when the correlation length is short and the field is rough. The traditional approach is to make use of a truncated Karhunen-Loève (KL) expansion, but the generation of even a single realisation of the field may then be effectively beyond reach (especially for 3-dimensional domains), because of the potentially very slow convergence of the KL expansion. In this talk, based on joint work with I. Graham, F. Kuo, D. Nuyens, and R. Scheichl, a completely different approach is used, in which the field is initially generated at a regular grid on a rectangle that contains the physical domain, and then if necessary interpolated to obtain the field at other points. In this approach there is no need for any truncation, rather the problem becomes the factorisation of a large dense matrix, for which we can use circulant embedding and FFT ideas. It then becomes feasible to tackle problems for which the random field needs hundreds of thousands of scalar random variables for its description.

Kronecker–Halton sequences in $\mathbb{F}_p((X^{-1}))$

Roswitha Hofer
Institute of Financial Mathematics and Applied Number Theory, Johannes Kepler University Linz, AUSTRIA, roswitha.hofer@jku.at

In this talk we discuss the distribution properties of hybrid sequences which are made by combining Halton sequences in the ring of polynomials and digital Kronecker sequences. We give a full criterion for the uniform distribution and results on the discrepancy of such hybrid sequences.

[1] R. Hofer Kronecker–Halton sequences in $\mathbb{F}_p((X^{-1}))$. *Finite Fields and Their Applications*, 50 : 154–177, 2018.

ON HIGHER-ORDER INTEGRATION ALGORITHMS IN (WEIGHTED) HERMITE SPACES

Josef Dick
UNSW Sydney, Australia, josef.dick@unsw.edu.au

Christian Irrgeher
RICAM Linz, Austria, christian.irrgeher@oeaw.ac.at

Gunther Leobacher
KFU Graz, Austria, gunther.leobacher@uni-graz.at

Friedrich Pillichshammer
JKU Linz, Austria, friedrich.pillichshammer@jku.at

Questions of tractability of integration for classes of functions on the $\mathbb{R}^s$ are of great practical importance. (Weighted) Hermite spaces were introduced in [1] as a class of functions on the $\mathbb{R}^s$ for which questions of tractability of integration – with respect to Gaussian measure – could be answered. Two particular types of weighted Hermite spaces were studied in depth: Hermite spaces of functions of finite smoothness and Hermite spaces of analytic functions. Shortly thereafter is was shown in [2] that for certain weighted Hermite spaces of analytic functions there are integration algorithms which admit exponential convergence, and many questions about related tractability concepts could be answered. In [3] an integration algorithm was developed for Hermite spaces of finite smoothnes with essentially optimal order of convergence.

In this talk we address the problem of the optimal ε-exponent of tractability of integration in weighted Hermite spaces of finite smoothness α.

[1] C. Irrgeher and G. Leobacher. High-dimensional integration on $\mathbb{R}^d$, weighted Hermite spaces, and orthogonal transforms. *J. Complexity*, 31(2):174–205, 2015.

[2] C. Irrgeher, P. Kritzer, G. Leobacher, and F. Pillichshammer. Integration in Hermite spaces of analytic functions. *J. Complexity*, 31(3):380 – 404, 2015.

[3] J. Dick, C. Irrgeher, G. Leobacher, and F. Pillichshammer. On the optimal order of integration in Hermite spaces with finite smoothness. *SIAM J. Numer. Anal.*, 2018.

Asymptotic Behaviour of the Sudler Product of Sines for Quadratic Irrationals

Sigrid Grepstad
Department of Mathematical Sciences, Norwegian University of Science and Technology, Norway,
sigrid.grepstad@ntnu.no

Mario Neumüller
Department of Financial Mathematics and Applied Number Theory, Johannes Kepler University, Austria, mario.neumueller@jku.at

We study the growth of the following sequence $P_n(\alpha) = \prod_{r=1}^{n} |2\sin(\pi r\alpha)|$ for real, irrational numbers α. More precise, we study the case where α is a quadratic irrational i.e. the continued fraction expansion of α is of the form $\alpha = [a_0; a_1, \dots, a_h, \overline{a_{h+1}, \dots, a_{h+l}}]$.
Similar products appear in many different fields of mathematics such as partition theory, Padé approximation, dynamical systems and recently also in connection with the analysis of the discrepancy of certain hybrid sequences in [1].
In particular, we study the subsequence $(Q_n(\alpha))_{n\geq 1}$ of $(P_n(\alpha))_{n\geq 1}$ defined as

$$Q_n(\alpha) := \prod_{r=1}^{q_n} |2\sin(\pi r\alpha)|\,.$$

Here $(q_n)_{n\geq 0}$ are the best approximation denominators of α. We show that this subsequence converges to a periodic sequence with period equal to that of the continued fraction expansion of α. This verifies a conjecture posed by Mestel and Verschueren in [2].

[1] R. Hofer and F. Puchhammer. On the discrepancy of two-dimensional perturbed Halton-Kronecker sequences and lacunary trigonometric products. *Acta Arith.*, 180: 365–392, (2017).

[2] P. Verschueren and B. Mestel. Growth of the Sudler product of sines at the golden rotation number. *J. Math. Anal. Appl.*, 433: 200–226, (2016).

Monday (14:45–15:45), Tuesday (14:45–15:45, 16:15–17:45)

Points on the Sphere and Other Manifolds: New Frontiers and Recent Progress

Organizer(s): Johann Brauchart

Numerical integration rely on “good point sets", say, digital nets and lattice rules in the unit cube. An alternative strategy is to select point sets that discretize the domain of interest of the underlying manifold the function is defined on with respect to a criterion suitable to measure the quality of the cubature. Optimizing an "energy" can provide such point sets. One example is the sum of all mutual distances. Maximizing configurations have minimal spherical cap L_2 discrepancy and minimal worst-case error (WCE) of a QMC rule for functions in the unit ball of the Sobolev space over the sphere that is also a reproducing kernel Hilbert space with respect to the distance kernel; see, Brauchart and Dick [A simple proof of Stolarsky's invariance principle. Proc. Amer. Math. Soc. 141 (2013)] and subsequent work. Spherical t-designs that define cubature rules that are exact for spherical polynomials of degree $\leq t$ can be characterized using certain energy functionals as well as so-called QMC designs that, in the asymptotic sense, still have the optimal rate of convergence of the WCE for functions with a certain Sobolev space smoothness; see, Brauchart et al. [QMC designs: optimal order quasi Monte Carlo integration schemes on the sphere. Math. Comp. 83 (2014)]. Generalizations have been made w.r.t. the manifold and the metric and the function spaces.

Recently, the finer structure of the point sets attracted interest by considering "hyperuniformity" in the compact setting (https://arxiv.org/abs/1709.02613v2). Hyperuniformity in the non-compact setting was introduced by Torquato and Stillinger [Local density fluctuations, hyperuniformity, and order metrics, Phys. Rev. E 68 (2003)] (see also L. Pietronero, A. Gabrielli, and F. S. Labini [Statistical Physics for cosmic structures, Physica A 306 (2002)] to describe idealized infinite point configurations, which exhibit properties between order and disorder.

A recent breakthrough was the proof that the E_8 and Leech lattice are the densest packing of congruent spheres in dimension 8 (Viazovska, Annals 2017) and 24 (Cohn et al., Annals 2017), respectively. Both proofs make ingenious use of linear programming bounds, a method which can be applied to a wide range of problems. In this minisymposia one of the speakers is reporting on results on the energy of codes and designs in various settings.

This special session aims at bringing together young and senior researchers who work on recent topics under the umbrella of energy of point configurations, their fine structure and generalizations of results and concepts for cubature on the sphere.

Monday 14:45–15:45, **Room: Salle 5**

Points on the Sphere and Other Manifolds: New Frontiers and Recent Progress (1)

Chair: Johann Brauchart

Overview / Hyperuniformity in the Compact Setting: Deterministic and Random Aspects

Johann S. Brauchart
Institute of Analysis and Number Theory, Graz University of Technology, Austria,
j.brauchart@tugraz.at

Peter J. Grabner
Institute of Analysis and Number Theory, Graz University of Technology, Austria,
peter.grabner@tugraz.at

Wöden Kusner
Department of Mathematics, Vanderbilt University, USA, wkusner@gmail.com

Jonas Ziefle
Fachbereich Mathematik, University Tübingen, Germany, jonas.ziefle@googlemail.com

We provide an overview of the topics of the special session "Points on the Sphere and Other Manifolds: New Frontiers and Recent Progress" and present results on 'hyperuniformity' in the compact setting.

Determinantal Point Processes and Optimality

Carlos Beltrán
Departamento de Matematicas Estadística y Computación, Universidad de Cantabria, Spain,
beltranc@unican.es

Jordi Marzo
Departament de Matemàtiques i Informàtica, Universitat de Barcelona, Spain, jmarzo@ub.edu

Joaquim Ortega-Cerdà
Departament de Matemàtiques i Informàtica, Universitat de Barcelona, Spain, jortega@ub.edu

The study of minimal (Riesz) energy points is an old subject with connections ranging from error correcting codes to the mathematical models for superconductors. In this talk I will discuss the use of some determinantal point processes to obtain well distributed points in terms of discrepancy, separation and expected energy.

Tuesday 14:45–15:45, **Room: Salle 5**

Points on the Sphere and Other Manifolds: New Frontiers and Recent Progress (2)

Chair: Johann Brauchart

Bounds for the Green Energy on SO(3)

Damir Ferizovič
Institute of Analysis and Number Theory, Graz University of Technology, Austria,
damir.ferizovic@tugraz.at

Carlos Beltrán
Departamento de Matematicas Estadística y Computación, Universidad de Cantabria, Spain,
beltranc@unican.es

Given N points from a set, and a real valued function f in two variables on this set, the f-energy of these points is defined as the sum of f-values at pairs of mutually different points.

We derive a simple formula for the Green function on SO(3) and, using a determinantal point process, we obtain an upper bound for the Green energy.

t-Designs on Manifolds: an Asymptotic Bound on the Number of Points.

Ujué Etayo
Departamento de Matemáticas, Estadística y Computación, Universidad de Cantabria,
etayomu@unican.es

A t-design, also called averaging set or Chebyshev quadrature formula is a set of N points of a manifold M such that

$$\int_M P(x)d\mu(x) = \frac{1}{N}\sum_{i=1}^{N} P(x_i) \tag{9.2}$$

for all polynomials $P(x)$ of total degree less or equal than t.

In [1] Bondarenko, Radchenko and Viazovska proved that there always exist t-designs with N points for $N \geq Ct^d$ in the $d-$dimensional sphere, i.e. that we can always have t-designs with a number of points comparable to the dimension of the space of polynomials in the sphere of dimension d. In this talk I will present a generalization of this result to any compact, connected, real algebraic manifold. In order to do so, we will use some recent results on area regular partitions together with some Marcinkiewicz-Zygmund inequalities.

[1] A. Bondarenko, D. Radchenko and M. Viazovska. Optimal asymptotic bounds for spherical designs. *Annals of mathematics*, 178 (2): 443–452, 2013.

[2] J. Marzo, U. Etayo and J. Ortega-Cerdà. Asymptotically optimal designs on compact algebraic manifolds. Accepted in Monatshefte für Mathematik, 2018. arXiv:1612.06729.

[3] G. Gigante and P. Leopardi. Diameter bounded equal measure partitions of Ahlfors regular metric measure spaces. *Discrete and Computational Geometry*, 57 (2): 419–430, 2017. arXiv:1510.05236.

[4] R. Berman and J. Ortega-Cerdà. Sampling of real multivariate polynomials and pluripotential theory. To appear in Amer. Jour. Math., 2018. arXiv:1509.00956.

Tuesday 16:15–17:15, **Room: Salle 5**

Points on the Sphere and Other Manifolds: New Frontiers and Recent Progress (3)

Chair: Johann Brauchart

Universal Bounds on Energy of Codes and Designs in Various Settings

Peter G. Boyvalenkov
Institute for Mathematics and Informatics, Bulgarian Academy of Sciences, Bulgaria,
peter@math.bas.bg

Peter D. Dragnev
Department of Mathematical Sciences, Purdue University Fort Wayne, USA, DragnevP@ipfw.edu

Doug P. Hardin
Center for Constructive Approximation, Department of Mathematics, Vanderbilt University, USA,
doug.hardin@vanderbilt.edu

Edward B. Saff
Center for Constructive Approximation, Department of Mathematics, Vanderbilt University, USA,
edward.b.saff@vanderbilt.edu

M. M. Stoyanova
Faculty of Mathematics and Informatics, University of Sofia, Bulgaria, stoyanova@fmi.uni-sofia.bg

The seminal paper Cohn and Kumar (JAMS, 2007) introduced the notion of universal optimal configurations on the unit sphere, projective spaces, and in a subsequent paper of Cohn and Zhao on Hamming spaces. In particular, sharp codes (spherical designs of strength $2m-1$ with m distinct inner products) are universally optimal; i.e., they minimize the potential energy for any absolutely monotone potential interaction h. In 2016 the authors introduced a universal lower bound on energy, for which given a dimension n and the cardinality of a code $C \subset S^{n-1}$ one can find suitable positive Radau/Lobato quadrature with nodes $\{\alpha_i\}$ and weights $\{\rho_i\}$, such that the energy is bounded below by $\sum \rho_i h(\alpha_i)$ for any such h. We shall consider the subsequent generalizations to various settings of this work.

Estimates for numerical integration errors on unit spheres of arbitrary dimension

Peter Grabner
Institute of Analysis and Number Theory, Graz University of Technology, Austria,
peter.grabner@tugraz.at

Tetiana Stepaniuk
Institute of Analysis and Number Theory, Graz University of Technology, Austria,
t.stepaniuk@tugraz.at

We consider the spaces of continuous functions $H^{(\frac{d}{2},\gamma)}(S^d)$, $\gamma > \dfrac{1}{2}$, on the unit sphere $S^d \subset R^{d+1}$, $d \geq 2$, which are boundary case as $s \to \frac{d}{2}$ for the classical Sobolev spaces $H^s(S^d)$ ($s > \frac{d}{2}$) and contain an extra logarithmic weight.

We study the worst case error of numerical integration on the unit sphere S^d, $d \geq 2$, for these classes of functions.

The worst-case (cubature) error of the cubature rule $Q[X_N, \omega]$ in a Banach space B of continuous functions on S^d with norm $\|\cdot\|_B$ is defined by

$$\mathrm{wce}(\mathrm{Q}[\mathrm{X_N},\omega];\mathrm{B}) := \sup_{\mathrm{f}\in\mathrm{B},\|\mathrm{f}\|_\mathrm{B}\leq 1} |\mathrm{Q}[\mathrm{X_N},\omega](\mathrm{f}) - \mathrm{I}(\mathrm{f})|,$$

where

$$\mathrm{I}(\mathrm{f}) := \int_{\mathrm{S^d}} \mathrm{f}(\mathbf{x})\mathrm{d}\sigma_\mathrm{d}(\mathbf{x}), \quad \mathrm{Q}[\mathrm{X_N},\omega](\mathrm{f}) := \sum_{\mathrm{i}=1}^{\mathrm{N}} \omega_\mathrm{i}\mathrm{f}(\mathbf{x}_\mathrm{i}), \qquad \sum_{\mathrm{j}=1}^{\mathrm{N}} \omega_\mathrm{j} = 1.$$

We prove that

$$C_{d,\gamma} N^{-\frac{1}{2}} (\ln N)^{-\gamma} \leq \mathrm{wce}(Q[X_N,\omega]; H^{(\frac{d}{2},\gamma)}(S^d))$$

for all quadrature rules $Q[X_N, \omega]$ and provide examples of quadrature rules which satisfy

$$C^{(1)}_{d,\gamma} N^{-\frac{1}{2}} (\ln N)^{-\gamma+\frac{1}{2}} \leq \mathrm{wce}(Q[X_N,\omega]; H^{(\frac{d}{2},\gamma)}(S^d)) \leq C^{(2)}_{d,\gamma} N^{-\frac{1}{2}} (\ln N)^{-\gamma+\frac{1}{2}},$$

where the positive constants $C_{d,\gamma}$, $C^{(1)}_{d,\gamma}$ and $C^{(2)}_{d,\gamma}$ depend only on d and γ, but are independent of the rule $Q[X_N]$ and the number of nodes N of the rule.

Tuesday (16:15–17:45), Thursday (14:45–15:45)

New applications of QMC in physics, energy and environment

Organizer(s): Frances Kuo & Karl Jansen

High dimensional integrals appear in many modern applications across physics, energy and environment. The gauge theories in high energy particle physics are evaluated by very high dimensional integrals over group elements from conjugacy classes. Optimization models for the operation of power systems often contain several uncertainties and require the evaluation of expectations or risk functionals represented by very high-dimensional integrals. Spatial and temporal estimation in air pollution monitoring and control involve modeling uncertainties that also require evaluation of very high dimensional integrals. This session will be devoted to these and other new and challenging applications of QMC methods.

Tuesday 16:15–17:45, **Room: Salle 8**

New applications of QMC in physics, energy and environment (1)

Chair: Frances Kuo

A DATA FUSION APPROACH FOR SPATIO-TEMPORAL PM2.5 ESTIMATION WITH QMC METHOD

Chieh-Han Lee
Department of Bioenvironmental Systems Engineering, National Taiwan University, Taiwan, d03622009@ntu.edu.tw

Dirk Nuyens
Department of Computer Science, KU Leuven, Belgium, dirk.nuyens@cs.kuleuven.be

Hwa-Lung Yu
Department of Bioenvironmental Systems Engineering, National Taiwan University, Taiwan, hlyu@ntu.edu.tw

Frances Y. Kuo
School of Mathematics and Statistics, University of New South Wales, Australia, f.kuo@unsw.edu.au

Air pollution is now the largest single environmental health risk in the world. Most exposure assessment of air pollution are based on ambien air quality monitoring system. The spatial and temporal resolution of current monitoring system are very poor. Recently, the rapid development and increase of low-cost sensors provide an unprecedented new way in understanding the space-time distribution of air quality. However, the low quality of low-cost sensors give no assurance to the measurements. There is an emergent need in investigating the potentials of the using these highly uncertain data. The present study proposed a framework in spatio-temporal environmental estimation with the use of uncertain data. The framework includes both data calibration for the uncertainty assessment and data fusion for the estimation. In some cases, the probabilistic distribution of data uncertainty can be complex and non-Gaussian. QMC method is applied with a great potential to incorporate into the data fusion framework. Hence, the high-dimensional data uncertainties can be evaluated efficiently. With the results, it is helpful for public to undstand and assess exposures of neighorhood.

Randomized QMC methods for two-stage stochastic optimization problems: Recent progress

Hernan Leövey
Axpo, Baden, Switzerland, hleovey@yahoo.com

Werner Römisch
Department Mathematics, Humboldt-University Berlin, Germany, romisch@math.hu-berlin.de

We extend our earlier work on applying randomized QMC methods for solving two-stage stochastic optimization models. Although the integrands are nondifferentiable or even discontinuous we show that lower order ANOVA terms of the integrands can be smooth if certain geometric conditions are satisfied and the marginal densities are sufficiently smooth. In case of a normal density we show that the geometric conditions are satisfied almost surely and that the effective dimension can be reduced by applying principal component factorization to the covariance matrix. The theory is illustrated by presenting numerical results for randomly shifted lattice rules and randomly scrambled Sobol' sequences to solve a mixed 0-1 electricity portfolio model with normal prices and demands.

Neutron Source Localization by Analyzing the Detector responses and Markov Chain Monte Carlo (MCMC) Method

Naser Vosoughi
Department of Energy Engineering , Sharif University of technology, Iran, nvosoughi@sharif.edu

Fatemeh Mechershavi
Department of Energy Engineering , Sharif University of technology, Iran,
fatemehmatureshavi@rocketmail.com

Localization of a point neutron source using a developed computer program namely MCMC_MATURE is performed. The computer program analyses several detector responses in some certain media by Markov Chain Monte Carlo (MCMC) method and an iteration algorithm. Identification of the possible regions of source position would be found by analyzing the initial fluxes generated by mesh tally of Monte Carlo N-Particle computer code. The MCMC_MATURE program is capable to estimate the flux between detectors. Regular-Sequential, Irregular-Sequential and Non-Sequential are three used methods for sampling the generated random number in two dimensions. Each sample is multiplied by a sampling function and leads to a random source position to make a location chain. The main assumed functions are uniform, sinusoidal, exponential and proportional which make the sampling function in three structures namely simple, symmetric, and asymmetric.The acceptance or rejection of generated random locations is based on the Metropolis-Hasting algorithm. The Metropolis-Hasting ratio will check the generated random positions by analyzing their importance. This ratio ensures that the chain tends to the main position of source whenever its length increases. In iteration algorithm, the medium was meshed and each chain would increase a count in the proportional mesh. The number of counting which is saved in a matrix namely probability counting matrix would be normalized after investigation of a group chain. Investigating the next group chain will generate the other probability counting matrix. Generating the probability counting would be stopped whenever the difference between the probability counting matrix and its previous one lead to a minimum quantity. This minimum quantity is determined for localization with notice to the maximum likelihood error and power analysis of the computer. The reasonable value of the minimum quantity in one and two dimensions is derived as 0.05 and 0.02 respectively. The sampling methods are investigated in two-dimensional uniform and Non-uniform media including water and wood. The best sampling method is obtained by Non-sequential sampling method. Amongst the sampler function, the exponential function with symmetric structure offers the best accuracy. Sensitivity analysis of mentioned algorithm is investigated for several factors namely number of meshes, number of group chain and error of detector. The results show that the increase in number of meshes or number of group

chain and decrease in maximum error of detector lead to less precision error and make the results more accurate.

Thursday 14:45–15:45, **Room: Salle 8**

New applications of QMC in physics, energy and environment (2)

Chair: Karl Jansen

Lattice Field Theory: a physics case for high dimensional integration

Karl Jansen
NIC, DESY, Platanenallee 6, 15738 Zeuthen, Germany, karl.jansen@desy.de

We give an introduction to lattice field theories and the underlying principle of gauge invariance. It is demonstrated that such systems lead to very high dimensional integrations such that new methods are very much in need. This holds in particular for problems which are described by complex integrands and where standard Markov chain Monte Carlo methods fail. These include research directions such as the matter anti-matter asymmetry in the universe which led to our sheer existence. At the examples of systems in zero, one and two dimensions we show how QMC, iterative numerical or symmetrized integration have the potential to overcome the problems of high dimensionality and complex integrands.

Improving Monte Carlo integration by symmetrization

Julia Volmer
NIC, DESY, Platanenallee 6, 15738 Zeuthen, Germany, julia.volmer@desy.de

The error scaling for Markov Chain-Monte Carlo (MCMC) techniques with N samples behaves like $1/\sqrt{N}$. This scaling makes it often very time intensive to reduce the error of calculated observables, in particular for applications in 4-dimensional lattice quantum chromodynamics as our theory of the interaction between quarks and gluons. Even more, for certain cases, where the infamous sign problem appears, MCMC methods fail to provide results with a reliable error estimate. It is therefore highly desirable to have alternative methods at hand which show an improved error scaling and have the potential to overcome the sign problem. One candidate for such an alternative integration technique we used is based on a new class of polynomially exact integration rules on U(N) and SU(N) which are derived from polynomially exact rules on spheres. We applied these rules successfully to a non-trivial, zero-dimensional model with a sign problem and obtained arbitrary precision results. We also tested a possible way to apply the integration rules for spheres for the case of a one-dimensional U(1) model, the topological rotor, which already leads to a problem of very high dimensionality. In the talk we introduce the method and show results.

Wednesday (10:00–12:00, 13:45–15:45), Thursday (10:00–12:00)

Stochastic Computation and Complexity

Organizer(s): Stefan Heinrich, Thomas Mueller-Gronbach & Raphael Kruse

The minisymposium is devoted to algorithms and complexity for

- quadrature of SDEs
- approximation of stochastic processes
- high and infinite dimensional integration and approximation

including, in particular, aspects of

- lower bounds
- connections to functional analysis.

Wednesday 10:00–12:00, **Room: Amphi 3**

Stochastic Computation and Complexity (1)

Chair: Stefan Heinrich

On the error rate of the Euler for SDEs with piecewise Lipschitz drift coefficient

Thomas Müller-Gronbach
University of Passau, Germany, thomas.mueller-gronbach@uni-passau.de

Larisa Yaroslavtseva
University of Passau, Germany; ETH Zürich, Switzerland, larisa.yaroslavtseva@uni-passau.de

We study strong approximation of the solution of a stochastic differential equation (SDE) at the final time based on evaluations of the driving Brownian motion at finitely many fixed times. For this problem, polynomial error rate results are typically achieved under the assumption that the first order derivatives of the coefficients of the equation satisfy a polynomial growth condition. We show that this condition is far from being sufficient for polynomial error rates, whatever numerical scheme is used.

Algorithms for the Approximation of Iterated Stochastic Integrals in Infinite Dimensions

Claudine Leonhard
Department of Computer Science, Christian-Albrechts-Universität zu Kiel, Germany, cleo@informatik.uni-kiel.de

Andreas Rößler
Institute of Mathematics, Universität zu Lübeck, Germany, roessler@math.uni-luebeck.de

We consider the problem of the approximation of two-times iterated stochastic integrals with respect to a Q-Wiener process in the infinite dimensional setting. Therefore, we present two algorithms that are based on a Fourier series expansion of the Q-Wiener process in case of a trace class operator Q given the increments of the Q-Wiener process. In contrast to the finite dimensional setting, which is contained as a special case, the error estimates given in the L^2-norm depend on the covariance operator Q. As a result of this, the optimal choice for the approximation algorithm is not uniquely determined as it depends on the eigenvalues of Q. The presented approximation algorithms open the door for the implementation of higher order numerical schemes for SPDEs that do not possess commutative noise.

Deep optimal stopping: Solving high-dimensional optimal stopping problems with deep learning

Sebastian Becker
Inst. of Mathematics, Frankfurt University, Germany, becker@math.uni-frankfurt.de

Patrick Cheridito
D-MATH, ETH Zürich, Switzerland, patrick.cheridito@math.ethz.ch

Arnulf Jentzen
D-MATH, ETH Zürich, Switzerland, arnulf.jentzen@sam.math.ethz.ch

Timo Welti
D-MATH, ETH Zürich, Switzerland, timo.welti@sam.math.ethz.ch

Optimal stopping problems suffer from the curse of dimensionality. This talk concerns a new deep learning method for solving high-dimensional optimal stopping problems. In particular, the introduced algorithm can be employed for the pricing of American options with a very large number of underlyings. The presented algorithm is in fact applicable to a very broad class of financial derivatives (such as path-dependent American, Bermudan, and Asian options) as well as multifactor underlying models (such as Heston-type stochastic volatility models, local volatility models, and jump-diffusion models). Moreover, the algorithm allows to approximatively compute both the price and an optimal exercise strategy for the American option claim. Numerical results on benchmark problems are presented which suggest that the proposed algorithm is highly effective in the case of many underlyings, in terms of both accuracy and speed.

A Randomized Time-Stepping Method for Differential Equations with Time-Irregular Coefficients

Monika Eisenmann
TU Berlin, Germany, m.eisenmann@tu-berlin.de

Mihály Kovács
Chalmers University of Technology, Sweden, mihaly@chalmers.se

Raphael Kruse
TU Berlin, Germany, kruse@math.tu-berlin.de

Stig Larsson
Chalmers University of Technology, Sweden, stig@chalmers.se

To obtain a rate of convergence for the quadrature of time-irregular functions, the Monte–Carlo integration technique has proven itself to be a very powerful tool. It is a well known fact that for a function which is merely L^2-integrable, one can prove that the rate of convergence is still 0.5. Deterministic quadrature rules, as for example the mid point rule, on the other hand are known to fail for non-smooth data.

We extend this concept to the numerical approximation of differential equations with non-smooth temporal data. Under these weak regularity assumptions, ordinary one step methods, as the backward Euler scheme, can diverge if simple point evaluations of the data are used. A new approach to avoid this problem is to dismiss point evaluations of the data at the predestined grid points and instead use evaluations at random points in between. Similar to the Monte–Carlo algorithm for the approximation of integrals, it is then possible to prove that this modification converges in a mean-square sense with respect to the underlying probability space.

This concept can be applied to a wide range of problems, containing for example possibly nonlinear infinite-dimensional evolution equations. The practicability of the randomized scheme is also illustrated through several numerical experiments.

Wednesday 13:45–15:45, **Room: Amphi 3**

Stochastic Computation and Complexity (2)

Chair: Thomas Mueller-Gronbach

Central limit theorems for multilevel stochastic approximation algorithms

Steffen Dereich
Institute for Mathematical Stochastics, Universität Münster, Germany, steffen.dereich@wwu.de

Thomas Müller-Gronbach
Faculty of Computer Science and Mathematics, Universität Passau, Germany,
thomas.mueller-gronbach@uni-passau.de

In this talk we present new central limit theorems for multilevel adaptations of stochastic approximation algorithms for the computation of a zero of a function $f : \mathbb{R}^d \to \mathbb{R}^d$, which is given as a parameterised family of expectations. We assume that approximate solutions satisfy certain bias and variance assumptions and carry out an optimisation over the parameters. Additionally, we essentially only require that f satisfies a classical contraction property from stochastic approximation theory.

On two quadrature rules for stochastic integrals with fractional Sobolev regularity

Monika Eisenmann
TU Berlin, Germany, m.eisenmann@tu-berlin.de

Raphael Kruse
TU Berlin, Germany, kruse@math.tu-berlin.de

In this talk we discuss the numerical quadrature of a stochastic integral, where the temporal regularity of the integrand is measured in the fractional Sobolev-Slobodeckij norm in $W^{\sigma,p}(0,T)$, $\sigma \in (0,2)$, $p \in [2,\infty)$. We introduce two quadrature rules: The first is best suited for the parameter range $\sigma \in (0,1)$ and consists of a Riemann-Maruyama approximation on a randomly shifted grid. The second quadrature rule applies to the case of a deterministic integrand of fractional Sobolev regularity with $\sigma \in (1,2)$. In

both cases the order of convergence is equal to σ with respect to the norm in $L^p(\Omega)$. As one application, we consider the stochastic integration of a Poisson process, which has discontinuous sample paths. The theoretical results are accompanied by numerical experiments.

Stochastic integration in quasi-Banach spaces: what Besov regularity does the stochastic heat equation posess?

Petru Cioica
Mathematics, University of Otago, New Zealand, pcioica@maths.otago.ac.nz

Sonja Cox
KdVI, University of Amsterdam, the Netherlands, s.g.cox@uva.nl

Mark Veraar
DIAM/EWI, TU Delft, the Netherlands, M.C.Veraar@tudelft.nl

I will explain how to set up a stochastic integration theory for quasi-Banach spaces. The motivation is to determine the spatial Besov regularity of solutions to stochastic partial differential equations, with Besov parameters in the range of quasi-Banach spaces. This is a relevant class of spaces when implementing adaptive approximation schemes.

Weak and strong approximation of fractional order elliptic equations with spatial white noise

David Bolin
Department of Mathematical Sciences, Chalmers University of Technology and University of Gothenburg, Sweden, david.bolin@chalmers.se

Kristin Kirchner
Department of Mathematical Sciences, Chalmers University of Technology and University of Gothenburg, Sweden, kristin.kirchner@chalmers.se

Mihály Kovács
Department of Mathematical Sciences, Chalmers University of Technology and University of Gothenburg, Sweden, mihaly@chalmers.se

The numerical approximation of solutions to stochastic partial differential equations with additive spatial white noise on a bounded domain in $\mathbf{R}^d$ is considered. The differential operator is given by the fractional power L^β, $\beta \in (0,1)$, of an integer order elliptic differential operator L and is therefore non-local. The inverse operator $L^{-\beta}$ is represented by a Bochner integral from the Dunford–Taylor functional calculus. By applying a quadrature formula to this integral representation, the inverse fractional power operator $L^{-\beta}$ is approximated by a weighted sum of non-fractional resolvents $(I + t_j^2 L)^{-1}$ at certain quadrature nodes $t_j > 0$. The resolvents are then discretized in space by a standard finite element method. This approach is combined with an approximation of the white noise based on the mass matrix of the finite element discretization. In this way an efficient numerical algorithm for computing samples of the approximate solution is obtained. For the resulting approximation, both the strong mean-squared error as well as the so-called weak error is analyzed and an explicit rate of convergence is derived in both cases. Numerical experiments for $L = \kappa^2 - \Delta$, $\kappa > 0$, on the unit cube $(0,1)^d$ in $d = 1, 2, 3$ spatial dimensions with different values of $\beta \in (0,1)$ attest the theoretical results. The method is particularly interesting for real-world applications in spatial statistics, where fractional order stochastic partial differential equations with spatial white noise play an important role owing to their relation to Gaussian Matérn fields.

Thursday 10:00–12:00, **Room: Salle 5**

Stochastic Computation and Complexity (3)

Chair: Raphael Kruse

Multilevel Monte Carlo for McKean-Vlasov SDEs

Alvin Tse
School of Mathematics, The University of Edinburgh, UK, A.T.H.Tse@sms.ed.ac.uk

Lukasz Szpruch
School of Mathematics, The University of Edinburgh, UK, L.Szpruch@ed.ac.uk

Shuren Tan
School of Mathematics, The University of Edinburgh, UK, Shuren.Tan@ed.ac.uk

Stochastic Interacting Particle System (SIPS) and their limiting stochastic McKean-Vlasov equations offer a very rich and versatile modelling framework. On the one hand, interactions allow us to capture complex dependent structure. On the other hand, they provide a great challenge for Monte Carlo simulations. The non-linear dependence of the approximation bias on the statistical error renders classical variance reduction techniques not applicable. In this talk, we will propose a strategy to overcome this difficulty. In particular, we will establish a Multilevel Monte Carlo estimator for SIPS and demonstrate its computational superiority over standard Monte Carlo techniques.

Solving parabolic rough partial differential equations using regression

Christian Bayer
Weierstrass Institute Berlin, Germany, christian.bayer@wias-berlin.de

Denis Belomestny
University of Duisburg-Essen, Germany, denis.belomestny@uni-due.de

Martin Redmann
Weierstrass Institute Berlin, Germany, redmann@wias-berlin.de

Sebastian Riedel
TU Berlin, Germany, riedel@math.tu-berlin.de

John Schoenmakers
Weierstrass Institute Berlin, Germany, schoenma@wias-berlin.de

In this talk, we discuss the numerical approximation of parabolic partial differential equations (PDEs) driven by a rough path which, e.g., can be the lift of a path of a fractional Brownian motion. By using the Feynman-Kac formula, the solution can be represented as the expected value of a functional of the corresponding hybrid Stratonovich-rough differential equation. A time-discretisation of this equation and a Monte Carlo regression in the spatial variable lead to an approximation of the solution to the rough PDE. We analyse the regression error and provide several numerical experiments to illustrate the performance of our method.

Randomized Numerical Schemes for SDE/SPDEs

Raphael Kruse
Institut für Mathematik, Technische Universität Berlin, Germany, kruse@math.tu-berlin.de

Yue Wu
School of Engineering, University of Edinburgh, Germany, yuewu57@gmail.com

It is demanding to approximate numerical solutions of non-autonomous SDEs where the standard smoothness and growth requirements of standard Milstein-type methods are not fulfilled. In the case of a non-differentiable drift coefficient function f, we, in [1], proposed a *drift-randomized Milstein method* to achieve a higher order approximation and discussed the *optimality* of our convergence rates.

We also investigated the numerical solution of non-autonomous semilinear stochastic evolution equations (SEEs) driven by an additive Wiener noise. Usually quite restrictive smoothness requirements are imposed in order to achieve high order of convergence rate. For instance, some general assumption asks for the semilinearity to be infinitely often Fréchet differentiable with bounded derivatives. This condition already excludes any truly nonlinear Nemytskii-type operator. Thus a novel numerical method for the approximation of the solution to the semilinear SEE that combines the drift-randomization technique from [1] with a Galerkin finite element method from [3] is introduced in [2] to address this. It turns out that the resulting method converges with a higher rate with respect to the temporal discretization parameter without requiring any differentiability of the nonlinearity. Our approach also relaxes the smoothness requirements of the coefficients with respect to the time variable considerably.

[1] R. Kruse and Y. Wu. A randomized Milstein method for stochastic differential equations with non-differentiable drift coefficients, *arXiv preprint*, arXiv:1706.09964, 2017.

[2] R. Kruse and Y. Wu. A randomized and fully discrete Galerkin finite element method for semilinear stochastic evolution equations, *arXiv preprint*, arXiv:1801.08531, 2018.

[3] V. Thomée. *Galerkin Finite Element Methods for Parabolic Problems*. Springer-Verlag, Berlin, 2006.

Lower bounds for stochastic integration in fractional Sobolev classes

Stefan Heinrich
University of Kaiserslautern, Germany, heinrich@informatik.uni-kl.de

We study the complexity of stochastic integration of functions from fractional Sobolev spaces. Algorithms and error estimates were obtained by Eisenmann and Kruse [1]. Here we prove matching lower bounds and determine this way the complexity of these problems. Our methods are extensions of those from [2].

[1] Monika Eisenmann, Raphael Kruse. Two quadrature rules for stochastic Itô-integrals with fractional Sobolev regularity. arXiv:1712.08152

[2] Stefan Heinrich. Lower Complexity Bounds for Parametric Stochastic Itô Integration. *Monte Carlo and Quasi-Monte Carlo Methods 2016, Proceedings*, to appear.

Thursday (10:00–12:00, 14:45-15:45)

Low discrepancy sequences and point sets–devoted to the 80th birthday of Henri Faure

Organizer(s): Friedrich Pillichshammer & Wolfgang Ch. Schmid

Quasi-Monte Carlo rules are based on nodes with excellent uniform distribution properties. A common measure for the quality of the distribution of a point set or a sequence is its discrepancy. This special session is devoted to the construction and analysis of point sets and sequences with low discrepancy such as the Halton sequence, (polynomial) lattice point sets or (t, s)-sequences and others. The session is organized in honor of Henri Faure who will celebrate his 80th birthday on July 12, 2018 and who is well-known for his pioneer work on "low discrepancy sequences".

Thursday 10:00–12:00, **Room: Amphi 3**

Low discrepancy sequences and point sets - devoted to the 80th birthday of Henri Faure (1)

Chair: Wolfgang Ch. Schmid

Counting Points in Boxes with Henri Faure: From Discrepancy Bounds to Dependence Structures

Christiane Lemieux
Department of Statistics and Actuarial Science, University of Waterloo, Canada,
clemieux@uwaterloo.ca

In this presentation, we will first review some of the results we obtained in collaboration with Henri Faure over the last 10–15 years. In particular, we will go over some of the tools that we have used to derive new discrepancy bounds for (t, s)-sequences. Many of these tools make use of combinatorial arguments and tricks involving digital expansions. We will then discuss new results assessing the dependence structure of scrambled $(0, m, s)$-nets. These point sets can be obtained from the so-called $(0, s)$-sequences introduced by Henri Faure over 35 years ago. These new results provide a way to assess the advantage of these constructions over Monte Carlo methods for multidimensional integration by considering the dependence structure of the point set rather than its discrepancy. However, digital expansions and careful counting arguments are also required to perform this type of analysis, and as such this provides a nice connection with the work discussed in the first part of the talk.

Discrepancy Bounds for Nets and Sequences

Peter Kritzer
Johann Radon Institute for Computational and Applied Mathematics (RICAM), Austrian Academy of Sciences, Austria, peter.kritzer@oeaw.ac.at

The concepts of (t, m, s)-nets and (t, s)-sequences are among the most essential in the theory of quasi-Monte Carlo methods. Both from a practical and a theoretical point of view, it is of great interest to study the distribution properties of such point sets, which is frequently done by considering various forms of discrepancy. In this talk, we give a short review of discrepancy bounds for (t, m, s)-nets and

(t, s)-sequences; any such account would be incomplete without mentioning Henri Faure's numerous contributions to this subject. Hence, we shall highlight some of his most important results on discrepancy bounds for nets and sequences, in addition to summarizing findings from the last years. In particular, we discuss discrepancy bounds that have been derived in joint work with Henri Faure.

Quasi-Monte Carlo integration over a triangle

Takashi Goda
School of Engineering, University of Tokyo, Japan, goda@frcer.t.u-tokyo.ac.jp

Kosuke Suzuki
Graduate School of Science, Hiroshima University, Japan, kosuke-suzuki@hiroshima-u.ac.jp

Takehito Yoshiki
Graduate School of Informatics, Kyoto University, Japan, yoyoy@live.jp

In this talk we discuss quasi-Monte Carlo integration of twice differentiable functions defined over a triangle. We provide an explicit construction of infinite sequences of points, which achieves the worst-case integration error of order $N^{-1}(\log N)^3$ uniformly for all $N \geq 2$. This rate of convergence is best possible apart from the $\log N$ factor for linear quadrature rules. The key ingredient of our approach is the dyadic Walsh analysis of such smooth functions over a triangle under a suitable recursive partitioning.

[1] T. Goda, K. Suzuki, T. Yoshiki. Quasi-Monte Carlo integration for twice differentiable functions over a triangle. *J. Math. Anal. Appl.*, 454: 361–384, 2017.

On the intriguing search for good permutations

Florian Pausinger
School of Mathematics & Physics, Queen's University Belfast, United Kingdom,
f.pausinger@qub.ac.uk

The intriguing search for permutations that generate generalised van der Corput sequences with exceptionally small discrepancy forms an important part of the research work of Henri Faure. The aim of this talk is to survey (some of) Henri's contributions over the last four decades which considerably improved our understanding of one-dimensional van der Corput sequences and inspired a lot of related work. We recall and compare the different approaches in the search for van der Corput sequences with low discrepancy, i.e., using a single generating permutation versus using a sequence of permutations. Throughout the talk we collect old and new open questions which all stem from the extensive work of Henri and his coworkers and which will hopefully inspire more work in the future.

Thursday 14:45-15:45, **Room: Amphi 3**

Low discrepancy sequences and point sets–devoted to the 80th birthday of Henri Faure (2)

Chair: Friedrich Pillichshammer

On discrepancy and pair correlation of sequences in the unit interval

Gerhard Larcher
Institute of Financial Mathematics and Applied Number Theory, University Linz, Austria,
gerhard.larcher@jku.at

Henri Faure has provided ground-breaking work on construction of sequences with low discrepancy. He holds the records for sequences with respect to various discrepancy measures.
In this talk we give some recent results on lower bounds for discrepancies of sequences in the unit interval, and we compare these results with the known upper bounds (many of them provided by Faure).
Further we give some remarks and recent results on pair correlation of sequences and the connection to discrepancy.

Richardson Extrapolation of Polynomial Lattice Rules

Josef Dick
School of Mathematics and Statistics, The University of New South Wales, Australia,
josef.dick@unsw.edu.au

We study multivariate numerical integration of smooth functions in weighted Sobolev spaces with dominating mixed smoothness $\alpha \geq 2$ defined over the unit cube. We propose a new quasi-Monte Carlo based quadrature rule, named extrapolated polynomial lattice rule, which achieves the almost optimal rate of convergence of the worst-case integration error in these spaces. Extrapolated polynomial lattice rules are constructed in two steps: i) construct classical polynomial lattice rules over $\mathbb{F}_b$ with α consecutive sizes of nodes, $N = b^{m-\alpha+1}, b^{m-\alpha+2}, \ldots, b^m$, and ii) recursive application of Richardson extrapolation to a chain of α approximate values of the integral obtained by consecutive polynomial lattice rules. We discuss the component-by-component construction of the underlying rules, convergence rates, the dependence on the dimension and the fast QMC matrix-vector multiplication.

This is joint work with Takashi Goda and Takehito Yoshiki.

Thursday (10:00–12:00, 14:45-15:45)

Non-uniform Random Variate Generation

Organizer(s): Josef Leydold

Thursday 10:00–12:00, **Room: Salle 6**

Non-uniform Random Variate Generation (1)

Chair: Josef Leydold

Optimal Importance Sampling Density 1: Approximation Methods

Wolfgang Hörmann
Department of Industrial Engineering, Boğaziçi University İstanbul, Turkey, hormannw@boun.edu.tr

Josef Leydold
Institute for Statistics and Mathematics, WU Wien, Austria, josef.leydold@wu.ac.at

Approximating a density function is an important task for designing random variate generation algorithms. There exists quite a lot of literature on the constructing of automatic acceptance-rejection algorithms for large classes of (e.g. log-concave or T-concave) distributions. For these methods the approximation is constructed such that the area below an upper bound of the density (the so called *hat function*) is minimized.

Another widely used example is the approximation of the inverse CDF of the distribution for random variate generation based on the inversion method. Here a very precise approximation is required as the minimization of the maximal distance of the approximate CDF from the true CDF is crucial for the accuracy of the inversion method.

The task of approximation of sampling from the optimal importance sampling density has, however, attracted rather little attention. In this case it is important to evaluate the area below the approximating density. Different to the situation for the rejection method the approximating function need not majorize the target density. In certain applications of importance sampling it is also important that the approximation is continuously differentiable.

In this talk we present new approximation methods that try to automatically minimize the variance of one-dimensional importance sampling algorithms.

Optimal Importance Sampling Density 2: Evaluating CDF and PDF of the Sum of Lognormals

Wolfgang Hörmann
Department of Industrial Engineering, Boğaziçi University İstanbul, Turkey, hormannw@boun.edu.tr

Kemal Dinçer Dingeç
Altınbaş University, İstanbul, Turkey, kemal.dingec@altinbas.edu.tr

Josef Leydold
Institute for Statistics and Mathematics, WU Wien, Austria, josef.leydold@wu.ac.at

Evaluating CDF and PDF of the sum of lognormal random variates by Monte Carlo simulation is a topic discussed in several recent papers. Our experiments show, that in particular for variances smaller than one, *conditional Monte Carlo* (CMC) in a well chosen main direction leads already to a quite simple algorithm with large variance reduction.

For the general case the implementation of the CMC algorithm requires numeric root finding which can be implemented robustly using upper and lower bounds for the root. Adding *importance sampling* (IS) to the CMC algorithm can lead to large additional variance reduction. For the special case of IID lognormal random variates the root is obtained in closed form. It is important that for this case the optimal importance sampling density is very close to the product of its conditional densities. So the product of the approximate one-dimensional conditional densities is used as IS density.

Applying the different approximation methods discussed in Part 1 of this talk, it is possible to obtain a significant additional variance reduction over the pure CMC algorithm by means of importance sampling. When also the density of the lognormal sum is required, it is important that an approximating function with continuous first derivative is available.

In this talk the variance reduction factors obtained with the three different approximation methods and the necessary setup times for the random variate generation algorithm are compared. Also the influence of the selected main direction is analyzed.

Parsimonious Adaptive Rejection Sampling Schemes

Luca Martino
Signal Theory and Communications Department, Universidad Carlos III de Madrid, Spain,
lmartino@ing.uc3m.es

The adaptive rejection sampling (ARS) algorithms are well-known Monte Carlo techniques which draw efficiently independent samples from univariate target densities. The ARS schemes yield a sequence of proposal functions that converge toward the target, so that the probability of accepting a sample approaches one. However, sampling from the proposal pdf becomes more computationally demanding each time it is updated. We present different Parsimonious Adaptive Rejection Sampling (PARS) schemes, where an efficient trade-off between acceptance rate and proposal complexity is obtained. The resulting algorithms are faster than the standard ARS approach.

Acceptance Tail Sampling Method

Efraim Shmerling
Department of Mathematics, Ariel University, Israel, efraimsh@Yahoo.com; efraimsh@ariel.ac.il

A new simple and fast universal method called acceptance tail (AT) which can be utilized for sampling from continuous distributions with unbounded support (univariate and low-dimensional multivariate) and from discrete distributions with infinite support is introduced. For many distributions which are important in theory and practice random number generators (RNGs) implementing the AT method proved to perform significantly better than the currently available RNGs in terms of speed. The continuous version of the AT method is presented below (the discrete version is very similar and is based on the same idea).

The setup stage of the AT method looks as follows. First we divide the density (region under the graph of the probability density function) of the "target" random variable X into two unequal parts: a bounded main part ("bulk" or "head") of the density, the area (volume) of which is very close to one, and an unbounded remaining part called "tail", then define a set of m units of the same form (CUs) which cover the "head" of the density with common area (volume) $1/n$, $m = n$ or $m < n$, m/n very close to one, which enables us to initialize the tables required by the AT method. The CUs need to have the

form such that the selection of a random point from a CU is simple and convenient which is the case, for instance, for rectangular (hyperrectangular) CUs.

The generation stage consists of 4 steps (there are cases where some of the steps or parts of them can be omitted).

1. Select randomly a positive integer less or equal to n.
2. If the selected integer is less or equal to m, generate a random point $P(P_1; P_2)$ in the CU indexed by i; otherwise go to step 4.
3. If the generated point P belongs to the density "head", set $X = P_1$.
4. Generate a random point $T(T_1; T_2)$ from the "tail" and set $X = T_1$.

Note that the AT method is not a rejection method, though some specific versions of the AT method are similar to certain specific rejection methods, for instance, the ziggurat method.

Thursday 14:45-15:45, **Room: Salle 6**

Non-uniform Random Variate Generation (2)

Chair: Josef Leydold

A Table Method for Sampling from Multivariate Distrbutions with Unbounded Support

Moran Peri
Department of Mathematics, Ariel University, Israel, moran.jehasi@gmail.com

A table method for sampling from low-dimensional multivariate distributions with unbounded support are introduced. It is similar to the grid method proposed by Devroye (see [1], pp 278-282) for sampling from distributions with bounded support. It is based on covering the region under the density $f(x)$ of a random vector X of order d by n congruent hyperrectangles most of which lie completely below the density and only relatively few lie partly below the density. The Devroye method is simple and fast but does not extend to distributions with unbounded support. The total volume of the covering hyperrectangles is greater than one and the method belongs to the class of rejection methods.

The method proposel by us is based on covering by m congruent hyperrectangles not the entire region R under the density but only its main part H which lies over a hyperrectangle D (the remaining unbounded part of R is designated by T), and D is chosen so that the volume of H is very close to one. The common volume of the covering hyperrectangles equals $\frac{1}{n}$, $m \leq n$. An algorithm for finding the required covering hyperrectangles in the case where the partial derivatives of f are uniformly bounded is presented. Tables in which the coordinates of a vertex of each covering hyperrectangle are listed need to be initialized at the setup stage of the method. The generation of X looks as follows. We first select randomly a positive integer i less or equal to n. If the selected integer i is less or equal to m, we generate a point P from the hyperrectangle indexed by i and "accept" the first d coordinates of P as a generated value of X provided that the hyperrectangle indexed by i lies completely under the graph of f or the chosen hyperrectangle lies partly under the graph of f but the point P generated in it lies under the graph of f; otherwise we generate a point from T utilizing one of the sampling methods, for instance the rejection method.

[1] L. Devroye. *Non-Uniform Random Variate Generation.* Springer-Verlag, New York, 1986.

Generation of Waiting Time in a Markovian Trial Sequence

Yael Hagbi
Department of Mathematics, Ariel University, Israel, yael787@gmail.com

A new efficient method for generating infinite discrete random variables with an extremely complicated analytic expression for their probability mass function and an algorithm implementing the method for sampling from $W_\Lambda(r)$ distributions are introduced.

A random number generator implementing the algorithm was designed and tested in experimental sampling and the analysis of the obtained results is presented.

The $W_\Lambda(r)$ distribution is a distribution of the waiting time (number of trials) until the r-th occurrence of a compound pattern Λ in a Markovian trial sequence

$X_{-m+1}, X_{-m+2}, ..., X_0, X_1, X_2, ...$ with state space $S = \{b_1, ..., b_s\} (s \geq 2)$, where the outcome of each trial depends on the outcomes of the m directy preceding trials and depends only on these. A compound pattern Λ is a union of simple patterns Λ_i, $i = \overline{1, n}$, and a simple pattern Λ_i is a sequence of k_i symbols $b_{i_1}, b_{i_2}, ..., b_{i_{k_i}}$, where the symbols are allowed to be repeated.

In order to define a $W_\Lambda(r)$ distribution, the initial probabilities

$$\pi_0(x_{-m+1}, ..., x_0) = P(X_{-m+1} = x_{-m+1}, ..., X_0 = x_0)$$

and a time-invariant transition probabilities matrix

$$p(x_t \mid x_{t-m}, ..., x_{t-1}) \equiv P(X_t = x_t \mid X_{t-m} = x_{t-m}, ..., X_{t-1} = x_{t-1})$$

need to be specified.

The theory of waiting-time distributions for patterns in multi-state Markovian trial sequences has been developed during last decades and widely applied in diverse areas such as DNA sequence analysis, reliability, quality control and start-up demonstration testing.

In view of the wide field of applications of the $W_\Lambda(r)$ distributions it is extremely important to develop sampling methods and random number generators for such distributions which is necessary for simulation of the processes modeled by them.

However, algorithms and software for sampling from Markovian order m waiting-time distributions have not been developed and are not available to date, which enables us to predict that the random number generator implementing the presented algorithm will be incorporated in most widely used mathematical software platforms in the near future.

Abstracts in special sessions

Monday (16:15–17:45), **Room: Salle 5**

Rare event simulation

Organizer(s): Chang-Han Rhee **Chair: Chang-Han Rhee**

Rare event simulation has long been one of the central subjects of Monte Carlo simulation research. In this special session, three speakers will present recent progresses in rare event simulation.

Rare-event simulation in machine learning: Infinite swapping and restricted Boltzmann machines

Henrik Hult
Department of Mathematics, KTH Royal Institute of Technology, Sweden, hult@kth.se

Pierre Nyquist
Department of Mathematics and Computer Science, Eindhoven University of Technology, Netherlands, p.k.u.nyquist@tue.nl

Carl Ringqvist
Department of Mathematics, KTH Royal Institute of Technology, Sweden, carrin@kth.se

Infinite swapping is a Monte Carlo method, originally developed by Doll, Dupuis and co-authors, designed to overcome the problem of rare-event sampling, and thus speed up convergence, in the setting of computing integrals with respect to Gibbs measures [2, 3]. This talk will focus on the properties of the method and a non-standard application in machine learning. Specifically, we will discuss how infinite swapping can be applied in the training of restricted Boltzmann machines (RBMs): Training an RBM boils down to computing expectations with respect to a Gibbs measure on a high-dimensional space, a task for which standard Monte Carlo methods are not enough due to the issue of rare-event sampling. To overcome this problem we study a Gibbs-sampling version of infinite swapping (which is of independent interest) and compare its performance to other standard methods using a toy data set, similar to that of [1], and MNIST.

[1] Desjardins, G., Courville, A., Bengio, Y., Vincent, P., and Dellaleau, O. Parallel tempering for training restricted boltzmann machines. In *JMLR Workshop and Conference Proceedings: AISTATS 2010*, volume 9, pp. 145–152. 2010.

[2] Dupuis, P., Liu, Y., Plattner, N., and Doll, J. D. On the infinite swapping limit for parallel tempering. *Multiscale Model. Simul.*, 10 (3): 986–1022, 2012.

[3] Doll, J. D., Plattner, N., Freeman, D. L., Liu, Y., and Dupuis, P. Rare-event sampling: occupation-based performance measures for parallel tempering and infinite swapping Monte Carlo methods. *J. Chem. Phys.*, 137, 2012.

ALOE importance sampler for the union of rare events

Art B. Owen
Stanford University, USA, owen@stanford.edu

Jingyi Kenneth Tay
Stanford University, USA, kjytay@stanford.edu

Yury Maximov
Los Alamos National Laboratory, USA, yury@lanl.gov

Michael Chertkov
Los Alamos National Laboratory, USA, chertkov@lanl.gov

Here we estimate the probabilty μ of a union of J rare events defined in terms of a random variable $\boldsymbol{x}$. The algorithm begins by picking event j with probability proportional to its individual occurence probability. We then sample $\boldsymbol{x}$ conditionally on that event happening, and count the total number $S(\boldsymbol{x})$ of events that happen. The estimate of μ is then the union bound $\bar{\mu}$ times the average of $S(\boldsymbol{x})^{-1}$ over n repeated trials.

This importance sampler has been used by Frigessi & Vercellis (1985) for combinatorial enumeration, Naiman & Priebe (2001) for scan statistics in genomics and medical imaging, Shi, Siegmund & Yakir (2007) for linkage analysis, and Adler, Blannchet & Liu (2012) for exceedance probabilities of Guassian random fields. The literature does not name it. It always has At Least One rare Event, so we call it ALOE. We find upper bounds on the variance of the ALOE importance sampler. It always has $\mathrm{var}(\hat{\mu}) \le \mu(\bar{\mu} - \mu)/n$. It also has $\mathrm{var}(\hat{\mu}) \le (J + J^{-1} - 2)/(4n)$.

We consider power system reliability, where the phase differences between connected nodes have a joint Gaussian distribution and the J rare events arise from unacceptably large phase differences. In the grid reliability problems even some events defined by 5772 constraints in 326 dimensions, with probability below 10^{-22}, are estimated with a coefficient of variation of about 0.0024 with only $n = 10{,}000$ sample values. The algorithm extends beyond estimation of μ. We also use it to study the distribution of the number of false discoveries in genomics problems.

Efficient Rare-Event Simulation for Multiple Jump Events in Regularly Varying Random Walks and Compound Poisson Processes

Bohan Chen
Centrum Wiskunde & Informatica, Netherlands, b.chen@cwi.nl

Jose Blanchet
Stanford University, USA, jose.blanchet@stanford.edu

Chang-Han Rhee
Centrum Wiskunde & Informatica, Netherlands, rhee@cwi.nl

Bert Zwart
Centrum Wiskunde & Informatica, Netherlands, bert.zwart@cwi.nl

We propose a class of strongly efficient rare event simulation estimators for random walks and compound Poisson processes with a regularly varying increment distribution in a general large deviations regime. Our estimator is based on an importance sampling strategy that hinges on the heavy-tailed sample path large deviations result recently established in [1]. The new estimators are straightforward to implement and can be used to systematically evaluate the probability of a wide range of rare events with bounded relative error. They are "universal" in the sense that a single importance sampling scheme applies to a very general class of rare events that arise in heavy-tailed systems. In particular, our estimators can deal with rare events that are caused by multiple big jumps (therefore, beyond the usual principle of a single big jump) as well as multidimensional processes such as the buffer content process of a queueing network. We illustrate the versatility of our approach with several applications that arise in the context of mathematical finance, actuarial science, and queueing theory.

[1] Rhee, C.H., Blanchet, J. and Zwart, B., 2016. Sample Path Large Deviations for Heavy-Tailed Lévy Processes and Random Walks. arXiv preprint arXiv:1606.02795.

Monday (16:15–17:15, **Room: Salle 6**

Simulation in finance and operation management

Organizer(s): Guangxin Jiang **Chair: Guangxin Jiang**

Speeding Up Ranking and Selection Procedures for Large Scale Problems Using Cloud Computing

Jun Luo
Antai College of Economics and Management, Shanghai Jiao Tong University, China,
jluo_ms@sjtu.edu.cn

L. Jeff Hong
Department of Management Sciences, City University of Hong Kong, Hong Kong, China,
jeffhong@cityu.edu.hk

Ying Zhong
Department of Management Sciences, City University of Hong Kong, Hong Kong, China,
yzhong4-c@my.cityu.edu.hk

Classical sequential ranking-and-selection (R&S) procedures require all pairwise comparisons after collecting one additional observation from each surviving system, which is typically an $O(k^2)$ operation where k is the number of systems. When the number of systems is large (e.g., millions), these comparisons can be very costly and may significantly slow down the R&S procedures, especially using parallel simulation. In this paper we revise well-known KN procedure slightly and show that one may reduce the computational complexity of all pairwise comparisons to an $O(k)$ operation, thus significantly reducing the computational burden. We further notice that the computational complexity of communications could be a bottleneck. Then we proposed a divide-and-conquer approach to reduce the communicational complexity significantly. Numerical experiments demonstrate the benefit of our proposed procedure as implemented in the Cloud.

Constructing Surface for Derivative Pricing and Sensitivity Analysis

Guangxin Jiang
School of Management, Shanghai University, China, gxjiang@shu.edu.cn

L. Jeff Hong
Department of Management Sciences, City University of Hong Kong, Hong Kong, China,
jeffhong@cityu.edu.hk

Haihui Shen
Department of Management Sciences, City University of Hong Kong, Hong Kong, China,
haihui.shen@my.cityu.edu.hk

In financial industries, practitioners often need to know the derivative price and its Greeks in real-time trading. The analytical formula of the price surface is often unavailable and Monte Carlo simulation is used to estimate the price and Greeks at fixed values of the market parameters. However, simulations are often time consuming and cannot be used in making real-time decisions. In this paper we propose a new simulation regime, called offline-learning-online-application, in derivative pricing. It utilizes the time of the market close to learn the price surface of a derivative. The enhanced stochastic kriging is proposed to learn the price surface, then the Greek surfaces are obtained by taking partial derivatives directly on this price surface. We show that the obtained price and Greek surfaces are accurate and consistent. Numerical examples illustrate the effectiveness of the new simulation regime and the appropriateness of enhanced stochastic kriging in learning the surfaces.

Tuesday 10:00–12:00, **Room: Amphi 3**

Algorithms for High-Dimensional Approximation (and Integration) Problems

Organizer(s): Michael Gnewuch & Peter Kritzer **Chair: Michael Gnewuch & Peter Kritzer**

High-dimensional or even infinite-dimensional approximation and integration problems are ubiquitous in many areas of applications as, e.g., in computational finance, physics, or quantum chemistry. One reason for this is, e.g., that complex stochastic models are often based on a sequence of independent and identically distributed random variables, implying that expectations can be represented as infinite-dimensional integrals.

The last two decades saw huge progress in the analysis of high- and infinite-dimensional approximation and integration problems: Tractability studies essentially became the main

eld of research in information-based complexity, concepts as effective dimension and weights were introduced, and methods such as the component-by-component construction for rank-1-lattice rules and for polynomial lattice rules (which are special (t, m, s) -nets) or Smolyak's algorithm (also known as sparse grid methods) have been developed and/or studied with regard to their behavior with respect to increasing dimension. These developments provided algorithms that are effective for a moderately large number of dimensions. Later on "meta-algorithms" such as multilevel algorithms, multivariate decomposition methods (formerly known as changing dimension algorithms), or dimension-wise quadrature methods that used Monte Carlo, quasi-Monte Carlo, or, e.g., Smolyak algorithms as building blocks, led to even better results in theory and practice, and actually allowed to tackle infinite-dimensional problems. New concepts such as truncation dimension or embeddings of scales of function spaces allow for more effective approximation algorithms or to show that algorithms that are (provably) effective in one setting are also (provably) effective in other related settings. In the proposed special session we want to focus on new algorithms for high- and infinite-dimensional approximation and integration problems, and on their error analysis.

An Optimal Automatic Algorithm Employing Continuous Linear Functionals

Yuhan Ding
Department of Mathematics, Misericordia University, USA, yding@misericordia.edu

Fred Hickernell
Department of Applied Mathematics, Illinois Institute of Technology, USA, hickernell@iit.edu

Lluís Antoni Jiménez Rugama
Department of Applied Mathematics, Illinois Institute of Technology, USA, ljimene1@hawk.iit.edu

Automatic algorithms attempt to provide approximate solutions that differ from exact solutions by no more than a user-specified error tolerance. This talk describes an automatic adaptive algorithm for approximating the solution to a general linear problem on Banach spaces. The algorithm employs continuous linear functionals of the input function, specifically Fourier coefficients. We assume that the Fourier coefficients of the solution to be approximated decay sufficiently fast, but do not require the decay rate to be known a priori. We also assume that the Fourier coefficients decay steadily, although not necessarily monotonically. Under these assumptions, our adaptive algorithm is shown to achieve an approximation to the function satisfying the desired error tolerance, without prior knowledge of the norm of the function to be approximated. Moreover, the computational cost of our algorithm is shown to have

asymptotically the same cost as the best possible algorithm employing arbitrary linear functionals. For functions whose Fourier coefficients decay at some given rate, our algorithm is guaranteed to provide approximate solutions that satisfy the error tolerance. In addition to that, the computational cost of our algorithm and the lower complexity bound of this kind of problem is presented in the talk. The situation of multi-dimensional tensor product spaces is discussed.

How good is random information? - Approximation in the Hilbert space setting

Aicke Hinrichs
JKU Linz, Austria, `Aicke.Hinrichs@jku.at`

To find optimal algorithms for numerical problems like function approximation, one has to choose the information optimally, and this might be difficult in practice. Choosing the information functionals as iid random functionals and using the optimal algorithm for the chosen information may be almost as good as using optimal information or may be much worse, depending on the problem setting. We analyse this problem in the Hilbert space setting and give a partial answer in this case about the power of random information.

This is joint work with David Krieg and Erich Novak.

Integration and L_2-Approximation on Hermite Spaces of Functions of Infinitely-Many Variables

Klaus Ritter
Dept. of Mathematics, TU Kaiserslautern, Germany, `ritter@mathematik.uni-kl.de`

Hermite spaces of functions of a single variable are defined in terms of decay conditions of the Fourier coefficients w.r.t. the Hermite basis. Under appropriate assumptions such spaces and their finite tensor products are reproducing kernel Hilbert spaces. In this talk we consider countably-infinite tensor products of Hermite spaces. We investigate whether these tensor products are reproducing kernel Hilbert spaces of functions of infinitely many variables, and we present results for the corresponding integration and L_2-approximation problem.

Joint work with Michael Gnewuch (Kiel), Mario Hefter (Kaiserslautern), Aicke Hinrichs (Linz), and Greg Wasilkowski (Lexington).

Tractability of Multivariate Approximation over Weighted Standard Sobolev Spaces

Henryk Woźniakowski
Institute of Applied Mathematics, University of Warsaw, Poland, `hwozniak@mimuw.edu.pl`

We study multivariate approximation defined over weighted standard Sobolev spaces in the worst case setting. We consider algorithms that use arbitrary linear functionals. We analyze a number of tractability notions. We prove that no matter how positive weights are chosen multivariate approximation is *not* quasi-polynomial tractable. On the other hand, we find necessary and sufficient conditions on the decay of product weights to achieve various levels of weak tractability. Joint work with A. G. Werschulz.

Tuesday 10:00–12:00, **Room: Salle 8**

Uncertainty Quantification and Sensitivity Analysis in Computational Finance

Organizer(s): Giray Okten **Chair: Giray Okten**

Application of QMC and Global Sensitivity Analysis to Option Pricing and Greeks

Sergei Kucherenko
Imperial College London, UK, s.kucherenko@imperial.ac.uk

Stefano Scoleri
Iason Ltd, Italy, stefano.scoleri@gmail.com

Marco Bianchetti
Intesa Sanpaolo, Italy, marco@bianchetti.org

Superior efficiency of QMC in many financial applications is usually explained by a reduced effective dimension of the problem, with respect to its nominal dimension [1]. Various techniques can be used to reduce effective dimension: this is possible because the effective dimension can vary by changing the order in which the variables are sampled. One popular choice for reduction of effective dimensions is to apply the Brownian bridge discretization (BBD). However, it was shown that, in some cases, the Brownian bridge can perform worse than the standard discretization (SD) in QMC simulation. The big question is how to know with certainty which numerical scheme will provide superior efficiency in QMC simulation. Global Sensitivity Analysis (GSA) based on Sobol' indices can provide this answer [2]. In the context of quantitative finance, GSA can be applied to estimate effective dimensions of a given problem. We tested the efficiency of QMC with respect to standard MC in computing prices and greeks (delta, gamma, vega) for selected option payoffs with increasing degree of complexity and path-dependency. Both single asset and multi-asset test cases were considered. We compared SD vs BBD and Principal Component Analysis (PCA) schemes of the underlying stochastic diffusion process, as well as Cholesky vs PCA factorization of the covariance matrix. GSA revealed that effective dimensions associated with BBD and PCA simulations are generally lower than those associated with SD and Cholesky decomposition (in the case of multi-asset options), and showed how much such dimension reduction acts for different payoffs and greeks. Effective dimensions, being linked with the structure of ANOVA decompositions (the number of important inputs, importance of high order interactions) fully explain the superior efficiency of QMC with BBD and PCA due to the specifics of Sobol' sequences. An exception is a Cliquet option for which SD has lower effective dimension than BBD or PCA.

We also applied Adjoint Algorithmic Differentiation (AAD) to compute greeks with both MC and QMC. As AAD requires a considerable implementation effort our results suggest that the use of finite differences coupled with QMC remains competitive in many realistic applications.

[1] R. E. Caflish et al. Valuation of mortgage-backed securities using Brownian bridges to reduce effective dimension. *The Journal of Computational Finance*, 1 (1): 27–46, 1997.

[2] I.M. Sobol', S. Kucherenko. Global Sensitivity Indices for Nonlinear Mathematical Models. Review. *Wilmott Magazine*, 1:56–61, 2005.

Sensitivity Analysis of Quasi-Monte Carlo methods for the Heston Model

Emanouil Atanassov
IICT-BAS, emanouil@parallel.bas.bg

Sergei Kucherenko
Imperial College London, UK, s.kucherenko@imperial.ac.uk

Dimitar Dimitrov
IICT-BAS, d.slavov@bas.bg

Stochastic volatility models like the Heston model allow for better simulation of the market dynamics at the expense of increased complexity and computational requirements. The quasi-Monte Carlo(QMC) methods can be applied successfully for pricing European, Asian and Exotic options under this model. We evaluate the use of scrambled Halton and Sobol sequences for pricing such options and compare it with the use of the standard MC method, while applying different numerical integration schemes such as *Euler, Milstein, Kahl-Jäckel, Andersen*. We apply Global Sensitivity Analysis (GSA) based on Sobol sensitivity indices in order to assess the effective dimensionality of the resulting integrals for the various types of options and schemes using an approach developed in [1]. We show that application of GSA fully explains efficiencies of using QMC with the different numerical integration schemes.

[1] M. Bianchetti, S. Kucherenko, S. Scoleri. Pricing and Risk Management with High-Dimensional Quasi Monte Carlo and Global Sensitivity Analysis. *Wilmott*, July, 46-70, 2015.

Sensitivity and Robustness of Financial Models

Giray Ökten
Florida State University, okten@math.fsu.edu

A practical use of the Sobol' sensitivity indices of a model is identifying and then freezing the unimportant parameters. However, what if the pattern of the Sobol' sensitivity indices itself is sensitive across different statistical estimations (calibrations), so that it is not clear which parameters are important? In this talk I will discuss examples from financial models where we observe such behavior, and present an approach to quantify the robustness of a model using randomized Sobol' sensitivity indices.

Sensitivities of Exotic Portfolios

Alexander Kreinin
Quantitative Research, Risk Analytics, Canada, alex.kreinin@ca.ibm.com

We consider the computation of sensitivities of financial portfolios containing exotic options. We demonstrate that the sensitivities of the exotic portfolios can be efficiently analyzed with a combination of the Quasi MC and change of measure techniques.

Tuesday 16:15–17:45, **Room: Salle 6**

Design and testing of random number generators

Organizer(s): Pierre L'Ecuyer **Chair: Pierre L'Ecuyer**

xoshiro/xoroshiro: new families of high-quality, high-speed PRNGs

David Blackman

Independent researcher, david.john2060@gmail.com

Sebastiano Vigna
Dipartimento di Informatica, Università degli Studi di Milano, Italia, sebastiano.vigna@unimi.it

xoshiro/xoroshiro are linear transformations that have been handcrafted to have good statistical properties and at the same time to be programmable very efficiently on superscalar processors, or even directly in hardware. We describe the linear transformations and their properties. Then, we suggest suitable combinations of linear transformations and nonlinear scramblers yielding extremely fast pseudorandom number generators of high quality. The generators we obtain improve under every aspect the popular xorshift+/xorshift* families, providing high-quality, high-speed generators that use a few hundred bits of memory, have provable equidistribution properties and pass strong statistical tests such as the classic BigCrush suite from TestU01. In addition, we describe a strong test based on dependencies in Hamming weights that is able to find bias in some generators which pass BigCrush.

Testing the Reliability of Statistical Tests for Pseudorandom Number Generators

Hiroshi Haramoto
Faculty of Education, Ehime University, Japan, haramoto@ehime-u.ac.jp

Statistical testing of pseudorandom number generators (PRNGs) are indispensable for their evaluation. It is necessary for PRNGs to pass various statistical tests. For example, TestU01, the most extensive test package, applies more than one hundred tests to a tested PRNG at a time. However, there is no universal test or battery of tests, a variety of testing procedures have been developed continuously.

Checking the reliability of each test is an important task. On the other hand, mathematical investigations are often difficult. Hence it appears that experimental methods to check soundness of statistical tests are useful. In this situation, we proposed an experimental three-level test to check soundness of statistical tests [1]. Since this three-level test is based on the uniformity of p-values of a test statistic, any tests whose distributions are continuous can be evaluated by this criterion.

In this talk, we show some defects of tests in the NIST test suite and modifications of those by using the three-level test. We will also reveal the reliability of several recent proposed tests: the LIL test [3], the arcsine test [2], and some discrete Fourier transform tests. In addition, we will consider how to extend the target of the three-level test to tests whose distributions are not continuous, especially the Birthday Spacing test in TestU01, that conforms to a Poisson distribution.

[1] Hiroshi Haramoto. Checking the Soundness of Statistical Tests for Random Number Generators by Using a Three-Level Test. arXiv:1710.06033, 2017.

[2] Paweł Lorek, Grzegorz Łoś, Filip Zagórski, and Karol Gotfryd. On testing pseudo random generators via statistical tests based on arcsine law. http://www.math.uni.wroc.pl/~lorek/papers/main_arcsine_AMC_SUBMITTED.pdf, 2017.

[3] Yongge Wang and Tony Nicol. On Statistical Distance Based Testing of Pseudo Random Sequences and Experiments with PHP and Debian OpenSSL. *Comput. Secur.*, 53(C):44–64, September 2015.

On the Lattice Structure of MIXMAX Random Number Generators

Pierre L'Ecuyer
DIRO, Université de Montréal, Canada, lecuyer@iro.umontreal.ca

Paul Wambergue
École Centrale, Châtenay-Malabray, France, paul.wambergue@gmail.com

Erwan Bourceret
École Polytechnique, Palaiseau, France, ebourceret@hotmail.fr

We study the lattice structure of the MIXMAX family of random number generators [1, 5], which produce a vector of random numbers at each step. They were initially justified as close approximations to ergodic dynamical systems having the K-mixing property, which implies a chaotic (fast-mixing) behavior. But for a K-mixing system, the matrix that defines the linear recurrence must have irrational entries, whereas for the MIXMAX it has only integer entries. As a result, the MIXMAX has a lattice structure like linear and multiple recursive congruential generators [3]. Its matrix entries were also selected in a special way to allow a fast implementation and this has an impact on the lattice structure. We study this lattice structure for vectors of successive and non-successive output values in various dimensions. We show in particular that for coordinates at specific lags not too far apart, in three dimensions, all the nonzero points lie in only two hyperplanes. This is reminiscent of the behavior of lagged-Fibonacci and AWC/SWB generators [2]. And even if we skip the output coordinates involved in this bad structure, other highly structured projections often remain, depending on the choice of parameters. A technical report is available in [4].

[1] N. Z. Akopov, G. K. Savvidy, and N. G. Ter-Arutyuntan Savvidy. Matrix generators for pseudorandom numbers. *Journal of Computational Physics*, 97:573–579, 1991.

[2] P. L'Ecuyer. Bad lattice structures for vectors of non-successive values produced by some linear recurrences. *INFORMS Journal on Computing*, 9(1):57–60, 1997.

[3] P. L'Ecuyer and R. Couture. An implementation of the lattice and spectral tests for multiple recursive linear random number generators. *INFORMS J. Computing*, 9(2):206–217, 1997.

[4] P. L'Ecuyer, P. Wambergue, and E. Bourceret. Spectral analysis of the MIXMAX random number generators. Submitted, 2017.

[5] K. G. Savvidy and G. K. Savvidy. Spectrum and entropy of C-systems MIXMAX random number generator. *Chaos, Solitons and Fractals*, 91:33–38, 2016.

Tuesday 16:15–17:45, **Room: Salle 7**

Computational challenges in finance

Organizer(s): Jérôme Lelong **Chair: Jérôme Lelong**

Smoothing the payoff for computation of basket options

Christian Bayer
Weierstrass Institute, Berlin, Germany, christian.bayer1979@gmail.com

We consider the problem of pricing basket options in a multivariate Black-Scholes or Variance-Gamma model. From a numerical point of view, pricing such options corresponds to moderate and high-dimensional numerical integration problems with non-smooth integrands. Due to this lack of regularity, higher order numerical integration techniques may not be directly available, requiring the use of methods like Monte Carlo specifically designed to work for non-regular problems. We propose to use the inherent smoothing property of the density of the underlying in the above models to mollify the payoff function by means of an exact conditional expectation. The resulting conditional expectation is unbiased and yields a smooth integrand, which is amenable to the efficient use of adaptive sparse-grid cubature. Numerical examples indicate that the high-order method may perform orders of magnitude faster than Monte Carlo or Quasi Monte Carlo methods in dimensions up to 25. (Joint work with Markus Siebenmorgen und Raul Tempone).

Cubature method to solve BSDEs: error expansion and complexity control

Jean–François Chassagneux
LPSM, Université Paris Diderot, France, chassagneux@lpsm.paris

In this work, we prove error expansions for the approximation of BSDEs when using the cubature method. To profit fully from these expansions, e.g. to design high order approximation methods, we need however to control the complexity growth of the cubature method. In our work, this is achieved by using a sparse polynomial interpolation method. We present several numerical results that confirm the efficiency of our new method.

This is a joint work with C. Garcia (UCL)

The Parareal Algorithm for American Options

Gilles Pagès
LPMA, UPMC, France, gilles.pages@upmc.fr

With parallelism in mind we investigate the parareal method for American contracts both theoretically and numerically. Least-Square Monte-Carlo (LSMC) and parareal time decomposition with two or more levels are used, leading to an efficient parallel implementation which scales linearly with the number of processors and is appropriate to any multiprocessor-memory architecture in its multilevel version. We prove L^2 superlinear convergence for an LSMC backward in time computation of American contracts, when the conditional expectations are known, i.e. before Monte-Carlo discretization. The argument provides also a tool to analyze the multi-level parareal algorithm; in all cases the computing time is increased only by a constant factor, compared to the sequential algorithm on the finest grid, and speed-up is guaranteed when the number of processors is larger than that constant. A numerical implementation confirms the theoretical error estimates.

Wednesday 10:00–12:00, **Room: Salle 6**

Simulation of mean-field stochastic differential equations

Organizer(s): Benjamin Jourdain **Chair: Benjamin Jourdain**

Mean-field SDEs which present a nonlinearity in the sense of McKean appear in many application fields : Lagrangian modeling of turbulent flows, calibration of local and stochastic volatility models, neurosciences, synchronization of oscillators,... Due to the dependence of the coefficients on the unknown marginal law of the solution, they cannot be simulated directly. By propagation of chaos results, they can be approximated by systems of interacting diffusive particles where the nonlinearity is replaced by interaction. The computation of the interaction terms is in general quadratic in the number of particles and therefore very costly. The talks in this session will present and analyse numerical schemes for specific applications or generic procedures aimed at accelerating the simulation of particle systems

Particle algorithm for McKean SDE: rate of convergence for some non-smooth drift interaction kernel

Mireille Bossy
Inria, Sophia Antipolis, mireille.bossy@inria.fr

In this talk we will discuss the convergence analysis of particle approximation of a McKean SDE (also called mean-field SDE). In the Lipschitz kernels framework, the numerical analysis of the basic particle approximation is well understood (at least formally/ modulo some control on the solution of the the Fokker Planck PDE).

But for a better numerical efficiency or for non-smooth interaction kernel, the particle approximation may involves a cut-off or mollifier function that limit the interaction between particles according to an additional parameter. Motivated by some computational fluid dynamics (CFD) applications, we describe some recent results of numerical analysis in such situation.

Variance reduction for mean-field stochastic differential equations

Denis Belomestny
Duisburg-Essen University, denis.belomestny@uni-due.de

John Schoenmakers
WIAS, schoenma@wias-berlin.de

The McKean-Vlasov or mean-field stochastic differential equations are a class of stochastic differential equations (SDE) where the drift and diffusion depend on the current position along the path and on the current distribution. They were derived to describe propagation of chaos in a system of particles that interact only by their empirical mean in the limit of large number of particles.

Let $[0,T]$ be a finite time interval and $(\Omega,\mathcal{F},\mathrm{P})$ be a complete probability space, where a standard m-dimensional Brownian motion W is defined. We consider a class of McKean-Vlasov SDEs (MVSDE), i.e., stochastic differential equations (SDE), whose drift and diffusion coefficients may depend on the current distribution of the process, of the form:

$$\begin{cases} X_t &= \xi + \int_0^t \int_{\mathbb{R}^d} a(X_s,y)\mu_s(dy)ds + \int_0^t \int_{\mathbb{R}^d} b(X_s,y)\mu_s(dy)dW_s \\ \mu_t &= \mathrm{Law}(X_t), \quad t\geq 0, \quad X_0\sim\mu_0 \end{cases} \tag{10.1}$$

where μ_0 is a distribution in $\mathbb{R}^d$, $a:\mathbb{R}^d\times\mathbb{R}^d\to\mathbb{R}^d$ and $b:\mathbb{R}^d\times\mathbb{R}^d\to\mathbb{R}^{d\times m}$.

A popular way of simulating the MVSDE (10.1) is to sample from the N-particle interacting diffusion model

$$X_t^{i,N} = \xi^i + \frac{1}{N}\sum_{j=1}^N \int_0^t a(X_s^{i,N},X_s^{j,N})\,ds + \frac{1}{N}\sum_{j=1}^N \int_0^t b(X_s^{i,N},X_s^{j,N})\,dW_s^i \tag{10.2}$$

for $i=1,\dots,N$, where ξ^i, $i=1,\dots,N$, are i.i.d copies of a r.v. ξ, distributed according the law μ_0, and W^i, $i=1,...,N$, are independent copies of W. Suppose we want to compute the function

$$u(t,x)=\mathsf{E}[f(X_T^{t,x})],$$

where $X^{t,x}$ denotes the solution of (10.1) started at time t in point x. The particle Monte Carlo approach for computing $u(0,x)$ at a fixed point $x\in\mathbb{R}^d$ would deliver a Monte Carlo estimate V_N defined as the average of the values of f on particles:

$$V_N \doteq \frac{1}{N}\sum_{n=1}^N f\left(X_T^{n,N}\right). \tag{10.3}$$

The aim of the variance reduction methods is to reduce the statistical error, which results from the substitution of $\mathsf{E}[f(X_T)]$ with the sample average V_N. In this talk we discuss a possibility of using regression on particles to construct efficient and optimal variance reduction algorithms (see [1]). The main difficulty lies in the fact that particles are highly dependent and as a result, the standard regression analysis can not be applied.

[1] Belomestny, D., Häfner, S., Nagapetyan, T. and Urusov, M., Variance reduction for discretised diffusions via regression. *Journal of Mathematical Analysis and Applications*, 458(1), 1–12, 2018.

Weak error expansion for mean-field SDEs

Lukasz Szpruch
University of Edinburgh, l.szpruch@ed.ac.uk

Alvin Tse
University of Edinburgh, s1064213@sms.ed.ac.uk

Jean-Francois Chassagneux
Université Paris Diderot. , jean-francois.chassagneux@univ-paris-diderot.fr

We consider a stochastic process X, described by a mean-field stochastic differential equation, whose coefficients depend on the evolving law of the process itself. Such equations arise as a limit of the system of stochastic interacting particle systems $Y^{i,N}$, i.e SDEs that are coupled via the empirical law. In this talk, we show that under suitable regularity assumptions the weak error between X and $Y^{i,N}$ can be expressed as $\sum_{j=1}^{k-1} \frac{C_j}{N^j} + O(\frac{1}{N^k})$, for some constants $C_1, \ldots, C_{k-1}$ that do not depend on N. That is we formulate the weak-error particle expansion in the spirit of Talay and Tubaro. The expansion we propose relies on the powerful machinery of differentiation with respect to a probability measure, which was proposed by P. Lions in his lectures in Collège de France. At the core of our proof lies the study of the regularity of the PDE on measure spaces which might be of independent interest.

Numerical Analysis of a Particle Calibration Procedure for Local and Stochastic Volatility Models

Benjamin Jourdain
CERMICS, Ecole des Ponts ParisTech, France, benjamin.jourdain@enpc.fr

Alexandre Zhou
CERMICS, Ecole des Ponts ParisTech, France, alexandre.zhou@enpc.fr

The calibration of a local and stochastic volatility model to the market prices of vanilla options leads to a diffusion nonlinear in the sense of McKean, as the coefficients contain conditional expectations computed w.r.t. the coordinates of the solution. Guyon and Henry-Labordère introduced in [1] an efficient calibration procedure using kernel approximations of the conditional expectation and interacting particle systems. We show the weak convergence at order 1 for the explicit Euler scheme with constant time step discretizing the diffusion nonlinear in the sense of McKean, using the technique developed by Talay and Tubaro [2]. We then perform a numerical analysis of the calibration procedure from [1] and illustrate the efficiency of the method.

[1] J. Guyon and P. Henry-Labordère. Being particular about calibration. *Risk magazine*, 88-93, Jan. 1990.

[2] D. Talay and L. Tubaro. Expansion of the global error for numerical schemes solving stochastic differential equations. *Stochastic Analysis and Applications*, 8 (4): 483-509, 1990.

Wednesday 10:00–12:00, **Room: Salle 7**

Importance Splitting for Rare Event Simulation

Organizer(s): Frédéric Cérou & Arnaud Guyader **Chair: Frédéric Cérou**

The session will present recent theoretical advances on Importance Splitting , and its application for Monte-Carlo particle (neutron) transport.

New results concerning Adaptive Multilevel Splitting algorithms

Charles-Edouard Bréhier
CNRS and Univ. Lyon 1, brehier@math.univ-lyon1.fr

I will first introduce a general framework for the design and analysis of Adaptive Multilevel Splitting algorithms, and I will review the main consistency and efficiency results obtained in the last few years. Instead of considering the standard case of transitions between metastable states of Markov processes, I will present new algorithms when the observables depend on the values of the processes on a given, finite, time interval, and an application to estimation of return times in complex physical systems. To illustrate the behavior of these algorithms, numerical simulations will be discussed.

Numerical analysis and long time stability of Feynman-Kac dynamics

Gregoire Ferré
ENPC, France, gregoire.ferre@enpc.fr

Sequential Monte Carlo is a widely used family of algorithms that estimate rare event probabilities. They generally consist in running a set of replicas of a given system, and to select replicas depending on their position in the state space. With this procedure, one can enhance the number of replicas in regions that a single system would visit rarely. Observing the long time behavior of such algorithms provide information on the probabilities of rare events in a single system.

From a more general perspective, these algorithms rely on Feynman-Kac semigroups, that can be considered in continuous or discrete time. Compared to usual Markov chains, the trajectories of such system are weighted according to some function, which corresponds to the selection mechanism. If we consider a continuous dynamics we need, in practice, to perform a discretization that induces an error in the time step. In the context of rare events, it is important to understand the long time behavior of these dynamics and their discretizations, and to identify the leading error term induced by the numerical scheme.

However, the stability properties depend both on the dynamics itself and on the weighting function, so the stability of the dynamics depends on an interplay between these two features. We first present a new approach for studying the stability of Feynman-Kac semigroups and their discretizations. It draws idea from the theory of stochastic stability developed e.g. by Meyn and Tweedie, as well as from Large Deviation theory, which is a closely related field. The conditions we derive for the stability to hold have a natural interpretation, and we show that they apply to a number of interesting systems. We then provide a criteria to quantify the order of convergence of a numerical scheme to its continuous reference, and show relevant applications.

Convergence of Fleming-Viot particle systems to the mininal quasi-stationary distribution

Nicolas Champagnat
Inria Nancy, France, Nicolas.Champagnat@inria.fr

This is joint work with Denis Villemonais (IECL, Univ. Lorraine).

For absorbed Markov processes, a quasi-stationary distribution (QSD) is a distribution stationary conditionally on non-absorption. Fleming-Viot particle systems are designed to approximate conditional distribution of absorbed processes and hence quasi-stationary distributions. Contrary to non-absorbed processes, irreducibility does not entail in general uniqueness of QSD. This is typically true for processes which do not enter fast in compact sets when starting close to infinity, such as Galton-Watson processes of Ornstein-Uhlenbeck diffusions. In such cases, the question of which quasi-stationary distribution is approximated by the Fleming-Viot particle system remains largely open. We present general criteria ensuring the exponential convergence of conditional distributions to the so-called minimal QSD, at a speed depending on the initial distribution. We prove that, under these conditions, the Fleming-Viot particle system converges to the minimal QSD for any initial distribution. We finally present several examples of processes satisfying these conditions.

Application of an importance splitting method to radiation shielding simulations

Henri Louvin
CEA Saclay, Université Paris-Saclay, France, henri.louvin@cea.fr

The behaviour of neutral particles travelling through matter, such as neutrons and photons, can be described by a stochastic process, which can be simulated using Monte Carlo particle transport codes. These codes can be used to perform radiation shielding simulations, which are often characterized by strong radiation attenuation in matter, so that they fall within the scope of rare events analysis. In the most severe cases, the use of variance reduction methods is essential in order to get an acceptable flux estimation in a reasonnable computation time.

The wish to study particle methods as variance reduction techniques for radiation shielding simulations motivated the recent introduction of the Adaptive Multilevel Splitting (AMS)[1] to this application field. It has been implemented in the forthcoming version 11 of the Monte Carlo code TRIPOLI-4®[4] and successfully applied to a variety of cases [2, 3]. The specificities of this real life application required the development of extra features for the AMS, which efficiency have been extensively tested in physically challenging configurations.

[1] F. Cérou, A. Guyader Adaptive multilevel splitting for rare event analysis *Stoch. Anal. Appl.*, 25 (417), 2007.

[2] H. Louvin Development of an Adaptive Variance Reduction Technique for Monte Carlo Particle Transport *Ph.D. thesis*, Université Paris-Saclay, 2017.

[3] H. Louvin, E. Dumonteil, T. LeliÃ¨vre, M. Rousset, C.M. Diop Adaptive Multilevel Splitting for Monte Carlo Particle Transport *EPJ Nuclear Sci. Technol.*, 3 (29), 2017.

[4] TRIPOLI-4®Project Team TRIPOLI-4®, CEA, EDF and AREVA Reference Monte Carlo Code *Ann. Nucl. Energy*, 82 (151), 2015.

Wednesday 10:00–12:00, **Room: Salle 8**

Dispersion and Applications

Organizer(s): Aicke Hinrichs & Mario Ullrich **Chair: Aicke Hinrichs**

The dispersion of a point set, which is the volume of the largest axis-parallel box in the unit cube that does not intersect the point set, is an alternative to the discrepancy as a measure for certain (uniform) distribution properties. The computation of the dispersion, or even the best possible dispersion, in dimension two has a long history in computational geometry and computational complexity theory. Given the prominence of the problem, it is quite surprising that, until recently, very little was known about the size of the largest empty box in higher dimensions.

In the proposed special session we focus on recent developments in this area of research. Besides upper and lower bounds on the (minimal) dispersion that show, in particular, the surprising logarithmic dependence on the dimension of the inverse dispersion, we want to deal with recently established connections of the dispersion to numerical approximation problems, like the approximation of rank-one tensors.

The inverse of the dispersion depends logarithmically on the dimension

Mario Ullrich
Institut für Analysis, JKU Linz, mario.ullrich@jku.at

For $\epsilon \in (0, 1/2)$ and a natural number $d \geq 2$, let N be a natural number with

$$N \;\geq\; 2^9 \, \log_2(d) \, \left(\frac{\log_2(1/\epsilon)}{\epsilon} \right)^2 .$$

We prove that there is a set of N points in the unit cube $[0,1]^d$, which intersects all axis-parallel boxes with volume ϵ. That is, the dispersion of this point set is bounded from above by ϵ.

On the Dispersion of Sparse Grids

David Krieg
Mathematisches Institut, Universität Jena, Germany, david.krieg@uni-jena.de

For any $d \in \mathbf{N}$ and $\varepsilon \in (0,1)$, we present a point set in the d-dimensional unit cube $[0,1]^d$ that intersects every axis-aligned box of volume greater than ε. That is, its dispersion is at most ε. In a vast range of the parameters ε and d, we do not know any smaller point set with this property. The point set is an instance of a sparse grid and hence very easy to handle.

Recovery algorithms for high-dimensional rank one tensors

David Krieg
Mathematisches Institut, Universität Jena, Germany, david.krieg@uni-jena.de

Daniel Rudolf
Institut für Mathematische Stochastik, Universität Göttingen, Germany, daniel.rudolf@uni-goettingen.de

Rank one tensors are given as product of d univariate functions whose rth weak derivative is bounded by M. The problem of the uniform recovery of such functions based on function values suffers from the curse of dimensionality for $M \geq 2^r r!$. For smaller M, a randomized algorithm is known which breaks the curse. We construct a deterministic algorithm which is even less costly.

On further aspects of dispersion

Jan Vybiral
Czech Technical University, Praha, Czech Republic, jan.vybiral@fjfi.cvut.cz

We discuss recent developments connected with the dispersion of a point set. We define a k-dispersion of a point cloud as the volume of the largest box containing less than k points from this cloud. Minimal k-dispersion is then the infimum of k-dispersion over all point clouds with a prescribed number of points. We provide an upper and lower bound of this quantity. Furthermore, we discuss the possibility of deterministic constructions of point clouds with small minimal dispersion.

Wednesday 13:45–15:45, **Room: Salle 6**

When to stop a simulation

Organizer(s): Robert Kunsch & Daniel Rudolf **Chair: Robert Kunsch**

We discuss settings in which the number of observed samples or computed function values must be adapted to the problem instance in order to solve a given problem. Usual assumptions in numerical analysis and statistics lead to solutions with fixed sample size. Which assumptions enable for an adaptive sample size selection in order to solve a problem? What can be said on complexity?

Fast Adaptive Bayesian Cubature Using Low Discrepancy Sampling

Fred J. Hickernell
Applied Mathematics, Illinois Institute of Technology, USA, hickernell@iit.edu

Jagadeeswaran Rathinavel
Applied Mathematics, Illinois Institute of Technology, USA, jrathin1@hawk.iit.edu

Bayesian cubature approximates multiple dimensional integrals by assuming that the integrand is a Gaussian process. This approach leads to a credible interval for the value of the integral. The sample size used to approximate the integral may then be increased until the credible interval is small enough to meet the error tolerance. This talk addresses three aspects of Bayesian cubature when low discrepancy sampling is used. First, we show how well-chosen covariance kernel families that match the sampling scheme expedite the calculations needed to compute the credible intervals. Second, we explore cross-validation, as well as maximum likelihood estimation, for choosing the best parameters defining the covariance kernel of the Gaussian process. Third, we provide a deterministic worst-case interpretation of the Bayesian cubature formalism.

Solvable Integration Problems and Optimal Sample Size Selection

Robert J. Kunsch
Institute for Mathematics, Osnabrück University, Germany, robert.kunsch@uni-osnabrueck.de

Erich Novak
Institute for Mathematics, Friedrich Schiller University Jena, Germany, erich.novak@uni-jena.de

Daniel Rudolf
Institute for Mathematical Stochastics, Georg-August-Universität Göttingen, Germany, daniel.rudolf@uni-goettingen.de

A numerical problem is called *solvable* if for any given error threshold $\varepsilon > 0$ and any given confidence level $1-\delta \in (0,1)$ there exists a randomized algorithm which achieves an error smaller than ε at least with probability $1-\delta$ for all problem instances. We call such algorithms (ε, δ)-*approximating* for the respective class of inputs. An algorithm may adapt the sample size to the input based on observations but is required to terminate almost surely.

We provide examples of integration problems which are not solvable.

We also discuss a solvable problem where no solution with fixed sample size does exist. Namely, we aim to compute the mean $\mathbf{E}\,Y$ of a random variable Y with unknown statistical dispersion within an absolute error ε using i.i.d. observations. Under certain assumptions it is possible to select a sample size based on a variance estimate [1], or, more generally, based on an estimate of the central absolute p-moment $\mathbf{E}\,|Y-\mathbf{E}\,Y|^p$, with some $1 \le p \le 2$. A fixed number of observations is taken for the p-moment estimate, irrespective of the current input. In fact, this *fixed cost* is inevitable, the cost for the p-moment estimate within our algorithm matches a theoretical lower bound for the minimal number of observations needed by any (ε, δ)-approximating algorithm with any stopping rule. In addition, the expected cost of our algorithm has the optimal order in ε, δ and the scale of the input.

[1] F.J. Hickernell, L. Jiang, Y.W. Liu, A. Owen. Guaranteed conservative fixed width confidence intervals via Monte Carlo sampling. *Proceedings of the MCQMC 2012*, Springer, 2013.

[2] R.J. Kunsch, E. Novak, D. Rudolf. Solvable integration problems and optimal sample size selection. In preparation.

Improved Light Tailed Sample Averages for Robust Estimation of the Mean

Mark Huber
Mathematical Sciences, Claremont McKenna College, USA, mhuber@cmc.edu

Many randomized approximation algorithms operate by giving a procedure for simulating a random variable X which has mean μ equal to the true answer, and a relative standard deviation bounded above by a known constant c. Examples of this type of algorithm includes methods for approximating the number of satisfying assignments to 2-SAT or DNF, the volume of a convex body, or the partition function of a Gibbs distribution. Light tailed sample averages (LTSA) is a method that given $\epsilon > 0$ and $\delta > 0$, finds an estimate $\hat{\mu}$ that satisfies $\mathbf{P}(|\hat{\mu}/\mu - 1| > \epsilon) \le \delta$ using on average $(2 + g(\epsilon, \delta))c^2\epsilon^{-2}\ln(1/\delta)$ samples. The constant 2 in the number of samples is optimal, and g is a function where $g(\epsilon, \delta) \to 0$ as $(\epsilon, \delta) \to (0, 0)$. LTSA is a two phase method: in the first phase an initial estimate of μ is created, and that informs the number of samples needed in the second phase. The purpose of work presented here is to reduce the value of g (which can be large for usual values of ϵ and δ) by breaking the algorithm into three phases where the output of phase 1 informs the number of samples used in phase 2 which in turn informs the samples used in phase three. Adding more phases does not seem to improve the algorithm further.

Wednesday 13:45–15:45, **Room: Salle 7**

Recent advances in particle filtering

Organizer(s): Nicolas Chopin & François Le Gland **Chair: François Le Gland**

Interacting Particles for Online Inference on Static Parameters Using Streaming Data

Mathieu Gerber
School of Mathematics, University of Bristol, UK, mathieu.gerber@bristol.ac.uk

Kari Heine
Department of Mathematical Sciences, University of Bath, UK, K.M.P.Heine@bath.ac.uk

Let $(Y_t)_{t\geq 1}$ be a sequence of i.i.d. observations, $\{f_\theta; \theta \in R^d\}$ be a parametric model for a single observation and $\theta_\star = \text{argmax}_{\theta\in R^d} \text{E}[\log f_\theta(Y_1)]$. In this work we derive an algorithm that generates and propagates over time a finite set of interacting particles. These particles are used to define a sequence of estimators $(\theta_t)_{t\geq 1}$ having the following three appealing properties: (i) θ_t depends on the past observations $\{Y_s\}_{s=1}^{t-1}$ only through θ_{t-1}; that is, for some function F_t, $\theta_t = F_t(\theta_{t-1}, Y_t, U_t)$ where U_t is a vector of uniform random variables on $(0, 1)$, (ii) evaluating F_t exactly costs at most $C < +\infty$ operations, with C constant over t, and (iii) as $t \to +\infty$ the estimator θ_t converges to $\theta_\star$ at rate $\sqrt{\log(t)^{1+\beta} t^{-1}}$, with $\beta > 0$ arbitrary. These first two properties are particularly important when dealing with streaming data which require to update our estimate of θ in real time when a new observation is available. The proposed method also allows for performing statistical inference with data sets containing a large number of observations (the so-called "tall data" sets), which is known to be computationally challenging for current statistical methods.

Improving the particle filter in high dimensions using conjugate artificial process noise

Anna Wigren
Dept of Information Technology, Uppsala Univ, Sweden, anna.wigren@it.uu.se

Lawrence Murray
Dept of Information Technology, Uppsala Univ, Sweden, lawrence.murray@it.uu.se

Fredrik Lindsten
Dept of Information Technology, Uppsala Univ, Sweden, fredrik.lindsten@it.uu.se

The particle filter is one of the most successful methods for state inference and identification of general non–linear and non–Gaussian models. However, standard particle filters suffer from degeneracy of the particle weights for high–dimensional problems. We propose a method for improving the performance of the particle filter for certain challenging state space models, with implications for high-dimensional inference. First we approximate the model by adding artificial process noise in an additional state update, then we design a proposal that combines the standard and the locally optimal proposal. This results in a bias–variance trade–off, where adding more noise reduces the variance of the estimate but increases the model bias. The performance of the proposed method is evaluated on a linear Gaussian state space model and on the non–linear Lorenz'96 model. For both models we observe a significant improvement in performance over the standard particle filter.

On the stability and the uniform propagation of chaos properties of ensemble Kalman–Bucy filters

Pierre Del Moral
INRIA Bordeaux, France, pierre.del_moral@inria.fr

The ensemble Kalman filter is a sophisticated and powerful data assimilation method for filtering high dimensional problems arising in fluid mechanics and geophysical sciences. This Monte Carlo method can be interpreted as a mean–field McKean–Vlasov type particle interpretation of the Kalman–Bucy diffusions. Besides some recent advances on the stability of nonlinear Langevin type diffusions with drift interactions, the long–time behaviour of models with interacting diffusion matrices and conditional distribution interaction functions has never been discussed in the literature. One of the main contributions of the talk is to initiate the study of this new class of models. The talk presents a series of new functional inequalities to quantify the stability of these nonlinear diffusion processes. The second contribution of this talk is to provide uniform propagation of chaos properties as well as L^p–mean error estimates w.r.t. the time horizon.

Convergence of resampling algorithms

Nicolas Chopin
ENSAE Malakoff, France, nicolas.chopin@ensae.fr

Mathieu Gerber
School of Mathematics, Univ of Bristol, UK, mathieu.gerber@bristol.ac.uk

Nick Whiteley
School of Mathematics, Univ of Bristol, UK, nick.whiteley@bristol.ac.uk

This talk covers some new results on the asymptotic behaviour of the error introduced by the resampling step of a particle filter. General conditions are given for the consistency of the resampling scheme. It is also shown that, standard resampling schemes may be modified in a certain way so that the mean square error is $O(N^{-1})$. Interestingly, as far as the MCQMC audience is concern, some of the underlying ideas of these technical results rely on concepts borrowed from QMC theory.

Wednesday 13:45–15:45, **Room: Salle 8**

Variance reduction, estimator efficiency, rare-event probability

Organizer(s): Zdravko Botev **Chair: Zdravko Botev**

Variance Reduction Techniques for the Accurate Computation of the Distribution of the sum of Ordered Random Variables.

Nadhir Ben Rached
KAUST, Saudi Arabia, nadhir.benrached@kaust.edu.sa

Zdravko Botev
UNSW Sydney, Australia, botev@unsw.edu.au

Alba Kammoun
KAUST, Saudi Arabia, alba.kammoun@kaust.edu.sa

Mohammed-Slim Alouini
KAUST, Saudi Arabia, slim.alouini.edu.sa

Raul Tempone
KAUST, Saudi Arabia, raul.tempone@kaust.edu.sa

We aim to efficiently compute the rare event probability given by the left-tail of the distribution of the sum of ordered Random variables (RVs). This has an important application in the context of performance analysis of wireless communication systems. Given that the use of naive Monte Carlo (MC) simulations is inappropriate in the rare event setting, we employ instead two well-known variance reduction techniques; namely importance sampling and conditional MC. The importance sampling estimator is featured with bounded relative error under a mild condition that can be shown to be satisfied by many challenging distributions. Moreover, possible improvements of this unified estimator is provided for the Weibull and the Pareto cases. The conditional MC estimator has also the bounded relative error property for the Generalized Gamma variate and the logarithmic efficiency for the Log-normal RV. Numerical results show that the proposed estimators yield accurate estimates of the probability of interest with small computing cost. Also, we identify the setting where the importance sampling estimator outperforms the conditional MC one and vice versa. Finally, we show numerically that the convergence rate of the conditional MC estimator is improved via the use of the randomized quasi MC approach.

Infinite swapping using iid samples

Guo-Jhen Wu
Division of Applied Mathematics, Brown University, USA, paul_dupuis@brown.edu

Paul Dupuis
Division of Applied Mathematics, Brown University, USA, paul_dupuis@brown.edu

We propose a new method for estimating rare event probabilities when independent samples are available. It is assumed that the underlying probability measures satisfy a large deviations principle with a scaling parameter which we call temperature. We show that by combining samples at different temperatures, one can construct an estimator with greatly reduced variance. Although as presented here the method is not as broadly applicable as other rare event simulation methods, such as splitting or importance sampling, it does not require any problem-dependent constructions that can lead to worse performance than ordinary Monte Carlo if done incorrectly.

Exploiting Asymptotics and Polar Coordinates for Rare Tail Estimation

Thomas Taimre
School of Mathematics and Physics, The University of Queensland, Australia, t.taimre@uq.edu.au

Patrick Laub
School of Mathematics and Physics, The University of Queensland, Australia and Department of Mathematics, Aarhus University, Denmark, p.laub@uq.edu.au|p.laub@math.au.dk

We consider the problem of estimating the right tail probability of a sum of random variables when the density of the sum is not known explicitly, but whose asymptotic behaviour is known. We embed this asymptotic structure into a simple importance sampling estimator, in which we consider the radial and angular components of the distribution separately. The estimator and procedure are applicable in both the heavy- and light-tailed settings, as well as for dependent and independent summands. We illustrate the approach with a series of examples.

Data-Driven Distributionally Robust Optimization via Optimal Transport: Algorithms and Applications

Fan Zhang
Stanford University, USA, fzh@stanford.edu

In this talk, we explain why optimal transport based Distributionally Robust Optimization (DRO) is an important class of stochastic optimization problems by showing that it encompasses a large class of estimators in machine learning. Our discussion will motivate a more general and powerful class of machine learning estimators and DRO formulations. We will provide optimal algorithms for the solution of these general DRO formulations. Finally, we will demonstrate empirically that our pro- posed methodology is able to improve upon a wide range of popular machine learning estimators.

Thursday 10:00–12:00, **Room: Salle 7**

Approximating Markov chain Monte Carlo

Organizer(s): Daniel Rudolf **Chair: Daniel Rudolf**

In the last decades, Markov chain Monte Carlo (MCMC) methods have attracted more and more attention. This attention and new applications motivated various novel MCMC approximation schemes which become part of the algorithmic toolkit. This includes not only classical "exact" MCMC algorithms but also inexact algorithms which replace the true target distribution by an approximation. As examples one can mention the pseudo-marginal approach or Monte Carlo within Metropolis as well as the exchange algorithm. The goal of this session is to discuss new algorithmic approaches and theoretical results which provide guarantees.

Barkera's algorithm for Bayesian inference with intractable likelihoods

Krys Latuszynski
Department of Statistics, University of Warwick, UK, k.g.Latuszynski@warwick.ac.uk

Flavio Goncalves
Universidade Federal de Minas Gerais, Brazil, fbgoncalves@est.ufmg.br

Gareth Roberts
Department of Statistics, University of Warwick, UK, gareth.o.roberts@warwick.ac.uk

Jan Palczewski
School of Mathematics, University of Leeds, UK, J.Palczewski@leeds.ac.uk

The standard approach to inference for parametric diffusion processes relies on discretisation techniques (such as the Euler method) that introduce an approximation error difficult to quantify especially for discontinuous models, like jump-diffusions. In this talk, I will present methodology for exact inference that avoids discretisation errors and allows to design MCMC samplers targeting the exact posterior distribution of the diffusion parameters and diffusion path between observations. The approach is based on Bernoulli Factory type subroutines, and is a general alternative to pseudo-marginal inference. The talk will be based on http://imstat.org/bjps/papers/BJPS374.pdf and https://arxiv.org/pdf/1707.00332.pdf and ongoing projects.

On a new approach of the Unadjusted Langevin Algorithm via convex optimization

Alain Durmus
ENS Paris-Saclay, France, alain.durmus@cmla.ens-cachan.fr

Szymon Majewski
Institute of Mathematics, Polish Academy of Sciences, Poland, smajewski@impan.pl

Błażej Miasojedow
Institute of Mathematics, Polish Academy of Sciences, Poland, bmiasojedow@impan.pl

We provide new insights on the Unadjusted Langevin Algorithm. We show that this method can be formulated as a first order optimization algorithm of an objective functional defined on the Wasserstein space of order 2. Using this interpretation and techniques borrowed from convex optimization, we give a non-asymptotic analysis of this method to sample from logconcave smooth target distribution on R^d. Our proofs are then easily extended to the Stochastic Gradient Langevin Dynamics, which is a popular extension of the Unadjusted Langevin Algorithm. Finally, this interpretation leads to a new methodology to sample from a non-smooth target distribution, for which a similar study is done.

Approximation of geometrically ergodic Metropolis-Hastings algorithms

Felipe Medina-Aguayo
Department of Mathematics and Statistics, University of Reading, United Kingdom,
f.j.medinaaguayo@reading.ac.uk

Daniel Rudolf
Institute for Mathematical Stochastics, Georg-August-Universität Göttingen, Germany,
daniel.rudolf@uni-goettingen.de

Nikolaus Schweizer
Department of Econometrics and OR, Tilburg University, The Netherlands, n.f.f.schweizer@uvt.nl

In this paper, we provide explicit, non-asymptotic perturbation bounds for Metropolis-Hastings algorithms when the likelihood in the acceptance probability is subject to an approximation error. This type of approximation error may arise when the likelihood of the target density is intractable. A prominent example is the Monte Carlo within Metropolis algorithm (MCWM) for latent variable models. We focus on settings where the associated unperturbed chain is geometrically ergodic (but not necessarily uniformly ergodic). Our bounds on the difference between the n-th step distributions of the perturbed and the unperturbed chains require two inputs: First, we need to control the error made in approximating the likelihood, at least in the center of the state-space. Second, we need to verify a stability condition on the perturbed chain, either by proving a Lyapunov-type condition, or by restricting the perturbed chain to the center of the state space.

Importance Sampling Type Estimators based on Approximate Marginal MCMC

Matti Vihola
Department of Mathematics and Statistics, University of Jyväskylä, Finland, matti.s.vihola@jyu.fi

Jouni Helske
Department of Science and Technology, LinkÃ¶ping University, Sweden, jouni.helske@liu.se

Jordan Franks
Department of Mathematics and Statistics, University of Jyväskylä, Finland, jordan.j.franks@jyu.fi

We consider importance sampling (IS) type weighted estimators based on Markov chain Monte Carlo (MCMC) targeting an approximate marginal of the target distribution. In the context of Bayesian latent variable models, the MCMC typically operates on the hyperparameters, and the subsequent weighting may be based on IS or sequential Monte Carlo (SMC), but allows for multilevel techniques as well. The IS approach provides a natural alternative to delayed acceptance (DA) pseudo-marginal/particle MCMC, and has many advantages over DA, including a straightforward parallelisation and additional flexibility in MCMC implementation. We discuss briefly general theory, including consistency, central limit theorems, and guarantees against DA alternative. We then discuss several applications of the methodology, and our experimental results, which are promising. They show that the IS type approach can provide substantial gains relative to an analogous DA scheme, and is often competitive even without parallelisation.

The talk is based on the preprints [1, 2] and some new material.

[1] M. Vihola, J. Helske and J. Franks. Importance Sampling Type Estimators based on Approximate Marginal MCMC. *arXiv:1609.02541*, 2016.

[2] J. Franks and M. Vihola. Importance Sampling and Delayed Acceptance via a Peskun Type Ordering. *arXiv:1706.09873*, 2017.

Friday (10:00–12:00, **Room: Salle 5**

MCQMC in Computer Graphics

Organizer(s): Alexander Keller **Chair: Alexander Keller**

Fragmented Radix Trees for Efficient Sampling of Discrete Probability Distributions

Nikolaus Binder
NVDIA, Berlin, Germany, nikolaus.binder@gmail.com

We compare different methods for sampling discrete probability distributions with regard to computational efficiency, memory overhead, and sampling efficiency. While the fastest methods unfortunately do not preserve distribution properties of quasi-Monte Carlo sets and sequences, others suffer from bad average and/or worst-case performance. We introduce a variant of the cutpoint method that can be constructed very efficiently in parallel, retains the distribution properties, and improves the worst case performance. We demonstrate various applications in photorealistic image synthesis.

Reversible Jump Metropolis Light Transport using Inverse Mappings

Wenzel Jacob
EPFL, Lausanne, Swizerland, wenzel.jakob@epfl.ch

The field of computer graphics is currently undergoing a substantial shift to physics-based algorithms that carefully simulate the interaction of light and matter to create synthetic images of stunning realism. These methods all incorporate the idea of searching for "light paths", i.e. trajectories that carry light from the light sources to a virtual camera, typically using a number of different sampling strategies to improve convergence. An unfortunate side effect of this approach is that the presence of multiple strategies impedes exploration of the domain due to the low success rate of inter-strategy transitions. I will review Markov Chain-based rendering methods and then discuss recent work that re-casts rendering in the Reversible Jump MCMC framework to address this problem.

Joint work with Benedikt Bitterli, Jan Novák, and Wojciech Jarosz

On the Usage of Control Variates for Monte Carlo Direct Illumination in Movie Rendering

Christophe Hery
Pixar Animation Studios, Emeryville, California, USA, chery@pixar.com

When synthesizing images by light transport simulation, the simulation of direct illumination often is the dominant component when rendering movies. Deadlines in the movie industry are tight and hence efficiency is paramount. For that reason, we will survey the background on easy and fast control coefficients for direct illumination and analyze their benefits with respect to convergence. We will compare the two main stream approaches of fixed budget sampling and progressive rendering. Finally, we address some issues with quasi-Monte Carlo methods, leading to the question whether they matter in our production environment.

Friday (10:00–12:00, **Room: Salle 6**

Monte Carlo for rare events

Organizer(s): Gerardo Rubino **Chair: Gerardo Rubino**

Sharp Bounds for the Reliability of a k-out-of-n System Under Dependent Failures Using Cutoff Phenomenon Techniques

Javiera Barrera
Universidad Adolfo Ibáñez, javiera.barrera@uai.cl

Guido Lagos
Universidad Adolfo Ibáñez, guido.lagos.barrios@gmail.com

In this work we consider the reliability of a network where link failures are correlated. We define the reliability as the probability of the network to be working at a given time instant. Our main contribution is a collection of results giving a detailed analysis of a non-trivial scaling regime for the probability of the network being working at a certain time, as the time and size of network scales. Here we consider that the network fails when there are no links working, or more generally when less than k-out-of-n edges are working (with k close to n) like in [2] and [5]. Our results allow to study the common-cause failure models describe in [3] on networks in a realistic, relevant, yet practical, fashion: it allows to capture correlated components in the network; it allows to estimate and give error bounds for the failure probabilities of the system; and at same time only needs to specify a reduced family of parameters. Moreover, our results for the k-out-of-n failure model allow to give new scaling regimes for the probabilistic behavior of the last-ordinals in the theory of extreme values for dependent tuples. The techniques are similar to those used to estimate the asymptotic convergence profile for ergodic Markov chains [1] or [4].

[1] J. Barrera and B. Ycart Bounds for Left and Right Window Cutoffs. *ALEA, Lat. Am. J. Probab. Math. Stat.*, 11 (2): 445–458, 2014.

[2] I. Bayramoglu and M. Ozkut. The Reliability of Coherent Systems Subjected to Marshall?Olkin Type Shocks. *EEE Transactions on Reliability,*, 64 (1): 435–443, 2015.

[3] U. Cherubini, F. Durante, and S. Mulinacci. *Marshall - Olkin Distributions-Advances in Theory and Applications: Bologna, Italy, October 2013, volume 141.* Springer, 2015.

[4] B. Lachaud and B. Ycart Convergence Times for Parallel Markov Chains. *Positive systems*, 169–176, 2006.

[5] T. Yuge, M. Maruyama, and S. Yanagi Reliability of a k-out-of-n System with Common-Cause Failures Using Multivariate Exponential Distribution. *Procedia Computer Science*, 96: 968?976, 2016.

The Multi–Level Creation Process in Flow Network Reliability Estimation

HEctor Cancela
Facultad de Ingenieria, UDELAR, Montevideo, Uruguay, cancela@fing.edu.uy

Leslie Murray
Facultad de Ingenieria, UNR, Rosario, Argentina, lmurray@fceia.unr.edu.ar

Gerardo Rubino
INRIA Rennes – Bretagne Atlantique, Rennes, France, gerardo.rubino@inria.fr

The Creation Process [1] is an algorithm that transforms a static network model into a dynamic one. It is the base of different variance reduction methods like Permutation Monte Carlo and C–Spectrum [2], both designed to make efficient reliability estimations on highly reliable networks. In the Creation Process all the links are failed at time $t = 0$, and they become operational (repaired) at independent and exponentially distributed times. If the rates of the exponentials are properly selected, every link is repaired at time $t = 1$ with a probability equal to its own single reliability in the static framework. As a consequence, at time $t = 1$ the nodes of interest are connected

with a probability equal to the network reliability. This algorithm is restricted to networks in which links can only be found in two states (failed or operational).

In [3] we proposed a method that makes efficient estimations of network reliability. This method is based on an application of Splitting over the Creation Process for models in which links can only be failed or operational. Attempting to extend such method to Stochastic Flow Networks —in which the states of the links are, in general, multiple valued— we modified the Creation Process into a generalization that we have called Multi–Level Creation Process. The method that we are introducing now is an application of Splitting over the Multi–Level Creation Process. It performs just like Splitting over the standard Creation Process in [3] but it allows to address the problem of multiple valued network links. This proposal solves efficiently the estimation of reliability in Stochastic Flow Networks.

[1] Tov Elperin, Ilya B. Gertsbakh, and Michael Lomonosov. Estimation of network reliability using graph evolution models. *IEEE Transactions on Reliability*, (5):572–581, 1991.

[2] Ilya Gertsbakh, Reuven Rubinstein, Yoseph Shpungin, and Radislav Vaisman. Permutational methods for performance analysis of stochastic flow networks. *Probability in the Engineering and Informational Sciences*, 28(1):21–38, 2014.

[3] Leslie Murray, Hector Cancela, and Gerardo Rubino. A Splitting algorithm for network reliability estimation. *IIE Transactions*, 45(2):177–189, 2013.

Studying Metabolic Networks Through Monte Carlo Simulations

Pablo Teixeira
Operations Research Department, Universidad de la Republica, Uruguay, `pteixeir@alumni.lmu.edu`

Hector Cancela
Operations Research Department, Universidad de la Republica, Uruguay, `cancela@fing.edu.uy`

Luis Acerenza
Systems Biology Laboratory, Universidad de la Republica, Uruguay, `aceren@fcien.edu.uy`

Constraint-based models of metabolic networks can give insight into the different cellular functions of a certain microorganism. By imposing a steady-state condition on them, these models define a flux space that is comprised of all possible functional states of the system, and this space can be analyzed through the use of Monte Carlo simulations [1]. One of the important characteristics of this space is its volume, which can arguably be directly related to the microorganism's repertoire of responses to adapt to different external conditions. Developing a method to compute this high dimensional hypervolume proves to be a challenging problem, since a standard Monte Carlo algorithm would not suffice for most models given the high dimensionality of the space and the fact that the relevant flux configurations only account for a small portion of this space, making it necessary to employ variance reduction methods. So far, we have successfully calculated volumes of the solution spaces of toy models of metabolism, and are in the process of developing a method to assess larger models, which would make a contribution to the already broad set of tools [2] that are used to obtain information from these linear programming models.

[1] N. Price. Uniform Sampling of Steady-State Flux Spaces: Means to Design Experiments and to Interpret Enzymopathies. *Biophysical Journal*, 87 (4): 2172–2186, 2004.

[2] N. Lewis. Constraining the Metabolic Genotype-Phenotype Relationship Using a Phylogeny of In Silico Methods. *Nature Reviews Microbiology*, 10 (4): 291–305, 2012.

Availability Estimation of Markovian Reliability Systems with Logistics via Cross-Entropy

Ajit Rai
ALSTOM (Saint-Ouen) & INRIA (Rennes), France, `ajit.rai@irisa.fr`

Bruno Tuffin
INRIA, Rennes, France, `bruno.tuffin@inria.fr`

Rene C. Valenzuela
ALSTOM TIS, Madrid, Spain, rene.valenzuela@alstomgroup.com

Gerardo Rubino
INRIA, Rennes, France, gerardo.rubino@inria.fr

Pierre Dersin
ALSTOM TIS, Saint-Ouen, France, pierre.dersin@alstomgroup.com

Urban passenger rail systems are large scale systems comprising highly reliable redundant structures and logistics (e.g., spares or repair personnel availability, inspection protocols, etc). To meet the strict contractual obligations, steady state unavailability of such systems needs to be accurately estimated as a measure of a solution's life cycle costs. We use Markovian Stochastic Petri Nets (SPN) models to conveniently represent the systems [1].

We propose a multi-level Cross-Entropy (CE) optimization scheme [2], where we exploit the regenerative structure in the underlying continuous time Markov chain (CTMC) and to determine optimal Importance Sampling (IS) rates in the case of rare events [3]. The CE scheme is used in a pre-simulation and applied to failure transitions of the Markovian SPN models only. The proposed method divides a rare problem into a series of less rare problems by considering increasingly rare component failures. In the first stage a standard regenerative simulation is used for non-rare system failures. At each subsequent stage, the rarity is progressively increased (by decreasing the failure rates of components) and the IS rates of transitions obtained from the previous problem are used at the current stage. The final pre-simulation stage provides a vector of IS rates that are optimized and are used in the main simulation. The experimental results showed *bounded relative error* (BRE) property as the rarity of the original problem increases, and as a consequence a considerable variance reduction and gain (in terms of work normalized variance).

[1] G. Ciardo, and K.S. Trivedi. A Decomposition Approach for Stochastic Reward Net Models. *Performance Evaluation*, 18(1): 37–59, 1993.

[2] R.Y. Rubinstein, and D.P. Kroese. *The Cross-Entropy Method: A Unified Approach to Combinatorial Optimization, Monte-Carlo Simulation and Machine Learning Information Science and Statistics, ISSN 1613-9011.* Springer Science & Business Media, 2004.

[3] G. Rubino, and B. Tuffin, eds. *Rare Event Simulation using Monte Carlo Methods.* John Wiley & Sons, 2009.

Friday (16:15–17:45, **Room: Salle 5**

Handling Discontinuities in QMC with Applications to Computational Finance

Organizer(s): Xiaoqun Wang **Chair: Xiaoqun Wang**

Problems in mathematical finance often involve discontinuities in their performance functions. It is known that discontinuities are detrimental to the accuracy of quasi-Monte Carlo (QMC) method. Handling discontinuities therefore becomes a challenging task for the use of QMC. Recently, there have been some progresses on handling discontinuities in QMC. This special session will mainly focus on the following topics: (i) the use of smoothing methods in QMC; (ii) the importance of dimension reduction methods combined with smoothing methods; (iii) the theoretical investigation as to exactly how much the convergence grate of QMC can be improved if using some smoothing methods.

OVERCOMING THE CHALLENGES IN QMC METHODS FOR COMPUTATIONAL FINANCE

Xiaoqun Wang
Department of Mathematical Sciences, Tsinghua University, China, xwang@math.tsinghua.edu.cn

High dimensionality and discontinuity are two outstanding challenges in computational finance, such as in the problems of pricing and hedging of complex financial derivatives. We show how these challenges can be overcome (at least partially) by a new method of simulating the underlying processes and by suitably smoothing the payoff functions via transformations or preintegration. We show that an improved convergence order can be obtained and the effective dimension can be reduced. Numerical experiments demonstrate that the proposed methods significantly increase the efficiency of QMC methods for pricing options and estimating the Greeks (or sensitivities).

AN IMPORTANCE SAMPLING-BASED SMOOTHING APPROACH FOR QUASI-MONTE CARLO SIMULATION OF DISCRETE BARRIER OPTIONS

Fei Xie
Department of Mathematical Sciences, Tsinghua University, China, xief13@mails.tsinghua.edu.cn

Zhijian He
School of Mathematics, South China University of Technology, China, hezhijian@scut.edu.cn

Xiaoqun Wang
Department of Mathematical Sciences, Tsinghua University, China, xwang@math.tsinghua.edu.cn

Handling discontinuities in financial engineering is a challenging task when using quasi-Monte Carlo method. This paper develops a smoothing approach based on importance sampling (IS) to remove multiple discontinuity structures sequentially arising from pricing discrete barrier options. The IS-based method yields an unbiased estimate with reduced variance. Furthermore, we find that the order of path generation matters the variance of the estimator. We closely analyze the optimal choice for the first generation step when pricing different kinds of knock-out barrier options under the Black-Scholes (BS) model and the Variance Gamma (VG) model. As confirmed by numerical experiments, the IS method combined with a proper path generation can reduce significantly the variance with improved rate of convergence. It is a good practice to choose optimally the first generation step in both models. In addition, we show that the effective dimension is greatly reduced by the combined method, explaining the superiority of the proposed procedure from another perspective. The IS-based method can be extended to general models with the Euler discretization provided that the transition distributions are known explicitly.

ON THE ERROR RATE OF CONDITIONAL QUASI-MONTE CARLO FOR DISCONTINUOUS FUNCTIONS

Zhijian He
School of Mathematics, South China University of Technology, Guangzhou, China, hezhijian@scut.edu.cn

This paper studies the rate of convergence for conditional quasi-Monte Carlo (QMC), which is a counterpart of conditional Monte Carlo. We focus on discontinuous integrands defined on the whole of $\Re^d$, which can be unbounded. Under suitable conditions, we show that conditional QMC not only has the smoothing effect (up to infinitely times differentiable), but also can bring orders of magnitude reduction in integration error compared to plain QMC. Particularly, for some typical problems in options pricing and Greeks estimation, conditional randomized QMC that uses n samples yields a mean error of $O(n^{-1+\epsilon})$ for arbitrarily small $\epsilon > 0$. As a by-product, we find that this rate also applies to randomized QMC integration with all terms of the ANOVA decomposition of the discontinuous integrand, except the one of highest order.

Abstracts in contributed sessions

Monday 10:00–12:00, **Room: Salle 7**

Nuclear applications

Chair: Gerardo Rubino

Study of Source Term Estimators in Coupled Finite-Volume/Monte-Carlo Methods for Plasma Edge Simulations in Nuclear Fusion

Bert Mortier
Computer Science, KU Leuven, Belgium, bert.mortier@cs.kuleuven.be

Martine Baelmans
Mechanical Engineering, KU Leuven, Belgium, martine.baelmans@kuleuven.be

Giovanni Samaey
Computer Science, KU Leuven, Belgium, giovanni.samaey@cs.kuleuven.be

In many applications, such as plasma edge simulation of a nuclear fusion reactor, a coupled PDE/kinetic description is required, which is usually solved with a coupled finite volume/Monte Carlo method. Different procedures have been proposed to estimate the source terms in the finite volume part that appear from the Monte Carlo part of the simulation. We present a systematic (analytical and numerical) comparison of the variance and computational cost of a coherent set of such estimation procedures. The comparison is based on an invariant imbedding procedure, in which systems of ordinary differential equations (ODEs) are derived that quantify the statistical error and computational cost of each estimator. We analyze in detail a scenario with forward-backward scattering in a one-dimensional slab. For this test case, we perform a parametric study of the expected statistical error and computational cost by numerically solving the ODEs, revealing the non-trivial dependence of the optimal choice of estimator on the problem parameters. We provide numerical experiments that extend the discussion to a one-dimensional slab with realistic scattering, representative for a single finite volume grid cell of the coupled code. Our results provide a means to select appropriate Monte Carlo estimation procedures throughout the, potentially strongly varying, domain.

Monte Carlo simulation of multiple particle spectra with energy and momentum conservation

Dmitry Savin
Dukhov Research Institute of Automatics, sd57@inbox.ru

Mikhail Kosov
Dukhov Research Institute of Automatics, Kosov@vniia.ru

We are developing CHIPS-TPT [1, 2] physics library for exclusive Monte Carlo simulation of neutron-nuclear reactions below 20 MeV. Exclusive modeling reproduces each separate scattering and thus requires conservation of energy, momentum and quantum numbers in each reaction. Inclusive modeling reproduces only selected values while averaging over the others and imposes no such constraints. Therefore the exclusive modeling allows to simulate additional quantities like secondary particle correlations and gamma-lines broadening and avoid artificial fluctuations. CHIPS-TPT is based on the formerly included in Geant4 CHIPS library, which follows the exclusive approach, and extends it to incident neutrons with the energy below 20 MeV. Unfortunately, the available data in this energy region is mostly presented in ENDF-6 format and semi-inclusive: while the cross sections are usually split in different channels, the secondary distributions are given for each secondary particle independently.

Independent randomization of secondary particle energies leads to violation of the conservation laws because of the possibility that the sum of randomized energies lies higher than the kinematic threshold. Simply discarding such pairs modifies the resulting spectrum which no longer reproduces the one given in the database. This is the reason why one needs to construct an energy distribution for the second particle for each fixed energy of the first particle. To narrow the problem we find the distributions as a family of functions that differ by the kinematic

thresholds only. We develop a procedure to construct these functions. For the third and subsequent particle the previous particles are treated as an effective "first" particle. We show that the spectra given in the databases are reproduced and that energy and momentum are strictly conserved in each interaction.

[1] M. Kosov and D. Savin. New exclusive CHIPS-TPT algorithms for simulation of neutron-nuclear reactions. *J. Phys. Conf. Ser.*, 608(1):012050, 2015.

[2] Savin, Dmitry and Kosov, Mikhail Exclusive data-based modeling of neutron-nuclear reactions below 20 Mev. *EPJ Web Conf.*, 146:12028, 2017.

A Batch on Patch Parallel Scheme in Monte Carlo Particle Transport Program

Zhicheng Ji
Institute of Applied Physics and Computational Mathematics, China, ji_zc@139.com

Ruihong Wang
Institute of Applied Physics and Computational Mathematics, China, wang_ruihong@iapcm.ac.cn

Modern parallel computer has made the large scale simulation of particle transport problems using Monte Carlo method possible, but also brought big challenge in code programming. This paper shows a “Batch on Patch" parallel computing scheme in Monte Carlo particle transport program. Then an allocate method of random numbers and an addition method of tallies is illustrated for getting consistent outcomes while using different number of parallel processes.

High Precision Shielding Calculation For Qinshan-I Reactor Model With Monte Carlo Particle Transport Code JMCT

Gang Li
Beijing Institute of Applied Physics and Computational Mathematics, China, lgseven@126.com

Li Deng
Beijing Institute of Applied Physics and Computational Mathematics, China, deng_li@iapcm.ac.cn

BaoYin Zhang
Beijing Institute of Applied Physics and Computational Mathematics, China, zby@iapcm.ac.cn

Rui Li
CAEP Software Center of High Performance Numerical Simulation, China, li_rui@iapcm.ac.cn

DunFu Shi
CAEP Software Center of High Performance Numerical Simulation, China, 254821416@qq.com

Yan Ma
Beijing Institute of Applied Physics and Computational Mathematics, China, mayan2301@126.com

LingYu Zhang
CAEP Software Center of High Performance Numerical Simulation, China, zhanglingyu09@163.com

YuanGuang Fu
CAEP Software Center of High Performance Numerical Simulation, China, fyg1989@126.com

JMCT(J Monte Carlo Transport code) is a general 3D Monte Carlo particle transport simulation software. It is developed by Institute of Applied Physics and Computational Mathematics (IAPCM) and CAEP Software Center for High Performance Numerical Simulation(CAEP-SCNS) in China, and has its own intellectual property. The aim of JMCT is providing high precision numerical simulation tool based on high performance parallel computers for the demand of high resolution numerical simulations of particle transport in nuclear field. Now JMCT includes: coupled transport of neutrons and gammas through a background medium/geometry, supporting for both multi-group and continuous energy treatment of cross sections, static keff eigenvalue calculations for criticality problems, neutron and gamma shielding calculation, running on a variety of large scale, parallel computing platforms. Especially there is a visual, interactive modeling tool JLAMT(J Large-scale Auto Modeling Tool) developed for JMCT code, which can help to build geometry models easily and quickly. In this paper, JMCT is used to simulated neutron fluxes of two points in irradiation monitoring pipe and four points in one main pipe of the Qinshan-I reactor in China. The results of JMCT are compared with the measured data.).

Monday 14:45–15:45, **Room: Amphi 3**

Improving MC and QMC integration

Chair: Sergei Kucherenko

Density estimation by randomized quasi-Monte Carlo

Amal Ben Abdellah
DIRO, Université de Montréal, Canada, amalbenabdellah@gmail.com

Pierre L'Ecuyer
DIRO, Université de Montréal, Canada, lecuyer@iro.umontreal.ca

Art B. Owen
Department of Statistics, Stanford University, USA, owen@stanford.edu

Florian Puchhammer
DIRO, Université de Montréal, Canada, florian.puchhammer@umontreal.ca

Randomized quasi-Monte Carlo (RQMC) is commonly used to estimate the mathematical expectation of a random variable expressed as an integral over the s-dimensional unit cube $(0,1)^s$. Under certain conditions the RQMC estimator converges faster than the crude Monte Carlo (MC) estimator. In this talk we examine how RQMC can improve the convergence rate of the mean integrated square error when estimating the density of a random variable X, defined as a function over $(0,1)^s$, in comparison to MC. We provide both theoretical and empirical results on the convergence rates of density estimators defined by histograms, averaged shifted histograms, and kernel density estimators, when the observations are generated via RQMC. We also investigate the combination of RQMC with a *conditional Monte Carlo* approach to density estimation, in which each observation of X is replaced by a conditional density given less information, and the estimator is the average of those conditional densities.

Variance reduction in Monte Carlo integration via function approximation

Yuji Nakatsukasa
National Institute of Informatics, Japan, nakatsukasa@nii.ac.jp

This work revisits Monte Carlo integration from a numerical analyst's viewpoint.

Classical algorithms for numerical integration (quadrature/cubature) follow the guiding principle: approximate the integrand with a simple function (e.g. a polynomial), and integrate the approximant exactly. In high-dimensional integration, such methods quickly become infeasible due to the curse of dimensionality.

Monte Carlo integration (MC) is a widely-used alternative, which simply takes the average of random samples, improving the estimate as more and more samples are taken. The main issue with MC is its slow "$\sqrt{\text{variance}/N}$" convergence, and various techniques have been proposed to reduce the variance.

In this work we reveal a numerical analyst's interpretation of MC: it approximates the integrand with a constant function, and integrates the constant exactly. This observation leads naturally to MC-like methods that combines MC with (high-dimensional) function approximation theory, including polynomial approximation, sparse grids, sparse approximation and low-rank approximation. The resulting method can be regarded as another variance reduction technique for Monte Carlo.

Monday 14:45–15:45, **Room: Salle 6**

Practice of QMC methods (1)

Chair: Friedrich Pillichsammer

Efficient usage and construction of QMC methods

Adrian Ebert
NUMA, KU Leuven, Belgium, adrian.ebert@cs.kuleuven.be

Dirk Nuyens
NUMA, KU Leuven, Belgium, dirk.nuyens@cs.kuleuven.be

Peter Kritzer
Johann Radon Institute for Computational and Applied Mathematics (RICAM), Austria,
peter.kritzer@oeaw.ac.at

The success of quasi-Monte Carlo rules is based on the variety of different methods and point sets, and thus grants flexibility which method to choose for a particular problem. The efficient construction and implementation of quasi-Monte Carlo methods is therefore of high interest in numerical integration. In this talk we review popular construction schemes for QMC methods ranging from digital nets and sequences to lattice rules, focusing on the efficient implementation in a compiled language such as C++. On the one hand, we focus on digital nets and sequences including randomization techniques such as nested uniform and affine matrix scrambling, see [5] and [3], respectively, as well as interlacing of digital nets. For efficiency reasons, these constructions are implemented for the base 2 case. On the other hand, we consider construction algorithms for lattice rules and polynomial lattice rules in weighted reproducing kernel Hilbert spaces. This includes component-by-component (CBC) constructions as well as the successive coordinate search construction, see [2], which can be seen as a generalization of the CBC method. For the latter we present bounds on the worst-case error in the considered function spaces and introduce a reduced version in the spirit of [1] to allow for an even faster construction. Implementations of the presented construction methods are publicly accessible via [4].

[1] J. Dick, P. Kritzer, G. Leobacher, F. Pillichshammer, A reduced fast component-by-component construction of lattice points for integration in weighted spaces with fast decreasing weights. *J. Comput. Appl. Math., Volume 276, 1–15.* 2015

[2] A. Ebert, H. Leövey, D. Nuyens, Successive Coordinate Search and Component-by-Component Construction of Rank-1 Lattice Rules. *In Glynn, P. (Ed.), Owen, A. (Ed.), Monte Carlo and Quasi-Monte Carlo Methods 2016.* Springer, 2018.

[3] J. Matoušek, Geometric Discrepancy: An Illustrated Guide. Springer, 1998.

[4] D. Nuyens, Magic Point Shop. [online] Available at:
https://people.cs.kuleuven.be/~dirk.nuyens/qmc-generators/
and https://bitbucket.org/dnuyens/qmc-generators

[5] A. B. Owen, Randomly permuted (t, m, s)-nets and (t, s)-sequences. *In Monte Carlo and Quasi-Monte Carlo Methods in Scientific Computing (H. Niederreiter and P. J.-S. Shiue, eds.) 299–317.*, Springer, 1995.

Probabilistic discrepancy bounds for Latin hypercube sampling with and without padding

Michael Gnewuch
Mathematisches Seminar, Christian-Albrechts-Universität Kiel, Germany, gnewuch@math.uni-kiel.de

Nils Hebbinghaus
Institut für Informatik, Christian-Albrechts-Universität Kiel, Germany, hebbing@gmx.de

Discrepancy measures are commonly used to quantify how uniformly a sample is distributed in the d-dimensional unit cube. Discrepancy theory is, e.g., intimately related to numerical integration and provides the theoretical foundation for quasi-Monte Carlo methods. A very important discrepancy measure is the star discrepancy.

In this talk we consider the star discrepancy, discuss its relation to numerical integration and the known upper and lower bounds for it. Here we focus on probabilistic bounds where the dependence on the sample size and the dimension d is explicitly stated. Such bounds are, e.g., important for high-dimensional and infinite-dimensional numerical integration.

We provide upper discrepancy bounds for Latin hypercube samples that match the probabilistic lower discrepancy bounds from the recent paper [1]. We discuss extensions of our result to padding Latin hypercube samples by Monte Carlo and some other types of sampling.

[1] B. Doerr, C. Doerr, M. Gnewuch. Probabilistic lower discrepancy bounds for Latin hypercube samples. *To appear in: J. Dick, F. Y. Kuo, H. Woźniakowski (Eds.), Contemporary Computational Mathematics – a Celebration of the 80th Birthday of Ian Sloan, Springer-Verlag*, 2018.

Monday 14:45–15:45, **Room: Salle 7**

Feynman-Kac

Chair: Arnaud Guyader

The Numerical Approximation Of Polynomial-Growth Backward Stochastic Differential Equations

Arnaud Lionnet
CMLA, Ecole Normale Supérieure de Cachan, France, arnaud.lionnet@cmla.ens-cachan.fr

Backward Stochastic Differential Equations (BSDEs) provide a systematic way to obtain Feynman–Kac formulas for linear as well as nonlinear partial differential equations (PDEs) of parabolic and elliptic type, and the numerical approximation of their solutions thus provide Monte-Carlo methods for PDEs. BSDEs are also used to describe the solution of path-dependent stochastic control problems, and they further arise in many areas of mathematical finance.

In this talk, I will discuss the numerical approximation of BSDEs when the nonlinear driver is not Lipschitz, but instead has polynomial growth and satisfies a monotonicity condition. The results presented cover and extend those found in [1], [2], [3], [4]. The time-discretization is a crucial step, as it determines whether the full numerical scheme is stable or not. Unlike for Lipschitz driver, while the implicit Bouchard–Touzi–Zhang scheme is stable, the explicit one is not and explodes in general. I will then present a number of remedies that allow to recover a stable scheme, while benefiting from the reduced computational cost of an explicit scheme. I will also discuss the issue of numerical stability and the qualitative correctness which is enjoyed by both the implicit scheme and the modified explicit schemes. Finally, I will discuss the approximation of the expectations involved in the full numerical scheme, and their analysis when using a quasi-Monte Carlo method.

[1] Arnaud Lionnet, Gonçalo dos Reis, and Lukasz Szpruch. Time discretization of FBSDEs with polynomial growth drivers and reaction-diffusion PDEs. *Annals of Applied Probability*, 25(5):2563–2625, 2015.

[2] Arnaud Lionnet, Gonçalo dos Reis, and Lukasz Szpruch. Convergence and stability of modified explicit schemes for BSDEs with polynomial growth drivers. *Annals of Applied Probability*, to appear.

[3] Arnaud Lionnet. An explicit scheme with adapted time-steps for monotone BSDEs. *Submitted*.

[4] Arnaud Lionnet, Gonçalo dos Reis, and Lukasz Szpruch. Full-projection explicit scheme for polynomial growth BSDEs. *Submitted*.

Stochastic methods for solving partial differential equations in high dimension

Marie Billaud-Friess
Laboratoire de Mathématiques Jean Leray, Centrale Nantes, France, marie.billaud-friess@ec-nantes.fr

In this talk, we present a probabilistic approach to solve high-dimensional partial differential equations (PDEs). The idea is to combine estimation of pointwise evaluations of the solution together with interpolation in the lines of [2, 3].
Feynman-Kac representation allows to interpret the solution $u(x)$ of a given PDE, at point x, as the expectation of a functional of a stochastic process X^x, solution of the Stochastic Differential Equation related to the PDE, starting from x. In practice, a Euler-Maruyama scheme is used together with a Monte-Carlo method to estimate this expectation, which yields an approximation $\tilde{u}(x)$ of $u(x)$. Then, an adaptive sparse interpolation method [1] is used to construct a global approximation $\tilde{u}$ of u from a few pointwise evaluations. Here, the interpolation is associated with a sparse grid obtained by tensorization of unidimensional grids made of magic points associated with given univariate functional bases [4]. The convergence of this standard approach with respect to time discretization of the stochastic process and the number of Monte-Carlo samples is very slow. To overcome this issue, we use a sequential control variate algorithm [2, 3] (where the current approximation $\tilde{u}$ is used to create a control variate for the next iteration) to compute successive pointwise evaluations of $u(x)$ with bias and variance decreasing geometrically with the number of steps of the algorithm.
Some numerical example will illustrate the behavior of the proposed method for the solution of high-dimensional elliptic problems.

This is a joint work with A. Macherey (LMJL, Centrale Nantes / LJK, Université Grenoble Alpes, Inria AIRSEA), A. Nouy (LMJL, Centrale Nantes), and C. Prieur (LJK, Université Grenoble Alpes, Inria AIRSEA).

[1] A. Chkifa, A. Cohen, and C.Schwab. High-dimensional adaptive sparse polynomial interpolation and applications to parametric PDEs, Found. Comput. Math., 14 (2014), pp. 601–633. 5

[2] E. Gobet and S. Maire. A spectral Monte Carlo method for the Poisson equation, Monte Carlo Methods Appl., 10 (2004), pp. 275–285.

[3] E. Gobet and S. Maire. Sequential control variates for functionals of Markov processes, SIAM Journal on Numerical Analysis, 43 (2009), pp. 1256–1275. 3

[4] Y. Maday, N. C. Nguyen, A. T. Patera, and G. S. H. Pau. A general multipurpose interpolation procedure: The magic points, Commun. Pure Appl. Anal., 8 (2009), pp. 383–404. 46

Monday 16:15–17:45, **Room: Amphi 3**

Markov Chain QMC

Chair: Bruno Tuffin

An Implementation of Short-Period Tausworthe Generators for Markov Chain quasi-Monte Carlo Methods

Shin Harase
College of Science and Engineering, Ritsumeikan University, Japan, harase@fc.ritsumei.ac.jp

We want to estimate $E_\pi[f(X)]$ using Markov chain Monte Carlo (MCMC) methods for a probability distribution π and to improve the accuracy by replacing IID uniform random points with quasi-Monte Carlo points. For this, Chen et al. [1] proved that MCMC remains consistent when the driving sequences are completely uniformly distributed (CUD). Here, the (infinite) sequence $u_i \in [0,1]$ for $i \geq 1$ is CUD, if

$$\lim_{n\to\infty} D_n^{*s}\left((u_1,\ldots,u_s),(u_2,\ldots,u_{s+1}),\ldots,(u_n,\ldots,u_{s+n-1})\right) = 0$$

holds for every integer $s \geq 1$, where D_n^{*s} is the star discrepancy. To construct such sequences approximately, Chen et al. [2] implemented short-period Tausworthe generators (i.e., linear feedback shift register generators) optimized in terms of the equidistribution property, which is a criterion used in pseudorandom number generation.

In this talk, we design good short-period Tausworthe generators in terms of the t-value of (t,m,s)-nets. It is known that Tausworthe generators can be viewed as polynomial Korobov lattice rules with a denominator $p(x)$ and a numerator $q(x)$ over $GF(2)$. For dimension $s = 2$, there is a connection between the t-value and continued fractions, that is, the t-value is optimal (i.e., $t = 0$) if and only if the partial quotients in the continued fraction of $q(x)/p(x)$ are all of degree one. Following the definition of CUD sequences, we want to search parameters whose t-values are optimal for $s = 2$ and as small as possible for $s \geq 3$. Tezuka and Fushimi [3] proposed an algorithm for finding such parameters by using the polynomial arithmetic analogue of Fibonacci numbers. Thus, we modify their algorithm and search the parameters again. In addition, we report some numerical experiments in MCMC.

[1] S. Chen, J. Dick, and A. B. Owen. Consistency of Markov chain quasi-Monte Carlo on continuous state spaces. *Ann. Statist.,* 39 (2): 673–701, 2011.

[2] S. Chen, M. Matsumoto, T. Nishimura, and A. B. Owen. New inputs and methods for Markov chain quasi-Monte Carlo. *Monte Carlo and quasi-Monte Carlo methods 2010*: 313–327, Springer Proc. Math. Stat., 23, Springer, Heidelberg, 2012.

[3] S. Tezuka and M. Fushimi. Calculation of Fibonacci polynomials for GFSR sequences with low discrepancies. *Math. Comp.,* 60 (202): 763–770, 1993.

Adaptive Importance Sampling for Markov Chain Quasi-Monte Carlo

Tobias Schwedes
Department of Mathematics, Imperial College London, UK, `t.schwedes14@imperial.ac.uk`

Ben Calderhead
Department of Mathematics, Imperial College London, UK, `b.calderhead@imperial.ac.uk`

Quasi-Monte Carlo (QMC) methods for estimating integrals are attractive since the resulting estimators often converge at a faster rate than pseudo-random Monte Carlo. However, they can be difficult to set up on arbitrary posterior densities within the Bayesian framework, in particular for inverse problems. We consider the use of completely uniformly distributed QMC numbers, previously applied in the context of standard Markov Chain Monte Carlo methods in [3, 2], within the general parallel MCMC framework developed in [1]. In each MCMC iteration, multiple proposals are generated according to a kernel which is updated adaptively depending on previous iterations. Suitably weighting proposals gives rise to an adaptive importance sampling method, for which we prove consistency. It is demonstrated numerically that the resulting approach scales close to N^{-1} as we increase parallelisation instead of $N^{-1/2}$ as is usual for standard Monte Carlo. For Bayesian inverse problems of dimensionality up to 25, including highly correlated Gaussian posteriors and real-world Bayesian logistic regression problems, we observe up to 2 orders of magnitude improvement compared with pseudo-random methods.

[1] Calderhead, Ben. A general construction for parallelizing Metropolis-Hastings algorithms. *Proceedings of the National Academy of Sciences*, 111 (49): 17408–17413, 2014.

[2] Chen, S and Dick, Josef and Owen, Art B. Consistency of Markov chain quasi-Monte Carlo on continuous state spaces. *The Annals of Statistics*, 673–701, 2011.

[3] Owen, Art B and Tribble, Seth D. A quasi-Monte Carlo Metropolis algorithm. *Proceedings of the National Academy of Sciences*, 102 (25): 8844–8849, 2005.

[4] Roberts, Gareth O and Rosenthal, Jeffrey S. Coupling and ergodicity of adaptive Markov chain Monte Carlo algorithms. *Journal of Applied Probability*, 44 (2): 458–475, 2007.

Sudoku Sampling For Markov Chains Simulation

Rami El Haddad
UR Mathématiques, Université Saint-Joseph, Liban, rami.haddad@usj.edu.lb

Joseph El Maalouf
UR Mathématiques, Université Saint-Joseph, Liban, maalouf.joseph1@hotmail.com

Christian Lécot
LAMA, Université Grenoble Alpes, France, christian.lecot@univ-smb.fr

Pierre L'Ecuyer
DIRO, Université de Montréal, Canada, lecuyer@iro.umontreal.ca

Markov chains modelling is used in many domains of science and technology. We are interested in discrete time Markov chains on discrete state spaces. In most interesting situations, matrix-based numerical methods would require huge memory capacity and computational time, so that Monte Carlo simulations provide valuable alternatives.

In order to improve simulation efficiency, a technique was introduced in [4] in the context of quasi-Monte Carlo methods. A large number N of samples are drawn from the initial distribution and the states evolve according to the Markov chain. The difference with usual Monte Carlo simulation is that two-dimensional quasi-random points are used in place of one-dimensional pseudo-random points, and the N copies of the chain are reordered at each time step. A version of the scheme, using randomized points, was analyzed in [2] and further exemplified in [3].

A stratified sampling technique was presented in [1]. A sample of N points is as follows: (1) if the unit square is uniformly divided into N equal sub-squares, one point lies in each sub-square, and (2) for each axis, the N projections of the points are distributed in each of the N subintervals that uniformly divide the unit interval. This approach was called *Sudoku sampling*. In this presentation, we prove that the variance of the Monte Carlo simulation of a Markov chain on a one-dimensional discrete state space using Sudoku sampling is of order $\mathcal{O}(N^{-3/2})$ when N chains are simulated in parallel. In addition, the variance reduction is analyzed on the results of numerical tests.

[1] R. El Haddad, R. Fakhereddine, C. Lécot, G. Venkiteswaran. Extended Latin hypercube sampling for integration and simulation. *Monte Carlo and Quasi-Monte Carlo Methods 2012*. Springer, 2013.

[2] P. L'Ecuyer, C. Lécot, B. Tuffin. A randomized quasi-Monte Carlo simulation method for Markov chains. *Operations Research*, 56: 958–975, 2008.

[3] P. L'Ecuyer, D. Munger, C. Lécot, B. Tuffin. Sorting methods and convergence rates for Array-RQMC: Some empirical comparisons. *Mathematics and Computers in Simulation*, 143: 191–201, 2018.

[4] C. Lécot, B. Tuffin. Quasi-Monte Carlo methods for estimating transient measures of discrete time Markov chains. *Monte Carlo and Quasi-Monte Carlo Methods 2002*. Springer, 2004.

Monday 16:15–17:45, **Room: Salle 7**

MCMC and large size

Chair: Alain Durmus

Scalable MCMC for Bayes Shrinkage Priors

Paulo Orenstein
Statistics, Stanford University, USA, pauloo@stanford.edu

James Johndrow
Statistics, Stanford University, USA, johndrow@stanford.edu

Anirban Bhattacharya
Statistics, Texas A&M University, USA, anirbanb@stat.tamu.edu

Gaussian scale mixture priors are frequently employed in Bayesian analysis of high-dimensional models, and a theoretical literature exists showing optimal risk properties of several members of this family when the truth is sparse. While optimization-based algorithms for the extremely popular Lasso and elastic net procedures can scale to dimension in the hundreds of thousands, corresponding Bayesian methods that use Markov chain Monte Carlo (MCMC) for computation are limited to problems at least an order of magnitude smaller. This is due to high computational cost per step of the associated Markov kernel and decay to zero of the convergence rate of the Markov chain as dimension grows. Here we propose an MCMC algorithm for computation in these models that combines block updating and approximations of the Markov kernel to directly combat both of these factors. Our algorithm dominates the best existing alternatives in both computational cost per step and convergence properties in high-dimensional applications. We give theoretical guarantees for the accuracy of the kernel approximation. The scalability of the algorithm is illustrated in simulations and applications with up to 100,000 predictors.

Scaling MCMC to Large Problem Sizes

James Johndrow
Department of Statistics, Stanford University, USA, johndrow@stanford.edu

Anirban Bhattacharya
Department of Statistics, Texas A&M University, USA, anirbanb@tamu.edu

Paulo Orenstein
Department of Statistics, Stanford University, USA, pauloo@stanford.edu

Jonathan Mattingly
Department of Mathematics, Duke University, USA, jonm@math.duke.edu

A common complaint among applied statisticians and researchers is that MCMC is "too slow" for many applications, limiting the relevance of Bayesian inference in high dimensional and large sample problems. We choose two challenging applications – high-dimensional regression models and crossed design random effects models – to illustrate the challenges faced by generic algorithms, and propose solutions that scale to the required problem size. In both cases, our solutions combine the use of approximations of expensive numerical linear algebra to reduce computational cost per step and block updating/explicit integration to reduce autocorrelation. We give some general results on the bias induced by use of approximations in MCMC, and show that our approximations meet the required conditions. We argue that the failure of MCMC to keep pace with algorithms commonly used in frequentist statistics can be partly overcome by increased use of approximations and reduced reliance on conditional conjugacy and Gibbs sampling.

Tuesday 14:45–15:45, **Room: Amphi 3**

Acceleration of MCMC

Chair: Hector Cancela

Sequential Tempered Markov Chain Monte Carlo for Bayesian Inference

Thomas Catanach
Computing and Information Science, Sandia National Laboratories, USA, tacatan@sandia.gov

Bayesian methods are critical for capturing our uncertainty about a system's behavior using probability distributions and for updating this understanding as new information becomes available. This uncertainty can then be incorporated into the prediction and decision-making processes. While Bayesian methods are very useful, they are often computationally intensive and can be difficult to tune. This necessitates the development of better

algorithms and methods. In this work, we discuss a group of population Markov Chain Monte Carlo (MCMC) methods that we refer to as Sequential Tempered MCMC (ST-MCMC) algorithms. ST-MCMC algorithms are a form of Sequential Monte Carlo[1, 2] and include algorithms like Transitional Markov Chain Monte Carlo[3] and Subset Simulation[4]. In general, these algorithms combine 1) a notion of tempering to gradually transform a population of samples from the prior to the posterior through a series of intermediate distributions, 2) importance resampling to rebalance the sample population when moving between intermediate distributions, and 3) MCMC to explore the intermediate distributions and decorrelate the sample population. ST-MCMC methods can be used to solve many Bayesian uncertainty quantification problems including parameter estimation, model selection, and posterior reliability analysis for estimating the probability of rare events, such as failures in a system.

Beyond motivating the importance of ST-MCMC methods, we will discuss the tuning and implementation of these algorithms to increase their performance and robustness. We can adaptively tune the proposal distribution for each intermediate distribution level to better suit the global structure of the distribution, which we learn from the sample population. Based on information from how the samples evolve through the intermediate distributions, we can tune the tempering rate and MCMC chain lengths to maintain a high effective sample size throughout the algorithm. These results make ST-MCMC easy to use and effective when solving Bayesian inference and uncertainty quantification problems.

[1] Pierre Del Moral, Arnaud Doucet, and Ajay Jasra. Sequential monte carlo samplers. *Journal of the Royal Statistical Society. Series B (Statistical Methodology)*, 68(3): 411–436, 2006.

[2] Nikolas Kantas, Alexandros Beskos, and Ajay Jasra. Sequential monte carlo methods for high-dimensional inverse problems: A case study for the navier-stokes equations. *SIAM/ASA Journal on Uncertainty Quantification*, 2(1): 464–489, 2014.

[3] Jianye Ching and Yi-Chu Chen. Transitional markov chain monte carlo method for bayesian model updating, model class selection, and model averaging. *Journal of Engineering Mechanics*, 133(7): 816–832, 2007.

[4] Siu-Kui Au and James L. Beck. Estimation of small failure probabilities in high dimensions by subset simulation. *Probabilistic Engineering Mechanics*, 16(4): 263–277, 2001.

Fast Maximum Likelihood estimation via Equilibrium Expectation for large network data

Maksym Byshkin
Universita della Svizzera italiana, Switzerland, `maksym.byshkin@usi.ch`

Alex Stivala
Universita della Svizzera italiana, Switzerland, `alex.d.stivala@gmail.com`

Antonietta Mira
Universita della Svizzera italiana, Switzerland, `antonietta.mira@usi.ch`

Alessandro Lomi
Universita della Svizzera italiana, Switzerland, `alessandro.lomi@usi.ch`

Network data may be analyzed by constructing statistical models that accurately reproduce the structural properties that may be of theoretical relevance or empirical interest. Examples of such models are Exponential Random Graph Models for social networks and Markov Random Field for image processing. Typically, Markov chain Monte Carlo (MCMC) methods are used when normalizing constants of statistical models cannot be computed. We improve on a recently developed Auxiliary Parameter Markov Chain Monte Carlo method [1] and propose a new MCMC approach for the Maximum Likelihood Estimation (MLE) of parameters of statistical models from exponential family. The existing MCMC approaches for parameter estimation (Bayesian, MCMCMLE of Geyer and Thompson [2], and stochastic approximation for Method of Moments [3]) require many MCMC simulations until convergence. The approach we propose does not require many converged simulations and, in a result, is much less computationally expensive. It relies on properties of Markov chains at equilibrium and, for this reason, we call it Equilibrium Expectation (EE). Using this approach we design a simple and efficient algorithm to find the MLE when it exists and is unique. The EE algorithm is similar to the Metropolis-Hastings algorithm, but allows MCMC simulation to be performed while constraining the desired networks properties.

We demonstrate the performance of the EE algorithm in the context of Exponential Random Graph Models (ERGMs) - a family of statistical models for network data. The EE algorithm is first tested on simulated networks. We compute bias and variance of the estimates and show that the estimates obtained with the proposed method are not less accurate than those obtained with stochastic approximation. Thus far, the lack of efficient computational methods has limited the empirical scope of ERGMs to relatively small networks with a few thousand nodes. Good scaling properties of the EE algorithm allow a dramatic increase in the size of networks that may be analyzed with ERGMs. This is illustrated in an analysis of several biological and one social network with 104,103 nodes.

[1] Byshkin, M., Stivala, A., Mira, A., Krause, R., Robins, G., Lomi, A. Auxiliary Parameter MCMC for Exponential Random Graph Models *Journal of Statistical Physics* 165(4), 740-754, 2016.

[2] Geyer, C. J., Thompson, E.A. Constrained Monte Carlo maximum likelihood for dependent data *Journal of the Royal Statistical Society. Series B (Methodological)* 657-699, 1992.

[3] Snijders, T.A. Markov chain Monte Carlo estimation of exponential random graph models *Journal of Social Structure* 3(2), 1-40, 2002.

Tuesday 14:45–15:45, **Room: Salle 6**

MC in finance

Chair: Gilles Pagès

MONTE CARLO PATHWISE SENSITIVITIES FOR BARRIER OPTIONS

Thomas Gerstner
Department of Mathematics, Goethe University Frankfurt, Germany, gerstner@math.uni-frankfurt.de

Bastian Harrach
Department of Mathematics, Goethe University Frankfurt, Germany, harrach@math.uni-frankfurt.de

Daniel Roth
Department of Mathematics, Goethe University Frankfurt, Germany, roth@math.uni-frankfurt.de

The Monte Carlo pathwise sensitivities approach is well established for smooth payoff functions. In this work, we present a new Monte Carlo algorithm that is able to calculate the pathwise sensitivities for discontinuous payoff functions. Our main tool is the one-step survival idea of Glasserman and Staum. As an application we use the results for a two-dimensional calibration of a CoCo-Bond, which we model with different types of discretely monitored barrier options.

Detecting the Presence of Jumps in Option Prices

Warren Volk-Makarewicz
Research Group Area Operations Research and Management, RWTH Aachen University, Germany, wm.volk.makarewicz@ada.rwth-aachen.de

Svetlana Borovkova
Department of Finance, Vrije Universiteit Amsterdam, Netherlands, s.a.borovkova@vu.nl

Bernd Heidergott
Department of Econometrics and Operations Research, Vrije Universiteit Amsterdam, Netherlands, b.f.heidergott@vu.nl

In risk management, model risk is the source of error due to misspecified capital requirements, especially in adverse market conditions. A question that relates to model building is whether it is beneficial to incorporate additional detail in the asset price processes. This research considers a simplified scenario on the benefit to include a compound Poisson process, with a specified jump size distribution, to a diffusion-based asset model to price European options.

To determine whether these jumps are present, we develop a hypothesis test utilizing the simulated asset price paths of the competing models. Our hypothesis test is based on stochastic gradient estimation to evaluate the sensitivity on the option value due to modifying the Markov Chain representation of the diffusion price path.

Compared to the t-Test for equal means, the proposed hypothesis test is more sensitive concerning the detection of price jumps in scenarios where either volatility or jump behaviour is dominant. This inference is due to the increased probability of rejecting the null hypothesis given a fixed number of sample paths in computing both estimators.

This is joint work with Svetlana Borovkova and Bernd Heidergott.

Tuesday 14:45–15:45, **Room: Salle 7**

Practice of QMC methods (2)

Chair: Gunther Leobacher

Rank-1 lattices and higher-order exponential splitting for the multi-dimensional time-dependent Schrödinger equation

Yuya Suzuki
NUMA, KU Leuven, Belgium, yuya.suzuki@cs.kuleuven.be

Dirk Nuyens
NUMA, KU Leuven, Belgium, dirk.nuyens@cs.kuleuven.be

Gowri Suryanarayana
Energy Ville Genk and Vito Mol, Belgium, gowrisurya@gmail.com

We study approximating the solution of the multi-dimensional time-dependent Schrödinger equation:

$$\begin{aligned} \mathrm{i}\frac{\partial u}{\partial t} &= -\frac{1}{2}\epsilon^2\Delta u + vu, \\ u(\boldsymbol{x},0) &= g(\boldsymbol{x}), \end{aligned}$$

for $\boldsymbol{x} \in [0,1]^d$, $t > 0$ and ϵ a small positive parameter. The functions $v(\boldsymbol{x})$ and $g(\boldsymbol{x})$ describe the potential and initial conditions. We assume periodic boundary conditions. Our numerical method consists of two steps. First we use a pseudo-spectral Fourier collocation method that uses samples of the functions on rank-1 lattice points. We then get a system of ordinary differential equations in time, which we solve approximately by stepping in time using the higher-order exponential operator splitting method. We prove that the numerical scheme proposed converges with the desired order with respect to the time step size, given that the potential is sufficiently smooth. Particularly, we prove that the required amount of smoothness is independent of the dimension of the problem. Moreover, our method has a distinct computational advantage: due to the rank-1 lattice structure, we efficiently construct the interpolation and the time-stepping operator using one-dimensional Fast Fourier Transformations. We demonstrate our results with several numerical examples.

Generation of True Random Numbers using quasi-Montecarlo Methods

Ana I. Gomez
Department of Mathematics, University of Cantabria, Spain, domingo.gomez@unican.es

Domingo Gomez-Perez
Department of Mathematics, University of Cantabria, Spain, domingo.gomez@unican.es

Florian Pausinger
School of Mathematics & Physics, Queen's University, United Kingdom, f.pausinger@qub.ac.uk

Random number generators are necessary in many areas, especially in cryptography. There are many physical sources for random numbers, however the strict requirements contained in the AIS31 evaluation criteria have to be met to be considered a reliable source of randomness. The design must provide a minimum throughput and a direct access to the source of bits, but also a stocastic model is necessary to estimate entropy per bit.

Technology in Field Programmable Gate Arrays (FPGA) allows fast and affordable implementation in programmable logic blocks. That is the reason why designs of true random number generators based in FPGA have gained a lot of attention [2]. In particular, one design which could be effective was given in [3]. However, it turned out that the underlying assumptions have not been adequately confirmed [1]. In this paper, we provide a stochastic model using Quasi-Monte Carlo point sets to guarantee a minimum entropy per bit.

[1] Markus Dichtl and Jovan Dj. Golic. High-speed true random number generation with logic gates only. In *International Workshop on Cryptographic Hardware and Embedded Systems*, pages 45–62. Springer, 2007.

[2] Oto Petura, Ugo Mureddu, Nathalie Bochard, Viktor Fischer, and Lilian Bossuet. A survey of ais-20/31 compliant trng cores suitable for fpga devices. In *Field Programmable Logic and Applications (FPL), 2016 26th International Conference on*, pages 1–10. IEEE, 2016.

[3] Berk Sunar, William J Martin, and Douglas R Stinson. A provably secure true random number generator with built-in tolerance to active attacks. *IEEE Transactions on computers*, 56(1), 2007.

Tuesday 14:45–15:45, **Room: Salle 8**

Multilevel Monte Carlo methods

Chair: Giovanni Samaey

Multilevel Monte Carlo for Asymptotic-Preserving Particle Schemes

Emil Løvbak
Department of Computer Science, KU Leuven, Belgium, emil.loevbak@cs.kuleuven.be

Stefan Vandewalle
Department of Computer Science, KU Leuven, Belgium, stefan.vandewalle@cs.kuleuven.be

Giovanni Samaey
Department of Computer Science, KU Leuven, Belgium, giovanni.samaey@cs.kuleuven.be

In many applications of particle simulation we encounter issues with timescale separation, where fast system behavior forces us to take small time steps, but slow system behavior forces us to simulate long time-horizons. When considering hyperbolic transport equations, a scaling parameter ε is commonly used to represent the relevant time-scale. The value of ε can vary by many orders of magnitude in different regions of the simulation, causing stiffness in the problem formulation. As this scaling parameter tends to zero, the hyperbolic equation converges, in the limit, to a parabolic diffusion equation which is clearly defined. Simulations of the transport equation in the small ε region however, suffer from extreme time step reduction constraints in order to maintain stability, hindering convergence to this equation.

In [1] a new Monte Carlo scheme was developed that converges in the diffusive limit without these stability constraints, at the cost of a linear model error in the time step size. In this new work we have implemented a multilevel Monte Carlo expansion to this scheme. This scheme aims to reduce simulation costs by computing a first estimate with a large time step size, and correspondingly large bias. This estimate can then be improved upon with more accurate Monte Carlo simulations with finer time step sizes. The bias corrections and the variances of

differences in this scheme have an interesting new structure. This structure differs from that which is typically observed in multilevel Monte Carlo applications.

In this talk we will present some new theoretical insights, resulting from the analysis of this multilevel scheme. These results will be validated with numerical simulations.

[1] G. Dimarco, L. Pareschi and G. Samaey. Asymptotic-Preserving Monte Carlo methods for transport equations in the diffusive limit. *SIAM Journal on Scientific Computing*, 40: A504–A528, 2018.

Robust Optimization of PDE Constrained Systems

Andreas Van Barel
Department of Computer Science, KU Leuven, Belgium, andreas.vanbarel@cs.kuleuven.be

Stefan Vandewalle
Department of Computer Science, KU Leuven, Belgium, stefan.vandewalle@cs.kuleuven.be

We consider PDE constrained optimization problems where the partial differential equation has uncertain coefficients modeled by means of random variables or random fields. The goal of the optimization is to determine a robust optimum, i.e., an optimum that is satisfactory in a broad parameter range, and as insensitive as possible to parameter uncertainties. Many goal functions can be defined that attempt to solve this problem. They vary in computational cost and in the robustness of the solution. In this talk, we focus on optimizing the expected value of a tracking type objective with an additional penalty on the variance of the state. The gradient and Hessian corresponding to such a cost functional also contain expected value operators. Since the stochastic space is often high dimensional, a multilevel (quasi-) Monte Carlo method is presented to efficiently calculate the gradient and the Hessian. If one is careful, the resulting estimated quantities are the exact gradient and Hessian of the estimated cost functional, which is important in practice for some optimization algorithms.

The convergence behavior is illustrated using a gradient and a Hessian based optimization method for a model elliptic diffusion problem with lognormal diffusion coefficient and optionally an additional nonlinear reaction term. The evolution of the variances on each of the levels during the optimization procedure leads to a practical strategy for determining how many and which samples to use. We also investigate the necessary tolerances on the mean squared error of the estimated quantities. Finally, a practical algorithm is presented and tested on a problem with a large number of optimization variables and a large number of uncertainties.

Tuesday 16:15–17:15, **Room: Amphi 3**

Rare events

Chair: Art Owen

Sequential Monte Carlo for efficient sampling of rare trajectories in reverse time

Jere Koskela
Statistics, Warwick University, UK, j.koskela@warwick.ac.uk

Paul Jenkins
Statistics & Computer Science, Warwick, UK, p.jenkins@warwick.ac.uk

Dario Spanò
Statistics, Warwick University, UK, d.spano@warwick.ac.uk

Sequential Monte Carlo methods are a powerful tool for simulating rare trajectories from stochastic processes. Samples are drawn from incorrect proposal dynamics, under which the desired event is no longer rare, and appropriately reweighted to compensate for the mismatch between the proposal and target distributions. The choice of proposal is crucial to the efficiency of the algorithm: well chosen proposals can result in many orders

of magnitude of variance reduction, while poor choices can lead to worse performance than naive Monte Carlo. Typically, analytic guidance for tuning proposal distributions depends on intractable quantities, such as the probability of the rare event of interest. Hence, such guidelines are difficult to use in practice, particularly when the problem is high dimensional.

I will show that specifying proposal dynamics in reverse time can substantially reduce the difficulty of the design task in settings where a typical initial condition leads to a rare terminal state. Practical examples include overflows of stable queues, or large epidemics under subcritical infection models. I will give a precise categorisation of settings in which time reversal is effective, characterise the optimal but unimplementable reverse-time proposal, and show that implementable approximations leading to good performance are readily available for a wide variety of problems. The resulting efficiency gains are demonstrated by simulated complarisons with a state-of-the-art adaptive multilevel splitting algorithm.

Monte Carlo Methods for Insurance Risk Computation

Shaul Bar-Lev
Department of Statistics, University of Haifa, Israel, barlev@stat.haifa.ac.il

Ad Ridder
Department EOR, Vrije University of Amsterdam, Netherlands, ad.ridder@vu.nl

Let $Y_1, Y_2, \ldots$ be i.i.d. positive random variables representing the individual claims at an insurance company, and let $N \in \mathcal{N}_0$ designate the random number of claims occurring during some period, then $S_N = \sum_{i=1}^{N} Y_i$ models the total claim amount, therefore called the aggregated claim. We are interested in computing the risk factor $\mathcal{P}\big(S_N > s\big)$ for large levels s. This is a well-known and much studied problem because of its importance for insurance companies [3].

However, whereas usually the problem is modeled using the Poisson or negative binomial distribution for N, and the Gamma or inverse Gaussian distribution for Y (see [2]), we shall investigate the counting variable having the Abel distribution, the strict arcsine distribution, the large arcsine distribution, or the Takacs distribution. These distributions can be represented by natural exponential families with cubic variance functions. From available data of a case study, we noticed that the (empirical) variance of the claim number data shows indeed a power law with respect to the (empirical) mean, with a power close to three.

These are new distributions for insurance modelling, and have to our knowledge not been considered before in computation and simulation studies. Therefore, we have developed new Monte Carlo algorithms for generating samples from these distributions. The algorithms are based on an asymptotic analysis of the distributions.

Furthermore, for the claim size Y we consider distributions belonging to natural exponential families, such as the Gamma, positive stable, and inverse Gaussian distribution. In this way we are able to compute the risk factor by simulation. Finally, using the reproducibility property of these distributions [1] we can efficiently generate samples of sums of i.i.d. claims sizes. For large-tail probabilities we propose to apply importance sampling as variance reduction technique to speed up the simulation computional times.

[1] S.K. Bar-Lev and P. Enis. Reproducibility and natural exponential families with power variance functions. *Annals of Statistics*, 14: 1507–1522, 1986.

[2] N.L. Bowers, H.U. Gerber, J.C. Hickman, D.A. Jones, and C.J. Nesbitt. *Actuarial Mathematics.* 2nd ed, Society of Actuaries, Itasca, Illinois, 1997.

[3] R. Kaas, M. Goovaerts, J. Dhaene, and M. Denuit. *Modern Actuarial Risk Theory.* 2nd ed, Springer, Heidelberg, 2008..

Thursday 10:00–12:00, **Room: Salle 8**

Analysis of low-discrepancy sequences

Chair: Kosuke Suzuki

On Bounded Remainder Sets for Sequences $(\{a_n\alpha\})_{n\geq 1}$ with $(a_n)_{n\geq 1}$ a Lacunary Integer Sequence

Lisa Kaltenböck
Institute of Financial Mathematics and Applied Number Theory, JKU Linz, Austria, lisa.kaltenboeck@jku.at

Gerhard Larcher
Institute of Financial Mathematics and Applied Number Theory, JKU Linz, Austria, gerhard.larcher@jku.at

Let $(x_n)_{n\geq 1}$ be an arbitrary sequence in $[0,1)$. An interval $[a,b) \subseteq [0,1)$ is called *bounded remainder set* for $(x_n)_{n\geq 1}$ if

$$|\#\{1 \leq n \leq N : x_n \in [a,b)\} - (b-a)N| \leq c,$$

for all natural numbers N with a constant c independent of N. We study bounded remainder sets for uniformly distributed sequences of the form $(\{a_n\alpha\})_{n\geq 1}$ where (a_n) is a lacunary integer sequence. Although the pure Kronecker sequence $(\{n\alpha\})_{n\geq 1}$ has a dense collection of bounded remainder sets, we provide an explicit example such that $(\{a_n\alpha\})_{n\geq 1}$ does not have any non-trivial bounded remainder sets. However, we present a collection of results which show that this is in general not the case.

Characterization of Matrices B such that (I, B, B^2) Generates a Digital Net with t-value Zero

Hiroki Kajiura
Graduate School of Science, Hiroshima University, Japan, hikajiura@hiroshima-u.ac.jp

For QMC, (t,m,s)-nets are commonly used. Here the base field is assumed to be the two-elements field $\mathbf{F}_2$. A (t,m,s)-net P is generated by s of $m \times m$ matrices $A_1, \ldots, A_s$ with coefficients in $\mathbf{F}_2$, called the generating matrices. It is considered as a point set in $[0,1]^s$ of size 2^m. Smaller value of t is preferable for QMC, but the optimal $t = 0$ is possible only when $s \leq 3$ for $\mathbf{F}_2$.

We study 3-dimensional digital nets generated by matrices (I, B, B^2) where I is the identity matrix and B is a square matrix. We give a characterization of B for which the t-value of the digital net is 0: B is obtained from Fauré's construction in a simple manner. As a corollary, we prove that such B exists for each m, and satisfies $B^3 = I$.

The original interest came from Markov Chain Quasi-Monte Carlo (MCQMC): Consider a maximal periodic (i.e. the period $2^m - 1$) sequence by a recursion $x_n := Bx_{n-1}$, $x_0 \in \mathbf{F}_2^m \setminus \{0\}$. For a given s, if we take overlapping s-tupples from one whole period (giving $2^m - 1$ vectors in $(\mathbf{F}_2^m)^s$) and supplement 0, we obtain a linear subspace V_s of $(\mathbf{F}_2^m)^s$ of $\mathbf{F}_2$-dimension m. The generating matrix of V_s is $(I, B, B^2, \ldots, B^s)$. For Markov Chain Quasi-Monte Carlo (MCQMC) method, it is desirable that V_s has small t-value for every s (cf.[1][2]). The above result shows some trade-off in this demand: if we try to have $t = 0$ for $s = 3$, then B has order 3 and does not give a maximal periodic sequence.

This is a joint work with Makoto Matsumoto and Kosuke Suzuki.

[1] Su Chen, Josef Dick, and Art B. Owen. Consistency of Markov chain quasi-Monte Carlo on continuous state spaces. *Ann. Statist.*, 39(2):673–701 (2011).

[2] Su Chen, Makoto Matsumoto, Takuji Nishimura, and Art B. Owen. New inputs and methods for Markov chain quasi-Monte Carlo. In *Monte Carlo and quasi-Monte Carlo methods 2010*, volume 23 of *Springer Proc. Math. Stat.*, pages 313–327. Springer, Heidelberg (2012).

SOME NEGATIVE RESULTS RELATED TO POISSONIAN PAIR CORRELATION PROBLEMS

Gerhard Larcher
Institute of Financial Mathematics and Applied Number Theory, JKU, Austria, gerhard.larcher@jku.at

Wolfgang Stockinger
Institute of Financial Mathematics and Applied Number Theory, JKU, Austria, wolfgang.stockinger@jku.at

We say that a sequence $(x_n)_{n\in\mathbb{N}}$ in $[0,1)$ has Poissonian pair correlations if

$$\lim_{N\to\infty}\frac{1}{N}\#\{1\le l\neq m\le N : \|x_l - x_m\|\le \frac{s}{N}\} = 2s$$

for every $s\ge 0$. The aim of this talk is to present a gap theorem which allows to deduce that a sequence $(x_n)_{n\in\mathbb{N}}$ of real numbers in $[0,1)$ having a finite number of gaps, i.e., there is an $g\in\mathbb{N}$ and infinitely many N, such that the point set $x_1,\dots,x_N$ has at most g different distances between neighbouring sequence elements, cannot have Poissonian pair correlations. This result covers a broad class of sequences, e.g., Kronecker sequences, the van der Corput sequence and in more general LS-sequences of partitions and points. Additionally, this theorem enables us to derive pair correlation properties for sequences of the form $(\{a_n\alpha\})_{n\in\mathbb{N}}$, where (a_n) is a strictly increasing sequence of integers with maximal additive energy.

The Asymptotic Distribution of Riesz' Energy

Raffaello Seri
DiECO and Center for Nonlinear and Complex Systems, University of Insubria, Italy,
raffaello.seri@uninsubria.it

Consider a configuration of N points $\omega_N = \{\xi_i, i = 1,\dots,N\}$ on the hypersphere $\mathbf{S}^d$ in $\mathbf{R}^{d+1}$. We define the *Riesz' energy* of ω_N as:

$$E_s(\omega_N) = 2\sum_{i=1}^{N-1}\sum_{j=1}^{i-1}\frac{1}{|\xi_i-\xi_j|^s} = \sum_{i\neq j}\frac{1}{|\xi_i-\xi_j|^s}$$

for $s\in\mathbf{R}$. The case with $s=0$ is degenerate and the Riesz' energy is generally replaced by the *logarithmic energy*, defined as $E_{\ln}(\omega_N) = -\sum_{i\neq j}\ln|\xi_i-\xi_j|$. We consider the behavior of the distribution function of E_s (and $E_{\ln}$) when applied to a sample ω_N of independent and uniformly distributed points on the surface of the hypersphere. As the distribution for finite N is intractable, we resort to the so-called asymptotic distribution, obtained when a suitably scaled version of E_s converges in distribution as long as N diverges to infinity. We identify three asymptotic regimes. In the first regime, when $s<\frac{d}{2}$, both the mean and the variance of the energy exist. In the second regime, when $\frac{d}{2}\le s<d$, only the mean exists but the variance does not. At last, in the third regime, when $d\le s$, no integer moment of the energy exists. We characterize the asymptotic distribution of Riesz' energy in the three regimes, giving special attention to the behavior of the right tail of the distribution. The asymptotic behavior of the energy departs significantly from the one of other measures of discrepancy on the sphere and hypersphere (see [1]).

[1] C. Choirat and R. Seri. Computational Aspects of Cui-Freeden Statistics for Equidistribution on the Sphere. *Mathematics of Computation*, 82 (284): 2137–2156, 2013.

Thursday 14:45–15:45, **Room: Salle 7**

Jittered sampling

Chair: Bruno Tuffin

A Sharp Discrepancy Bound for Jittered Sampling

Benjamin Doerr
LIX, École Polytechnique, France, lastname@lix.polytechnique.fr

For $m, d \in \mathbf{N}$, a jittered sampling point set P having $N = m^d$ points in $[0,1)^d$ is constructed by partitioning the unit cube $[0,1)^d$ into m^d axis-aligned cubes of equal size and then placing one point independently and uniformly at random in each cube. We show that there are constants $c \geq 0$ and C such that for all d and all $m \geq d$ the expected non-normalized star discrepancy of a jittered sampling point set satisfies

$$c\, dm^{\frac{d-1}{2}} \sqrt{\log(\tfrac{m}{d})} \leq E[D^*(P)] \leq C\, dm^{\frac{d-1}{2}} \sqrt{\log(\tfrac{m}{d})}.$$

This discrepancy is thus smaller by a factor of $\Theta\big(\sqrt{\frac{\log(m/d)}{m/d}}\big)$ than the one of a uniformly distributed random point set of m^d points. This result improves both the upper and the lower bound for the discrepancy of jittered sampling recently given by Pausinger and Steinerberger [1] and also removes the asymptotic requirement that m is sufficiently large compared to d.

[1] F. Pausinger and S. Steinerberger. On the discrepancy of jittered sampling. *J. Complexity*, 33:199–216, 2016.

On p-frame potential of random point configurations on the sphere

Masatake Hirao
School of Information and Science Technology, Aichi Prefectural University, Japan, `hirao@ist.aichi-pu.ac.jp`

In this talk we deal with two types of random point configurations, determinantal point process (DPP) and the jittered sampling on the sphere. DPP is a random point process, which is used in a fermion model in quantum mechanics, and the jittered sampling is one of the famous random sampling method using uniformly distribution on each divided areas. We note that, both random point configurations induce QMC design sequences for some Sobolev spaces on the sphere ([1, 2]). From the viewpoint of p-frame potential, we compare these random point configurations and spherical designs which are deterministic point configurations on the sphere. We also discuss some other potential functions and applications of these point configurations if possible.

[1] J.S. Brauchart, E.B. Saff, I.H. Sloan, R.S. Womersley. QMC Designs: optimal order quasi-Monte Carlo integration schemes on the sphere. *Math. Comp.*, 83(290): 2821–2851, 2014.

[2] M. Hirao. QMC designs and determinantal point processes. To appear in *Monte Carlo and Quasi-Monte Carlo Methods 2016*, P. Glynn and A. Owen, eds., Springer-Verlag.

Thursday 16:15–17:45, **Room: Amphi 3**

Efficient Sampling

Chair: Ad Ridder

Sampling of probability measures in the convex order and approximation of Martingale Optimal Transport problems

Aurélien Alfonsi
CERMICS, ENPC, FRANCE, `aurelien.alfonsi@enpc.fr`

Jacopo Corbetta
ZELIADE SYSTEMS, FRANCE, `j.corbetta@zeliade.com`

Benjamin Jourdain
CERMICS, ENPC, FRANCE, `benjamin.jourdain@enpc.fr`

We propose sampling methods preserving the convex order between two probability measures. In particular, we introduce the Wasserstein projection of a first probability measure on the set of probability measures dominated by the second one for the convex order. For empirical measures, this projection can be computed by solving a quadratic optimization problem with linear constraints. We apply our techniques to approximate Martingale Optimal Transport problems with two of three marginals by using linear programming solvers.

[1] A. Alfonsi, J. Corbetta and B. Jourdain Sampling of probability measures in the convex order and approximation of Martingale Optimal Transport problems. Preprint arXiv:1709.05287

Adaptive Construction of Transport Maps for Efficient Sampling

Daniele Bigoni
Massachusetts Institute of Technology, USA, dabi@mit.edu

Alessio Spantini
Massachusetts Institute of Technology, USA, spantini@mit.edu

Youssef Marzouk
Massachusetts Institute of Technology, USA, ymarz@mit.edu

Bayesian statistical models often yield high-dimensional and non-standard distributions for which direct integration is intractable—in the sense that one cannot perform Monte Carlo integration with independent unweighted samples or deterministic integration via some quadrature rule. In this setting, the transport approach seeks a deterministic map that transforms the intractable target distribution into a tractable "reference" distribution. Paired with an appropriate integration scheme for the reference distribution, this map can then be used to generate i.i.d samples or efficient quadrature rules for the target. Alternatively, one can view an approximate map as a "preconditioner" and correct for potential errors by sampling the pullback of the target distribution through the map.

In this talk, we address the automatic and adaptive learning of such transport maps. In particular, we seek maps within an infinite-dimensional space of generalized triangular functions, which contains the Knothe–Rosenblatt rearrangement for a given reference–target pair. We seek the map by minimizing the Kullback–Leibler divergence between the pushforward of the reference distribution through the map and the intractable target distribution. The variations of this problem can be evaluated in closed form, and our approach uses these variations to incrementally enrich the numerical representation of the map. Accuracy and computational cost are balanced via an inexpensive asymptotic convergence criterion and a validation heuristic. The adaptation scheme reveals and exploits low-dimensional structure in the transport map resulting from independence relationships and from smoothness of the associated densities.

We demonstrate the methodology using target distributions produced by parameter inference problems, inverse problems, and state-space models from a variety of application areas. Software and references are collected at http://transportmaps.mit.edu.

Efficiently approximating Gaussian Process Emulators with Inequality Constraints using MC/MCMC

Andrés F. López-Lopera
Mines Saint-Étienne, LIMOS, France, andres-felipe.lopez@emse.fr

François Bachoc
IMT, Université de Toulouse, France, francois.bacho@math.univ-toulouse.fr

Nicolas Durrande
Prowler.io, UK, nicolas@prowler.io

Olivier Roustant
Mines Saint-Étienne, LIMOS, France, roustant@emse.fr

Gaussian processes (GP) are one of the most famous Bayesian frameworks for regression due to their ability to fit a great variety of real-world data. It was showed in [3] that more realistic regression models can be obtained by considering inequality conditions (e.g. boundedness, monotonicity, convexity). However, the resulting posterior distribution given both interpolation and inequality constraints is no longer Gaussian but follows a truncated multivariate normal for a finite number of observations. In [2], an exact Monte Carlo (MC) procedure known as Rejection Sampling from the Mode (RSM) was introduced to approximate the posterior distribution. It is

based on a generalization of Von Neumann's rejection technique. Despite the effort made in [2] to provide an exact sampler, it is necessary to explore more efficient MC and MCMC techniques since rejection methods entail expensive computations in high dimensions due to low acceptance rates. Therefore, we investigate different MC (e.g. via exponential tilting, expT) and MCMC (e.g. Gibbs; Hamiltonian Monte Carlo, HMC) samplers for truncated multinormals. We assess their accuracy using different performance indicators (e.g. computational cost, effective sample size), and we compare them with respect to RSM's results. According to experiments on synthetic data, we concluded that both expT and MCMC methods usually outperformed the RSM sampler. More precisely, expT and HMC provided more efficient results in terms of time and sample correlations. Finally, we made both MC/MCMC samplers available to the scientific community as an open source R package [1].

[1] Andrés Felipe López-Lopera. LineqGPR: Gaussian Process Regression Models with Linear Inequality Constraints, 2017. R package version 0.0.1. This package will be freely available in June.

[2] Hassan Maatouk and Xavier Bay. *A New Rejection Sampling Method for Truncated Multivariate Gaussian Random Variables Restricted to Convex Sets*, pages 521–530. Springer International Publishing, Cham, 2016.

[3] Hassan Maatouk and Xavier Bay. Gaussian process emulators for computer experiments with inequality constraints. *Mathematical Geosciences*, 49(5):557–582, 2017.

Thursday 16:15–17:45, **Room: Salle 5**

Stochastic Differential Equations

Chair: Mireille Bossy

FINITE VARIANCE UNBIASED ESTIMATION OF STOCHASTIC DIFFERENTIAL EQUATIONS

Ankush Agarwal
Adam Smith Business School, University of Glasgow, United Kingdom, ankush.agarwal@glasgow.ac.uk

Emmanuel Gobet
Centre de Mathématiques Appliquées, École Polytechnique, France, emmanuel.gobet@polytechnique.edu

We develop a new unbiased estimation method for Lipschitz continuous functions of multi-dimensional stochastic differential equations with Lipschitz continuous coefficients. This method provides a finite variance estimator based on a probabilistic representation which is similar to the recent representations obtained through the parametrix method and recursive application of the automatic differentiation formula. Our approach relies on appropriate change of variables to carefully handle the singular integrands appearing in the iterated integrals of the probabilistic representation. It results in a scheme with randomized intermediate times where the number of intermediate times has a Pareto distribution.

Micro-macro acceleration method with relative entropy moment matching for scale-separated SDEs

Kristian Debrabant
Department of Mathematics and Computer Science, University of Southern Denmark, Denmark, debrabant@imada.sdu.dk

Tony Lelièvre
CERMICS, Ecole des Ponts ParisTech, France, tony.lelievre@enpc.fr

Giovanni Samaey
Department of Computer Science, KU Leuven, Belgium, giovanni.samaey@kuleuven.be

Przemysław Zieliński
Department of Computer Science, KU Leuven, Belgium, przemyslaw.zielinski@kuleuven.be

Many systems of current interest exhibit behavior on a wide range of time scales, which cannot be simulated directly on long (macroscopic) time intervals. In this talk, I will present and discuss a multiscale method to efficiently simulate the macroscopic observables of SDEs having strong separation between time-scales.

The method couples the microscopic model, of which we have full knowledge, to a macroscopic level, of which we assume only limited information. The nature of the macroscopic model is described by a finite set of macroscopic state variables – averages over the microscopic distribution. To bypass the prohibitive cost of the direct Monte Carlo simulation, the method combines short bursts of path simulation with extrapolation of macroscopic states forward in time.

In the crucial step of the algorithm, we aim to obtain, after extrapolation, a new ensemble of particles/replicas compatible with given macroscopic states. To address and regularize this inference problem, we introduce the matching operator based on the minimization of suitable distance between probability distributions – logarithmic relative entropy.

I will discuss the relation of the method to coarse graining and its basic properties. Particularly, the ongoing study aims to establish the convergence in the numerically weak sense, inquire about the stability for appropriately chosen test models, and provide a convenient numerical approach.

Rapid covariance based sampling of finite element approximations of linear SPDE in MLMC

Andreas Petersson
Department of Mathematical Sciences, Chalmers University of Technology and the University of Gothenburg, Sweden, andreas.petersson@chalmers.se

We consider the efficient simulation of the mean value of a non-linear functional of the end-time solution to a linear stochastic partial differential equations (SPDE) with additive Gaussian noise. A Galerkin finite element method is employed along with an implicit Euler scheme to arrive at a fully discrete approximation of the mild solution to the equation. A scheme is presented to compute the covariance of this approximation, which allows for rapid sampling in a Monte Carlo method. This is then extended to a multilevel Monte Carlo method (MLMC), for which we present a scheme to compute the cross-covariance between the approximations at different levels. In contrast to traditional path-based methods it is not assumed that the Galerkin subspaces at these levels are nested. The computational complexities of the presented schemes are compared to traditional methods and simulations confirm that the costs of the latter are significantly greater than those of the former.

Thursday 16:15–17:45, **Room: Salle 6**

Sequential methods and efficiency

Chair: François Le Gland

Sequential Monte Carlo Methods for Time-Varying Parameter Estimation

Andrea Arnold
Department of Mathematical Sciences, Worcester Polytechnic Institute, USA, anarnold@wpi.edu

Many applications in modern day science involve unknown system parameters that must be estimated using little to no prior information. A subset of these problems includes parameters that are known to vary with time but have no known evolution model. We show how nonlinear sequential Monte Carlo filtering techniques can be employed to estimate time-varying parameters, while naturally providing a measure of uncertainty in the estimation. Results are demonstrated on several applications from the life sciences.

New Insights into History Matching via Sequential Monte Carlo

Christopher Drovandi
School of Mathematical Sciences, Queensland University of Technology, Australia, c.drovandi@qut.edu.au

David Nott
Department of Statistics and Applied Probability, National University of Singapore, Singapore, standj@nus.edu.sg

Daniel Pagendam
Data 61, CSIRO, Australia, dan.pagendam@data61.csiro.au

The aim of the history matching method is to locate non-implausible regions of the parameter space of complex deterministic or stochastic models by matching model outputs with data. It does this via a series of waves where at each wave an emulator is fitted to a small number of training samples. An implausibility measure is defined which takes into account the closeness of simulated and observed outputs as well as emulator uncertainty. As the waves progress, the emulator becomes more accurate so that training samples are more concentrated on promising regions of the space and poorer parts of the space are rejected with more confidence. Whilst history matching has proved to be useful, existing implementations are not fully automated and some ad-hoc choices are made during the process. This occurs especially when the non-implausible region becomes small and it is difficult to sample this space uniformly to generate new training points. Here we develop a sequential Monte Carlo (SMC) algorithm for implementation which is semi-automated. Our novel SMC approach offers a much more reliable sampling of the non-implausible space.

Rethinking the Effective Sample Size in Importance Sampling

Víctor Elvira
IMT Lille Douai & CRIStAL, France, victor.elvira@imt-lille-douai.fr

Luca Martino
University Carlos III of Madrid, Spain, luca@tsc.uc3m.es

Christian P. Robert
Université Paris-Dauphine, France, xian@ceremade.dauphine.fr

The *effective sample size* (ESS) is a relevant measure of efficiency in Monte Carlo methods [1]. In importance sampling (IS), the ESS represents how many samples drawn from the target distribution would be required to obtain the same performance as in the IS estimator built with N samples. Thus, the ESS is theoretically defined as the ratio of the variances between the plain estimator and the IS estimator. However, computing these variances involves intractable integrals, and then, the ESS must be approximated. The well-known approximation $\widehat{ESS} = \frac{1}{\sum_{n=1}^{N} \bar{w}_n^2}$, where $\{\bar{w}_n\}_{n=1}^{N}$ are the normalized IS weights, was partially derived in [2, 3]. Due to its simplicity, this approximation has become an essential tool in IS-based methods, including adaptive IS (AIS) and Sequential Monte Carlo (SMC) [4, 5].

In this talk, we review the assumptions and approximations in the derivation of $\widehat{ESS}$. Note that, as a result of the approximation, neither the moment of the targeted distribution nor the samples appear in $\widehat{ESS}$. We highlight its limitations and the danger of trusting this approximation in different scenarios, especially when it largely underestimates or overestimates the ESS. Moreover, we derive other possible ESS-kind functions, discussing the theoretical requirements that a generic ESS function should satisfy.

[1] C. P. Robert and G. Casella, *Monte Carlo Statistical Methods*, Springer, 2004.

[2] A. Kong, "A note on importance sampling using standardized weights," *University of Chicago, Dept. of Statistics, Tech. Rep*, vol. 348, 1992.

[3] A. Kong, J. S. Liu, and W. H. Wong, "Sequential imputations and Bayesian missing data problems," *Journal of the American Statistical Association*, vol. 9, pp. 278–288, 1994.

[4] M. F. Bugallo, V. Elvira, L. Martino, D. Luengo, J. Míguez, and P. M. Djuric. Adaptive importance sampling: The past, the present, and the future. *IEEE Signal Processing Magazine*, 34(4):60–79, 2017.

[5] A. Doucet, N. de Freitas, and N. Gordon, Eds., *Sequential Monte Carlo Methods in Practice*, Springer, New York, 2001.

Thursday 16:15–17:45, **Room: Salle 7**

Monte Carlo in physics (1)

Chair: DanHua ShangGuan

Convergence Analysis of a Coupled Monte-Carlo/Pseudo-Timestepping Scheme Arising in Plasma Edge Simulations

Matthias Baeten
Department of Computer Science, KU Leuven, Belgium, matthias.baeten@kuleuven.be

Martine Baelmans
Department of Mechanical Engineering, KU Leuven, Belgium, martine.baelmans@kuleuven.be

Giovanni Samaey
Department of Computer Science, KU Leuven, Belgium, giovanni.samaey@kuleuven.be

At present, the plasma and neutral transport in the plasma edge region of magnetic confinement fusion devices is numerically simulated using plasma edge codes such as B2-EIRENE [1]. The plasma is modeled with PDEs (Braginskii equations) that express conservation of mass, momentum and energy, while the neutral particles are described by kinetic equations (Boltzmann equations), that model the neutral distributions in a position-velocity phase space. These two sets of equations are strongly coupled because of mutual interactions between plasma and neutral particles. On the one hand, these interactions give rise to source terms in the plasma equations. On the other hand, the physical parameters in the kinetic equations strongly depend on plasma quantities.

In B2-EIRENE, a finite volume (FV) approach to handle the plasma equations is combined with a Monte Carlo (MC) method for the kinetic neutral equations. The FV and MC methods are iteratively coupled using a pseudo-timestepping scheme. Pseudo-timestepping is used as the iterative method to solve the nonlinear discretized plasma equations. For the MC simulation of the neutral transport, the current plasma state can be used as plasma background for the MC simulation in every iteration.

We studied the propagation of the MC noise throughout the iterative scheme and the resulting convergence error on the plasma solution. Because of the MC noise in every iteration, the plasma state converges to a statistically steady state characterized by a bias and variance. We analytically derived, in a simplified scalar setting, the effect of the simulation settings, like the timestep and the number of particles, on the bias and variance of the plasma states. Numerical experiments with a 1D plasma edge code show that these relations also hold for the simulation of realistic (though still 1D) plasma edge models. In the talk, we will present both the theoretical en numerical results.

[1] D. Reiter, M. Baelmans, P. Börner. The EIRENE and B2-EIRENE codes. *Fusion Science and Technology*, 47 (3): 172–186, 2005.

Efficient Strategy for Global Tallying in the Monte Carlo Criticality Calculation

DanHua ShangGuan
Beijing Institute of Applied Physics and Computational Mathematics, China, sgdh@iapcm.ac.cn

Gang Li
Beijing Institute of Applied Physics and Computational Mathematics, China, lgseven@126.com

Rui Li
CAEP Software Center of High Performance Numerical Simulation, China, li_rui@iapcm.ac.cn

Li Deng
Beijing Institute of Applied Physics and Computational Mathematics, China, deng_li@iapcm.ac.cn

Based on an on-the-fly diagnostic method for the convergence of Shannon entropy corresponding to the fission source distribution, the uniform fission site algorithm will be invoked after the iteration step whose serial number is the maximum of the first active step and the first converged step according to the above mentioned diagnostic rule. Then, the global efficiency index will be calculated to judge if the standard has been satisfied. If so, the whole calculation will be stopped in advance; If not, more steps will be estimated to run the uniform fission site algorithm. This process will be repeated until reaching at the appointed maximum step number. This strategy is helpful to increase the efficiency of global tallying in the Monte Carlo criticality calculation).

Monte Carlo method projective estimators for angular and temporal characteristics evaluation of polarized radiation

Natalya Tracheva
Laboratory of Monte Carlo methods, Institute of Computational Mathematics and Mathematical Geophysics SB RAS,
Mechanics and Mathematics Departmen, Novosibirsk State University, Russia, tnv@osmf.sscc.ru

Sergey Ukhinov
Laboratory of Monte Carlo methods, Institute of Computational Mathematics and Mathematical Geophysics SB RAS,
Mechanics and Mathematics Departmen, Novosibirsk State University, Russia, sau@osmf.sscc.ru

In this talk, we consider Monte Carlo-based projective methods in application to a number of problems of atmospheric optics. Both methods in consideration, are built on the basis of the orthonormal polynomial expansion of a special kind.

For a numerical estimation of bidirectional angular characteristics of the scattered polarized radiation, transmitted through optically thick layers of matter and reflected by them, we propose an algorithm of the numerical statistical modeling in which projective expansion of the density of the corresponding angular distribution is realized using the system of basic functions, namely hemispherical harmonics, orthogonal with the weight $|\cos\theta|$. Here θ is the latitude reckoned from the normal to the surface.

For a numerical estimation of the time distribution density of polarized radiation flux in the scattering and absorbing media, illuminated by an external or internal source, we construct a Monte Carlo weighted algorithm, that is based on the randomized projective evaluation of the functionals via the modified Laguerre polynomials expansion.

We conclude by presenting some numerical results, obtained for two-dimensional angular distributions of the intensity and the degree of polarization of radiation, transmitted and reflected by optically thick layers of a scattering and absorbing substance, as well as their time distributions on the boundary of a half space.

The reported study was partially funded by RFBR according to the research projects 17-01-00823, 18-01-00356, 18-31-00213, 18-41-540003.

Friday 10:00–12:00, **Room: Amphi 3**

QMC and quadrature strategies for integration

Chair: Fred J. Hickernell

A Multigrid Multilevel Quasi-Monte Carlo Method with Sample Reuse

Pieterjan Robbe
Department of Computer Science, KU Leuven, Belgium, pieterjan.robbe@kuleuven.be

Dirk Nuyens
Department of Computer Science, KU Leuven, Belgium, dirk.nuyens@kuleuven.be

Stefan Vandewalle
Department of Computer Science, KU Leuven, Belgium, stefan.vandewalle@kuleuven.be

Multilevel Monte Carlo methods are efficient variance reduction techniques that use a sequence of coarse approximations to reduce the computational complexity in uncertainty quantification applications. These methods typically require many samples to be taken on different levels of accuracy. When a Full Multigrid solver is used for every sample, we obtain free coarse samples that can be reused on coarser levels of accuracy. We extend the recently proposed heuristic method by Kumar, Oosterlee and Dwight [1] to include a more rigorous error estimate, by using the random shifting technique known from Quasi-Monte Carlo literature. We apply our new method to a model PDE with random coefficients and illustrate numerically that significant speedup can be obtained by reusing the samples.

[1] Prashant Kumar, Cornelis W. Oosterlee, and Richard P. Dwight. A Multigrid Multilevel Monte Carlo Method using High-Order Finite-Volume Scheme for Lognormal Diffusion Problems. *International Journal for Uncertainty Quantification*, 7(1), 2017.

Combining Multiple Rank-1 Lattice Rules for Approximation

Lutz Kämmerer
Faculty of Mathematics, Technische Universität Chemnitz, Germany, kaemmerer@mathematik.tu-chemnitz.de

Toni Volkmer
Faculty of Mathematics, Technische Universität Chemnitz, Germany,
toni.volkmer@mathematik.tu-chemnitz.de

The approximation of functions using sampling values along single rank-1 lattices leads to convergence rates of the approximation errors that are far away from optimal ones in spaces of dominating mixed smoothness. A recently published idea that uses sampling values along multiple rank-1 lattices in order to reconstruct multivariate trigonometric polynomials accompanied by efficient methods for the construction of these sampling schemes as well as available fast Fourier transform algorithms motivates considerations on the approximation properties of the emerging sampling operators applied on functions of generalized mixed smoothness.

For instance, when measuring the sampling error in the L_2-norm, we show sampling error estimates where the exponent of the main part reaches those of the optimal sampling rate except for an offset of $1/2 + \varepsilon$, i.e., the exponent is almost a factor of two better up to the mentioned offset compared to single rank-1 lattice sampling.

Experimental Comparison of Higher-Order Digital Nets for QMC

Makoto Matsumoto
Graduate School of Sci., Hiroshima Univ., Japan, mmat@hiroshima-u.ac.jp

Mutsuo Saito
Graduate School of Sci., Hiroshima Univ., Japan, sai10@hiroshima-u.ac.jp

There are number of quadrature rules for quasi-Monte Carlo integration (QMC). We deal with digital nets. Let us consider a sample point set $P \subset [0,s]^s$ of finite size N. Classical Koksma-Hlawka inequality gives a sequence of such P with error bounds of order $O(N^{-1}(\log N)^{s-1})$, for functions of finite total variation. For smoother function classes, this order can be improved to $O(N^{-\alpha}(\log N)^{s\alpha})$ (α depending on the smoothness of the integrand), by interlacing methods by Dick. Algorithm to find such point sets are studied by Dick, Kuo, Nuyens, and some implementations are available from Nuyens' homepage.

As a variant of Dick's investigation, Walsh figure of merit (WAFOM) of a digital net is introduced by Matsumoto-Saito-Matoba. Its parametered version is studied by Suzuki, Yoshiki, Ohori, and the error bound of order $O(N^{-C\log(N)/s})$ is proved for very smooth integrands. We discuss how to fix the parameter according to the dimension s. We searched for digital nets with low-WAFOM values starting from Sobol sequence by linear scrambling (Harase's idea, keeping the t-value small).

We are interested in whether such higher orders of conversion can be observed in experiments or not. We searched for low-WAFOM point sets for dimensions 2 to 30, with sizes 2^{10} to 2^{18}. We compare their performance

with higher-order digital nets called interlaced Sobol point sets (implemented by Nuyens), together with classical Sobol point sets (implemented by Joe-Kuo) and Niederreiter-Xing point sets (implemented by Pirsic).

We conducted numerical experiments for five kinds of Genz test-functions. The low-WAFOM point sets are advantageous for very smooth functions (cosine function and Gaussian function) over the other higher-order digital nets (and classical Sobol and Niederreiter-Xing sets) when the dimension s is not large (say $s \leq 10$), with convergence speed over $O(N^{-1})$ (for $s \leq 5$, over $O(N^{-2})$). Even for $s = 30$, low-WAFOM point sets perform better than the other point sets for these smooth functions. On the other hand, for some non-differentiable functions and non-continuous functions, no advantage is observed.

This is a joint work with Shinsuke Mori.

Quadrature Points via Heat Kernel Repulsion

Jianfeng Lu
Department of Mathematics, Department of Physics, and Department of Chemistry, Duke University, Box 90320, Durham NC 27708, USA, jianfeng@math.duke.edu

Stefan Steinerberger
Department of Mathematics, Yale University, New Haven, CT 06510, USA, stefan.steinerberger@yale.edu

Matthias Sachs
Department of Mathematics, Duke University, Box 90320, Durham NC 27708, USA, and The Statistical and Applied Mathematical Sciences Institute (SAMSI), msachs@math.duke.edu

We discuss the classical problem of how to pick N weighted points on a $d-$dimensional manifold so as to be able to obtain a reasonable quadrature rule

$$\frac{1}{|M|}\int_M f(x)dx \simeq \frac{1}{N}\sum_{n=1}^{N} a_i f(x_i).$$

This problem, naturally, has a massive history; the point of our paper is to point out that selecting points and weights in such a way that the energy functional

$$\sum_{\substack{i,j=1 \\ i\neq j}}^{N} a_i a_j \exp\left(-\frac{d(x_i,x_j)^2}{4t}\right) \rightarrow \min$$

is guaranteed, via spectral theoretic properties of the Laplacian $-\Delta_g$, to lead to point sets that are theoretically guaranteed to have good properties. One nice aspect is that this behavior is universal on all manifolds; we show efficiency on several numerical examples.

[1] S. Steinerberger, Exponential Sums and Riesz energies, Journal of Number Theory 182, p. 37–56 (2018).

[2] S. Steinerberger, Spectral Limitations of Quadrature Rules and Generalized Spherical Designs, arXiv:1708.08736

Friday 10:00–12:00, **Room: Salle 7**

MCMC : Model selection and convergence

Chair: Krys Latuszynski

Average (E)BIC-like Criteria for Bayesian Model Selection

Jingnan Xue
Department of Statistics, Texas A&M University, USA, jnxue@stat.tamu.edu

Ye Luo
Department of Economics, University of Florida, USA, kurtluo@gmail.com

Faming Liang
Department of Statistics, Purdue University, USA, fmliang@purdue.edu

Markov chain Monte Carlo (MCMC) has been an indispensable tool for Bayesian analysis of complex statistical models even for high-dimensional problems. However, there still lacks a consistent criterion for selecting models based on the outputs of MCMC. The existing deviance information criterion (DIC) is known to be inconsistent and non-invariant for reparameterization. We propose an *Average BIC-like* (ABIC) model selection criterion and an *Average EBIC-like* (AEBIC) model selection criterion for low and high-dimensional problems, respectively; establish their consistency under mild conditions; and illustrate their applications using generalized linear models. The proposed criteria overcome shortcomings of DIC. The numerical results indicate that the proposed criteria significantly outperform DIC as well as the maximum likelihood estimate-based model selection criteria, such as AIC, BIC and EBIC, in terms of model selection accuracy.

Spatial Segmentation via the Generalized Gibbs Sampler

Nishanthi Raveendran
Department of Statistics, Macquarie University, Australia, nishanthi.raveendran@mq.edu.au

Georgy Sofronov
Department of Statistics, Macquarie University, Australia, georgy.sofronov@mq.edu.au

Jonathan Keith
School of Mathematical Sciences, Monash University, Australia, jonathan.keith@monash.edu

Identifying spatial domains can be considered one of the important issues of statistical data analysis. Problems of this type arise in a wide range of areas, including disease surveillance, spatial epidemiology, population genetics, criminology and ecological applications. In this paper, we develop a new Markov chain Monte Carlo (MCMC) algorithm within the Generalized Gibbs Sampler (GGS) framework [1], [2], which is particularly useful for sampling from distributions defined on spaces in which the dimension may vary from point to point. The GGS is a general approach, which includes a large variety of the well-known MCMC algorithms such as the Gibbs Sampler, the Metropolis-Hastings algorithm, the Reversible-jump Sampler and the Slice Sampler. We provide examples with artificially generated and real spatial data sets to illustrate the usefulness of the proposed algorithm. In particular, we apply the spatial segmentation algorithm to identify domains in presence-absence plant data.

[1] J. M. Keith, D. P. Kroese, D. Bryant. A Generalized Markov Sampler. *Methodology and Computing in Applied Probability*, 6(1): 29–53, 2004.

[2] J. Keith, G. Sofronov, D. Kroese. The Generalized Gibbs Sampler and the Neighborhood Sampler. In: A. Keller, S. Heinrich, H. Niederreiter (eds) *Monte Carlo and Quasi-Monte Carlo Methods 2006*. Springer, Berlin, Heidelberg, 2008.

MCMC for Bayesian penalized regression

Dootika Vats
Department of Statistics, University of Warwick, UK, D.Vats@warwick.ac.uk

Inference in Bayesian penalized regression is often made using Markov chain Monte Carlo (MCMC) methods. For MCMC to perform well, its rate of convergence to the target distribution must be (relatively) fast. In particular, geometric rates are of specific interest since they allow for output analysis of the MCMC samples. We discuss some results on the rate of MCMC samplers for different Bayesian penalized regression and variable selection models. We focus on the need for relating the rates of convergence of MCMC samplers to controllable parameters of the model so as to allow practitioners to realistically improve speed of samplers.

On the convergence time of some non-reversible Markov chain Monte-Carlo methods

Marie Vialaret
ENSAE ParisTech, Université Paris-Saclay, France, marie.vialaret@ensae-paristech.fr
Florian Maire
The Insight Centre for Data Analytics, University College Dublin, Ireland, florian.maire@ucd.ie

Markov chain Monte-Carlo (MCMC) methods are widely used in numerous fields of applied mathematics as relevant methods for parameter estimation when analytical computations are not feasible, among several uses. They consist in simulating a stochastic process according to a Markov dynamics that is invariant with respect to the distribution of interest. Whereas the canonical MCMC algorithms (*e.g.* Metropolis-Hastings [5, 3] or the standard single site update Gibbs sampler [2]) aim at simulating reversible dynamics with respect to the target distribution, recent papers have focused on the design of new MCMC algorithms whose dynamic is non-reversible. The main motivation is that non-reversible Markov chains potentially yield more accurate Monte Carlo estimates than their reversible counterparts.

In our work, we investigate the performance of non-reversible algorithms using a different measure of efficiency, namely the mixing time of the Markov chain, *i.e.* the time required by the chain to converge to its stationary distribution. In particular, we explore the potential conflict between the mixing time and the asymptotic efficiency of some non-reversible MCMC algorithms that have been recently proposed in the literature, such as [7, 1, 4] and [6]. We show that, in some cases, these non-reversible algorithms will indeed yield Monte Carlo estimates with very low asymptotic variance but at a price of a very slow convergence rate. This point, which, to the best of our knowledge, is missing in the existent literature has obvious practical implications. We propose an non-reversible algorithm that aims at solving, in some capacity, this conflict, by introducing skew-symmetric perturbations in the acceptance ratio of the Metropolis-Hastings algorithm and enlarging the state-space at the same time. We give a proof of validity of our sampler and illustrate its efficient behaviour by taking into account simultaneously the convergence time and the asymptotic variance on several discrete examples.

References

[1] J. Bierkens. Non-reversible Metropolis-Hastings. *Statistics and Computing*, 26 (6): 1213–1228, 2016.

[2] S. Geman, D. Geman. Stochastic Relaxation, Gibbs Distributions, and the Bayesian Restoration of Images. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 6 (6): 721–741, 1984.

[3] W.K. Hastings. Monte Carlo Sampling Methods Using Markov Chains and their Applications. *Biometrika*, 57 (1): 97–109, 1970.

[4] Y.A. Ma, T. Chen, L. Wu, E.B. Fox. A Unifying Framework for Devising Efficient and Irreversible MCMC Samplers. *arXiv preprint arXiv:1608.05973*, 2016.

[5] N. Metropolis, A.W. Rosenbluth, M.N. Rosenbluth, A.H. Teller, E. Teller. Equation of state calculations by fast computing machines. Journal of Chemical Physics, 21 (6), 1953.

[6] R. Poncet. Generalized and hybrid Metropolis-Hastings overdamped Langevin algorithms. *arXiv preprint arXiv:1701.05833*, 2017.

[7] Y. Sakai, K. Hukushima. Eigenvalue analysis of an irreversible random walk with skew detailed balance conditions. *arXiv preprint arXiv:1511.08100*, 2015.

Friday 13:45–15:15, **Room: Salle 6**

Monte Carlo in physics (2)

Chair: Gerardo Rubino

Monte-Carlo methods for reconstructing the aerosol scattering matrix

Anna Korda

Laboratory of Monte Carlo methods, Institute of Computational Mathematics and Mathematical Geophysics SB RAS, asc@osmf.sscc.ru

Sergey Ukhinov
Laboratory of Monte Carlo methods, Institute of Computational Mathematics and Mathematical Geophysics SB RAS, Mechanics and Mathematics Departmen, Novosibirsk State University, Russia, sau@sscc.ru

In this talk the problem of determination the elements of the atmospheric matrix from groundbased solar almucantar observations of polirized radiation is considered. A number of iterative algorithms were constructed previously. In these algorithms, mathematical modeling is used to successively refine the phase function values by using available information about the angular distribution of brightness on the underlying surface under the assumption that the contribution to brightness of the single-scattered radiation is rather large ([1],[2],[3]).

The objective of this study was construct modifications of these algorithms which will allow to find more precise estimation of the aerosol matrix elements and to numerically substantiate the convergence of these methods. For this purpose, an algorithm of Jacobi matrices calculation for the iteration operators of the methods was developed, and calculations were carried out for various parameters of the atmosphere. Also a study of the influence of measurement errors on the reconstruction of the scattering matrix was carried out.

The reported study was partially funded by RFBR according to the research projects 17-01-00823, 18-01-00356, 18-31-00213.

[1] Mikhailov G.A., Ukhiniov S.A., Chimaeva A.S. Monte Carlo Algorithms for Reconstruction of the Scattering Indicatrix Adjusted for Polarization *Russian Journal of Numerical Analysis and Mathematical Modelling*, Vol. 24, N. 5, pp. 455-465, 2009.

[2] Antyufeev V.S., Ivanov A.I., Livshits G.Sh. and Mikhailov G.A. Determination of aerosol scattering indicatrices under clear sky conditions in the 0.55-2.4 μm spectral region *Izv. Akad. Nauk SSSR, Ser. Fiz. Atmos. Okeana*, 16, No. 2, 146-155, 1980..

[3] Antyufeev V.S., Nazaraliev M.A. Inverse problems in Atmospheric Optics *Comp. Center, Sib. Branch. Rus.Akad. Sci.,* Novosibirsk, 1978.

Some Applications of Dynamics Simulation for Multi-Particle Systems in the Kinetic Model Framework

Mariya Korotchenko
Institute of Computational Mathematics and Mathematical Geophysics SB RAS, Novosibirsk, Russia, kmaria@osmf.sscc.ru

Aleksandr Burmistrov
ICMMG SB RAS; Novosibirsk State University, Novosibirsk, Russia, burm@osmf.sscc.ru

In this talk, we summarize our approaches earlier developed in the field of multi-particle systems simulation for various problems such as: rarefied gas relaxation (see, for example, [1]), particle coagulation [1, 2, 4], vehicular traffic flow [3] and price formation [5]. All these problems are described by the Boltzmann type equations. To solve them numerically, we propose to use the integral equation of the second kind and the weighted simulation using the Markov chain, which is uniquely determined by the coefficients of this integral equation. It allows to extend the well-developed theory of weighted Monte Carlo methods to the problems under consideration. Moreover, this makes it possible to estimate the parametric derivatives of the solution.

For some problems we construct "value" modifications of the weighted statistical simulation to solve numerically the kinetic equation (for details see [1, 2, 4]), which leads to a considerable reduction of computational costs.

This work is partly supported by the Russian Foundation for Basic Research (grants 16-01-00530, 17-01-00698, 18-01-00356).

[1] Korotchenko M.A., Mikhailov G.A., Rogazinskii S.V. Value Modifications of Weighted Statistical Modelling for Solving Nonlinear Kinetic Equations. *Russ. J. Numer. Anal. Math. Model.*, 5 (22): 471–486, 2007.

[2] Korotchenko M. Value Monte Carlo Algorithms for Estimating the Solution to the Coagulation Equation. *Springer Proc. Math. Stat.*, 23: 511–522, 2012.

[3] Burmistrov A., Korotchenko M. Monte Carlo Algorithm for Simulation of the Vehicular Traffic Flow Within the Kinetic Model with Velocity Dependent Thresholds. *Springer Proc. Math. Stat.*, 114: 109–117, 2014.

[4] Burmistrov A.V., Korotchenko M.A. Weight Monte Carlo Algorithms for Estimation and Parametric Analysis of the Solution to the Kinetic Coagulation Equation. *Numer. Anal. Appl.*, 2 (7): 104–116, 2014.

[5] Burmistrov A.V., Novikov A.V. Stochastic Kinetic Model of Price Formation. *MNIZh (International Research Journal)*, 7-3 (61): 107–112, 2017. (in Russian)
doi: 10.23670/IRJ.2017.61.030

Friday 13:45–15:15, **Room: Salle 7**

SDE, solutions and convergence rate

Chair: Przemyslaw Zielinski

Mean Reflected Stochastic Differential Equations with jumps : Simulation by using Particle Systems

Philippe Briand
Univ. Grenoble Alpes, Univ. Savoie Mont Blanc, CNRS, LAMA, 73000 Chambéry, France,
philippe.briand@univ-smb.fr

Abir Ghannoum
Univ. Grenoble Alpes, Univ. Savoie Mont Blanc, CNRS, LAMA, 73000 Chambéry, France
&
Univ. Libanaise, LaMA-Liban, P.O. Box 37, Tripoli, Liban, abir.ghannoum@univ-smb.fr

Céline Labart
Univ. Grenoble Alpes, Univ. Savoie Mont Blanc, CNRS, LAMA, 73000 Chambéry, France,
celine.labart@univ-smb.fr

This paper is devoted to the study of reflected Stochastic Differential Equations with jumps when the constraint is not on the paths of the solution but acts on the law of the solution. This type of reflected equations have been introduced recently by Briand, Elie and Hu [**?**] in the context of BSDEs, when no jumps occur. In [**?**], the authors study a numerical scheme based on particle systems to approximate these reflected SDEs. In this paper, we prove existence and uniqueness of solutions to this kind of reflected SDEs with jumps and we generalize the results obtained in [**?**] to this context.

High precision solvers for autonomous systems of differential equations based on Markov jump processes

Flavius Guiaş
Dortmund University of Applied Sciences and Arts, Germany, flavius.guias@fh-dortmund.de

In this presentation we will show how solutions of large autonomous systems of ordinary differential equations can be efficiently approximated by using a suitable stochastic method. The scheme delivers the best results for systems with a sparse incidence matrix, for example in the case of spatially discretized partial differential equations. Its basic principle is the fact that the dynamics of Markov jump processes can be used as an approximation for solutions of systems of differential equations. Nevertheless, except for some particular cases, this approach is not really an efficient numerical scheme for ODE. We will present several improvements which eliminate the mentioned drawback. The step function $\tilde{X}$ computed by the simulation of the jump processes can serve as a predictor which is further improved by suitable correction steps. Given the improved approximation $X^*(t)$ at time t, we compute the corresponding approximation at time $t+h$ by an integral scheme of the form $X^*(t+h) = X^*(t) + \int_t^{t+h} Q(s)\,ds$.

One possibility is to take $Q(s) = F(\tilde{X}(s))$, i.e. to perform a Picard iteration based on the predictor $\tilde{X}(\cdot)$ in order to obtain a better approximation denoted in particular by $\bar{X}(\cdot)$. The integral can be computed exactly during the simulation of the jump process, since the integrand is a step function. A range of further possibilities for computing improved approximations $X^*(\cdot)$ opens if we take for the integrand Q a polynomial which interpolates some intermediate values of $F(\tilde{X}(\cdot))$ or $F(\bar{X}(\cdot))$. The jump process $\tilde{X}$ or the Picard–approximation $\bar{X}$ are evaluated here at some few equidistant points between t and $t + h$. By using an exact quadrature formula in order to compute the integral above, we can employ the principle of the Runge–Kutta method in order to further improve our approximation. The length h of the time interval can be taken either fixed, or adapted (controlled by a given number of jumps of the process). Numerical tests show that this improved stochastic method can be comparable or even more efficient than the standard deterministic ODE-solver *ode45* (Dormand–Prince Runge–Kutta method) implemented under the same conditions. This presentation is a further development of the results in [1].

[1] F. Guiaş, P. Eremeev. Improving the stochastic direct simulation method with applications to evolution partial differential equations. *Appl. Math. Comput.*, 289 : 353–370, 2016.

Approximation rate of BSDEs using random walk

Christel Geiss
Department of Mathematics and Statistics, University of Jyväskylä, Finland, christel.geiss@jyu.fi

Céline Labart
LAMA, Université Savoie Mont-Blanc, France, celine.labart@univ-smb.fr

Antti Luoto
Department of Mathematics and Statistics, University of Jyväskylä, Finland, antti.luoto@jyu.fi

For the FBSDE

$$X_t = x + \int_0^t b(r, X_r)dr + \int_0^t \sigma(r, X_r)dB_r,$$
$$Y_t = g(X_T) + \int_t^T f(s, X_s, Y_s, Z_s)ds - \int_t^T Z_s dB_s,\ 0 \le t \le T$$

Briand, Delyon and Mémin have shown in [1] a Donsker-type theorem. If one approximates the Brownian motion B by a random walk B^n, the according solutions (X^n, Y^n, Z^n) converge weakly to (X, Y, Z). We investigate under wich conditions (Y_t^n, Z_t^n) converges to (Y_t, Z_t) in L_2 and compute the rate of convergence in dependence of the Höder continuity of the terminal condition function g.

[1] P. Briand, B. Delyon, J. Mémin. Donsker-type theorem for BSDEs *Elec. Comm. Proba.*, 6, 1–14, 2001.

Friday 13:45–15:15, **Room: Salle 8**

Applications of MC

Chair: Lingbin Bian

Network structure change point detection by posterior predictive discrepancy

Lingbin Bian
School of Mathematical Sciences, Monash University, Australia, lingbin.bian@monash.edu

Jonathan Keith
School of Mathematical Sciences, Monash University, Australia, jonathan.keith@monash.edu

Tiangang Cui
School of Mathematical Sciences, Monash University, Australia, tiangang.cui@monash.edu

Georgy Sofronov
Department of statistics, MACQUARIE University, Australia, georgy.sofronov@mq.edu.au

Detecting changes in network connectivity is important for research into systems as diverse as financial trading networks and brain interactions. In this work we developed novel Bayesian inference methods for detecting network structure change-points, with particular reference to fMRI data indicative of brain functional connectivity. We used the stochastic block model [1] to quantify the likelihood of a network structure and methods based on the posterior predictive assessment[2] to detect multiple network structure change-points. The parameter space for this model is high-dimensional and involves unknown latent labels and unknown number of communities of interacting regions. Monte Carlo techniques based on Gibbs sampling and reversible jump strategy provided an efficient way for sampling a posterior distribution of the stochastic block models over this parameter space.

[1] A. F. McDaid, T. B. Murphy, N. Friel and N. J. Hurley Improved Bayesian inference for the stochastic block model with application to large networks. *Computational statistics and data analysis*, 60:12 to 31, 2012.

[2] A. Gelman, X. L. Meng and H. Stern Posterior predictive assessment of model fitness via realized discrepancies. *Statistica Sinica, 6:733 to 807, 1996.*

A Perturbative Approach to Control Variates in Molecular Dynamics

Julien Roussel
CERMICS, École des Ponts & Inria Paris, France, julien.roussel@enpc.fr

Gabriel Stoltz
CERMICS, École des Ponts & Inria Paris, France, gabriel.stoltz@enpc.fr

We propose in [1] a general variance reduction strategy for diffusion processes. Our approach does not require the knowledge of the measure that is sampled, which may indeed be unknown as for nonequilibrium dynamics in statistical physics. We show by a perturbative argument that a control variate computed for a simplified version of the model can provide an efficient control variate for the actual problem at hand. We illustrate our method with numerical experiments and show how the control variate is built in practice.

[1] J. Roussel and G. Stoltz. A perturbative approach to control variates. *arXiv:1712.08022*, 2017.

Laplace Surface Green's Function on a Spherical Surface for Last-passage Monte Carlo Methods

Chi-Ok Hwang
Division of Liberal Arts and Sciences, Gwangju Institute of Science and Technology, South Korea,
chwang@gist.ac.kr

Unjong Yu
School of Physics and Chemistry,Gwangju Institute of Science and Technology, South Korea,

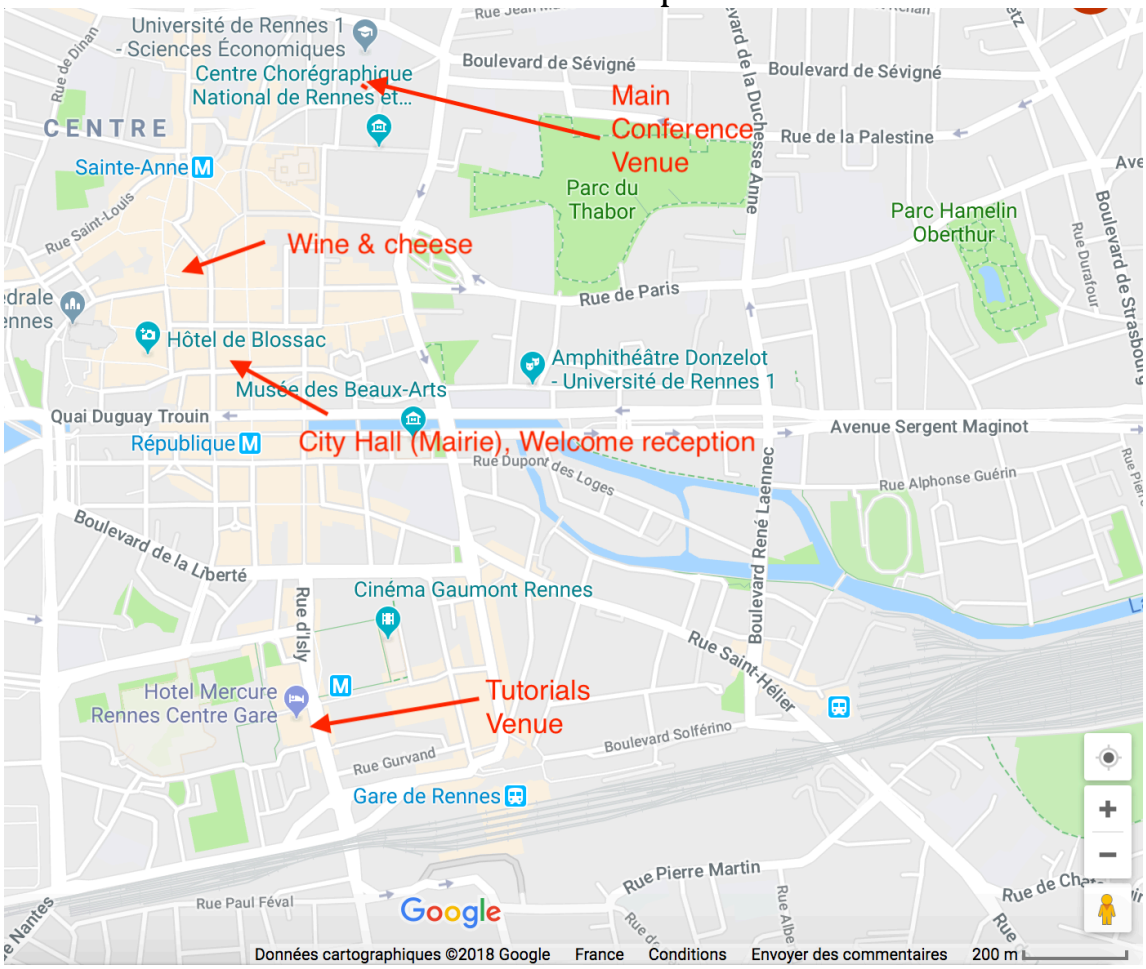
Youngmin Lee
Physics Concentration, GIST College, Gwangju Institute of Science and Technology, South Korea,

Last-passage Monte Carlo algorithms are good for obtaining charge density at a specific point on a conducting surface. In our previous research, we obtained Laplace Surface Green's function on a flat surface for Last-passage Monte Carlo methods. In this paper, we obtain Laplace Surface Green's function on a spherical surface for Last-passage Monte Carlo methods. We demonstrate this last-passage Green's function by obtaining charge density on a sphere held at a unit potential. In addition, using this last-passage Green's function we compute mutual capacitance and charge distribution of two conducting spheres. We compared them with analytic results of J. Lekner to find an excellent agreement.

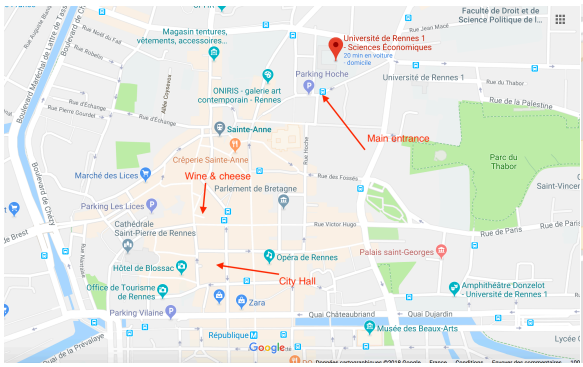
List of session chairs

Index

General Map



Conference Venue



Tutorials Venue